ENGN1620 – Spring 2016 Problem Set # 1 – Answers

1.1) 50K || 10 K = 8.33K @ 1 mA = 8.33 volts. $I_O = -0.833$ mA (Note: sign is from directions of current flow.)

1.2) 1K || 5K || 10 K = 769 ohms. @ 1 mA = 0.769 volts and $I_O = 0.0769$ mA.

1.3) The 100 ohm resistor does nothing and V_O = 5/11 = 0.455 V.

1.4) Convert Norton to Thevenin as 1 volt source with 100 K source resistance. The V_O = 1/12 = 0.0833 volts.

2.1) Remove the 100 K resistor. 50 K + (5K||100K) = 54.8K versus 55K a change of .2K in 55K or 0.363 %.

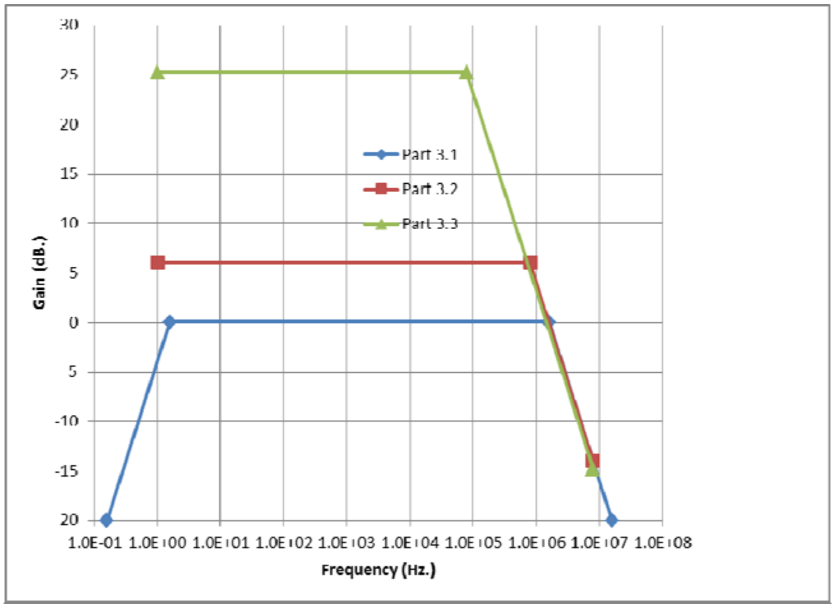
2.2) This is the hardest one as there are two candidates for the least important element: the 3uA source or the 10K resistor. The 3 uA source changes V_O by 0.3% if removed. 10K || 75 = 74.4 ohms a change of 0.6 ohms in 75 or 0.75 %. Take out the 3 uA source.

2.3) Remove the 100 K resistor. Combine both sources, the 10K and 1K resistors into a single Thevenin source with source resistance 10K || 1K = 909 ohms. The fractional change in voltage by adding or removing the 100 K resistor is 909/(100K+909) which is 0.90%.

3.1) At low frequency the source impedance is dominated by the 10 uF capacitor and with the 10 K resistor forms a high-pass filter with cutoff $f_C = \frac{1}{2\pi 1\cdot 10^{-5} 1\cdot 10^{+4}}$ or 1.59 Hz. At high frequency, the 100 ohm resistor forms a low pass filter with the 100 nF capacitor with cutoff of 1.59 MHz. Above and below these frequencies the transfer function changes by 20 dB per decade. At midband the loss is from the resistors acting as a voltage divider and is slightly under 1 % so we simply **say midband gain is 0 dB.**

3.2) The effect of the 100 K in parallel to the 2K is slightly under 2 % so we drop the 100 K resistor. The input voltage drives the controlled current source directly so that source with the 2K resistor is a Thevenin source of voltage 2*v_{in} and a 2K source resistance. That resistance together with the 100 pF capacitor **forms a low pass filter with cutoff of 795 KHz.** Above cutoff the **slope is -20 dB per decade**. Midband gain is 2X **or 6 dB.**

