## Engineering 1620 - Spring 2020

## Homework Set 4

1. We spent the first couple of days of the new mode of living to introduce transconductance and look at device capacitances. I gave you a handout that included how you derive these from data sheets for BJT devices. Discrete BJT transistors can sometimes give very good noise performance and so still find a market for special applications. Here is the URL for an Infineon RF BJT with a 40 GHz value of $\mathrm{F}_{\mathrm{T}}$ instead of the 250 MHz of your kit type 2N2222A: https://www.infineon.com/dgdl/Infineon-BFP640-DS-v03 00EN.pdf?ffileld=5546d462689a790c01690f03a9ca3928

Assume that the transistor is biased with $\mathrm{V}_{\mathrm{CE}}=2.5$ volts and $\mathrm{I}_{\mathrm{CQ}}=20 \mathrm{~mA}$. Draw a small signal, hybrid-pi model for this device and give numeric values for the transconductance, $C_{p i}, C_{C B}$, and $r_{p i}$. All you should need from the data sheet is the collector-base capacitance and the graph of $\mathrm{F}_{\mathrm{T}}$ as a function of operating conditions.
2. Here is a non-linear opamp circuit related to the diode problems you have seen and which may help remind you of the characteristics of sinusoids. Use ideal models for the opamps.
2.1) Sketch the waveform at point ' $A$ ' for a sinusoid input of 620 Hz with $V_{P}=2$ volts.
2.2) What is the purpose of $D 2$ ?
2.3) If there were no filter capacitor, i.e., $C_{F}=0$, what would the output look like? Sketch the waveshape and show its peak value.
2.5) What would the average output voltage (DC) be with the 0.16 ufd capacitor in the circuit? Make a crude estimate of the peak to peak voltage of the time-varying part.

3.) We are spending much of April on small signal analysis and the basic properties of both BJT and MOSFET standard circuits for use in analog integrated circuits.
3.1) What is this circuit? What is the purpose of $M 3$ and $M 1$ ? What do $M 2$ and $M 4$ do?

3.2) Derive the output impedance of this circuit, that is, the impedance at the drain of M2 relative to VSS in terms of whatever device small signal parameters of M 1 and M 2 you need.
3.3) What is the input impedance at low frequency in terms of the same small signal parameters?
3.4) Suppose $I_{\text {BIAS }}=20$ uamp. The circuit has all identical transistors and the overvoltage on M 4 is 0.33 volts. The manufacturing process leads to $\lambda=0.025$ for these devices. Should you need it, use $V_{T H}=0.5$ volts. What is the numerical value of the low-frequency output impedance?
3.5) There is probably some capacitance from the output node to ground from the drain to substrate area (call that $C_{D S S}$ ) but there is also some from the gate to drain capacitance, $C_{G D}$, of $M 2$. One end of that capacitor connects to the input node of the circuit. What is the effective output capacitance of the circuit in terms of these to capacitances? Hint: the current through the gate-drain capacitance of M2 goes into the input to the mirror and the mirror copies that into an additional current in the output.
4.) We also spent some time looking semi-quantitatively at opamps in use. Here is a current integrator circuit. When the digitally-operated switch is closed, then the capacitor is discharged and the circuit is insensitive to the incoming current signal. When it is open, the output voltage follows the integrated input current. Resettable integrators are used in optical systems, smoke detectors, radiation counters and quite a number of other places. This problem looks at the performance of the circuit while the switch is open.

The operational amplifier has a gain-bandwidth product of 24 MHz . The capacitor Copamp in the figure represents the input impedance of the amplifier which is simply a capacitance of 10 pf . The DC gain is 92 $\mathrm{DB}\left(4 \cdot 10^{4} \mathrm{X}\right)$. Assume that $\mathrm{C}_{\mathrm{INT}}$ is 20 pf and that it does not change the amplifier gain appreciably.

4.1) Neglecting the 10 pf . of the amplifier input, find an expression for the impedance seen by the current source, that is, the input impedance of the integrator circuit? This is the Miller effect in its simplest form. Include the actual frequency dependent gain of the amplifier. (The answer may not be exactly what you expect.)
4.2) At 10 KHz , what is the numeric value? What is the phase angle of the impedance? What single passive component has impedance closest to this value?
4.3) What is the impedance of the amplifier input capacitance at 10 KHz ? Is it significant compared to the Miller impedance?
4.4) Repeat for 1 MHz . The impedance probably does not change very much in this range of frequency why?
5.) Here is a Wilson current mirror with identical transistors with current gain, $\beta$. Neglect the Early effect and find the difference between the input and output currents expressed as a fraction of the input current, $i_{I N}$. Hint: The circuit has only two Kirchhoff current nodes and the collector currents of Q1 and Q2 are equal because the transistors are identical.

6.) (I am not sure whether I will reach active filters this year but this is a typical such circuit. I think you should be able to work it out using the perfect opamp model. I won't grade this one if I don't get there.) The Sallen and Key positive gain low pass filter is a popular way to make continuous time active filters. It is an economical way to make a low-Q pole pair. One can show that the dependence of the filter's transfer function on component tolerance is reduced if the gain of the filter is about 1.4 X . Here is the basic circuit:

6.1) Derive the transfer function of this circuit. You may leave the opamp closed-loop gain as a single constant but also write that constant in terms of the component values.
6.2) Suppose that you really do want a a 1 rad/second, second-order, low-pass filter with gain at DC of 1.4 and the transfer function:

$$
H(s)=\frac{1.4}{s^{2}+\sqrt{2} s+1}
$$

(This is called a Butterworth filter and has some nice properties.) Select component values to do this. Make R = 1 megohm to make capacitors reasonable values. Choose R1 and R2 on any reasonable basis.
6.3) You change your mind and want a 3.1 KHz filter of the same type. What are the capacitor values in this case if at the same time you lower R to 100 K . Hint: you don't recalculate the transfer function. Instead you scale all capacitors down by the ratio of $1 \mathrm{rad} / \mathrm{s}$ to 3.1 KHz . Then if you change R by a factor of 10 down, you scale the capacitors up by the same factor.