3.3) This is a little harder. By a KVL loop, $v_{in} = v_g + 1\cdot 10^3 \cdot 1\cdot 10^{-2} v_g = 11 v_g$ Then the current source is $\frac{0.01\cdot v_{in}}{11} = 0.909\cdot v_{in}$ mA. Through 20 K this is a low frequency gain of 18.18 **or 25.2 dB.** This is a low pass filter with cutoff from the 20 K resistor and 100 pF capacitor at a cutoff frequency of 79.5 KHz. Above cutoff the **slope is -20 dB per decade**.

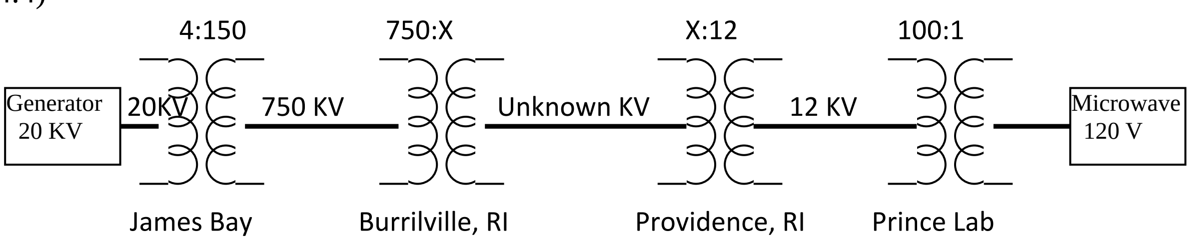


4.1) To make up for the 6% loss, 638 watts has to leave the hydroelectric plant on a 750 KV line or a current of 0.851 mA. Over 3000 Km (2 wires!) the resistive loss is 12.76 watts and the resistance must be no more than 17.4 megohms. Solving for A_C with the usual resistance formula $A_c = \dfrac{\rho l}{R}$ gives a cross section of $4.83 \cdot 10^{-5}$ cm^2**. The edge of a square of that size is 0.00694 cm (0.0027 inches).**

4.2) The volume of the wire is the cross section times its length or 14,480 cc. At 2.7 grams per cc this is 39.1 Kg or 86.7 pounds and \$ 134. **The capital investment is \$ 538.** The return on investment must be \$ 80.76. Dividing by the number of five minute intervals in a 365 day year gives the **cost of this line to Brown as 0.077 cents**. The microwave oven drew 0.6 KW for 1/12 hour at **a cost of 0.60 cents. The fraction is 12.8 %.**

4.3) Not counting the transformer at Albany, there has to be **at least four transformers** in the chain, one step- between the generator and the 750 KV line (**turns ratio: 4:150**), one in Burrilville of unknown ratio since we do not know the voltage used from there to Providence (**turns ratio: 750:X** where is the unknown line voltage in KV), one at Franklin Square (**turns ratio: X:12**), and one in Prince Lab to go down from 12 KV to 120 volts (**turns ratio: 100:1**).

4.4)

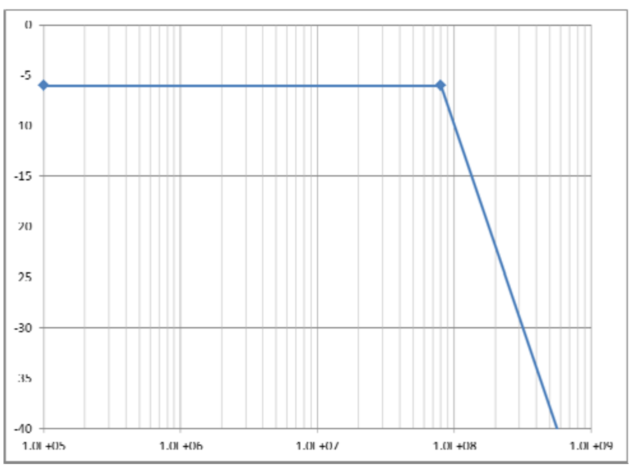


5.) Set the voltage to zero and it follows immediately that the Thevenin equivalent source impedance is the sum of the two capacitances C1 + C2. **The open circuit transfer function is** $H(s) = \frac{sC1}{sC1 + sC2} = \frac{C1}{C1 + C2}$ **and is independent of frequency from DC to daylight**. For most people the idea that a capacitor network can pass DC is very odd but is correct in this case. The slightest resistor load will, of course, convert this into a high pass filter in keeping with the idea that there should not be signal at DC. In practice there are circuits with so little resistive load that their time constants are measure in at least tens of years. The high end is more problematic as the input impedance decreases with increasing frequency and at high enough frequency, the source will no longer be ideal.

6.)

$$H(s) = \frac{R}{1+sRC}\left[R + sL + \frac{R}{1+sRC}\right]^{-1} = \frac{R}{R + sL + sR^2C + s^2LRC + R} = \frac{1}{2}\frac{1}{s^2LC/2 + s(RC + sL/R)/2 + 1}$$

This is a second order, low-pass filter with gain 0.5 (-6 dB) at DC. The phase is 90 degrees when the real part of the denominator $(1 + s^2LC)$ vanishes at $f_C = \frac{1}{2\pi\sqrt{LC/2}}$, at which frequency **the gain (magnitude of H) is** $\sqrt{\frac{LC}{2}} \cdot \frac{1}{RC + L/R}$. For this particular **circuit the cutoff frequency is 80 Mhz and the gain at that frequency is 0.354 or -9.03 dB**, essentially -3 dB from the DC value. The Bode plot has the high frequency asymptote intersecting the -6 dB line at 80 MHz and decreasing 40 dB per decade above that cutoff.



7..1) The linear part of the circuit consists of the two sources, RG, and C. It is **a high-pass filter for v_{in} to v_{GS}.** The cutoff **frequency is** $\frac{1}{2\pi RGC} = 10.6$ **Hz**.

7.2) Since both of the signal frequencies are at least an order of magnitude larger than the high-pass cutoff frequency, **I assume that they pass through the filter unattenuated, then**
$v_{GS} = 0.77 + 0.25\sin(\omega_1 t) + 0.25\sin(\omega_2 t)$

7.3) Use the identities: $\sin^2(\omega t) = 0.5\left(1-\cos(2\omega t)\right)$ and
$\sin(\omega_1 t)\sin(\omega_2 t) = 0.5(\cos((\omega_1 - \omega_2)t) - \cos((\omega_1 + \omega_2)t))$ to reduce the product terms to simple sinewaves. I am interested in the frequency components of the drain current so I will write the arguments of the sine and cosine functions as the frequency of the signal rather than the $\omega_n t$ form. I will also leave the constant K_N as a premultiplier and do the conversion to numerical values of the peak current in part 7.4.

$$i_D = K_N \begin{bmatrix} 0.5929 + 0.385\sin(1KHz) + 0.385\sin(1.5KHz) + 0.03125 + 0.03125\cos(2KHz) + 0.03125 \\ +0.03125\cos(3KHz) + 0.0625\cos(500Hz) - 0.0625\cos(2.5KHz) \end{bmatrix}$$

(Note: the sum of the three DC terms is 0.6554. The number of significant digits is probably more than is reasonable.)

7.4) The peak current for both linear signal **terms is 192 μA.**

7.5) The peak currents at each component of the drain current require multiplying the appropriate term in brackets by the K_N constant. Thus for **components other than the 1 and 1.5 KHz terms, we have: 31.25 μA at 500 Hz and 2.5 KHz; and 15.6 μA at 2 KHz and 3 KHz.** The fundamental at 1 KHz is 192 μA. So the **second harmonic distortion is 8.1 %.** This is over an order of magnitude **worse than what is marginally acceptable** in a consumer audio device**. Not a good amplifier.**