ENGN1620 – Spring 2020 Problem Set # 1 – Answers

1.1) 50K || 10 K = 8.33K @ 1 mA = 8.33 volts. $I_O = -0.833$ mA (Note: sign is from directions of current flow.) Alternately the current through 10 K is 5 times that through 50 K so I_o = 5/6 mA = 0.833 mA or 50K and 1 mA is Thevenin 50 V 50 K and output is 50*(10/60) = 8.33 volts.

1.2) 1K || 5K || 10 K = 769 ohms. @ 1 mA = 0.769 volts and $I_O = 0.0769$ mA. Alternately the current through 1K is 10 times that through 10 K, and through 5 K is twice through 10 K so current through 10 K is 1/13 of input or 0.0769 mA

1.3) The 100 ohm resistor does nothing and V_O = 5/11 = 0.455 V.

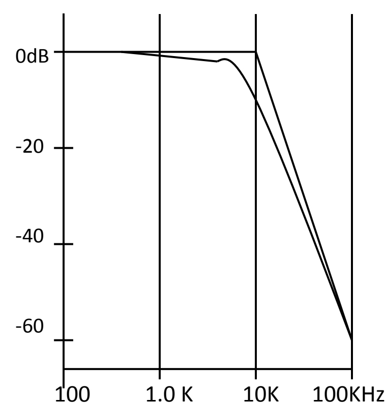
1.4) Convert Norton to Thevenin as 1 volt source with 100 K source resistance. The V_O = 1/12 = 0.0833 volts.

2.1) The sections are identical and the amplifiers assure that there are no loading effects. The magnitude will be the cube of the magnitude of one section so we equate:

$\left|H\left(f_{-3dB}\right)\right| = \dfrac{1}{\left(1+\left(f_{-3dB}/f_C\right)^2\right)^{3/2}} = \dfrac{1}{\sqrt{2}}$ where $f_C = \dfrac{1}{2\pi RC}$ is the -3dB point of one section. The cube root of 2 is 1.26 so $\left(f_{-3dB}/f_C\right)^2 = 0.26$ and $f_{-3dB} = 0.51 f_C$.

2.2) For R = 1K and C = 16 nfd, the value of f_C = 10 KHz and the 3 dB cutoff is 5.1 KHz.

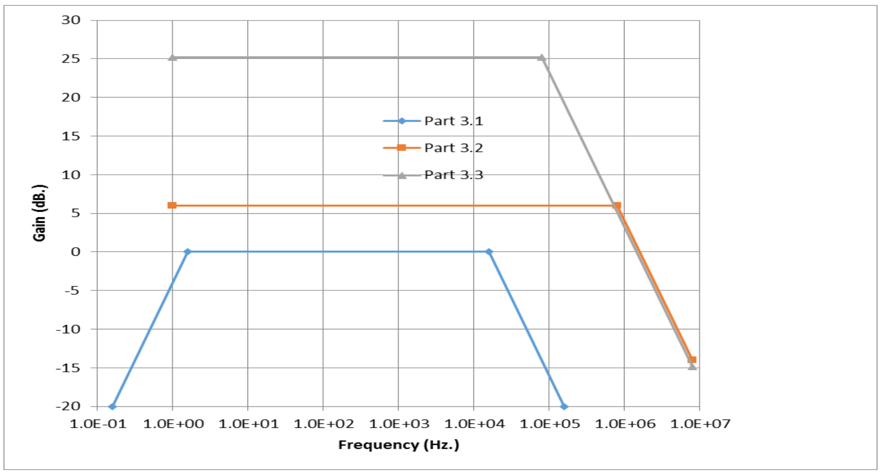
Frequency	Magnitude	Phase
510 Hz	-0dB	-8.3 deg.
5.1 KHz	-3dB	-81deg.
10 KHz	-9dB	-135 deg.
100 KHz	-60dB	-253 deg.



3.1) At low frequency the source impedance is dominated by the 10 uF capacitor and with the 10 K resistor forms a high-pass filter with cutoff $f_C = \dfrac{1}{2\pi 1\cdot 10^{-5} 1\cdot 10^{+4}}$ or 1.59 Hz. At high frequency, the 100 ohm resistor forms a low pass filter with the 100 nF capacitor with cutoff of 15.9 KHz. Above and below these frequencies the transfer function changes by 20 dB per decade. At midband the loss is from the resistors acting as a voltage divider and is slightly under 1 % so we simply **say midband gain is 0 dB.**

3.2) The effect of the 100 K in parallel to the 2K is slightly under 2 % so we drop the 100 K resistor. The input voltage drives the controlled current source directly so that source with the 2K resistor is a Thevenin source of voltage 2*v_{in} and a 2K source resistance. That resistance together with the 100 pF capacitor **forms a low pass filter with cutoff of 796 KHz.** Above cutoff the **slope is -20 dB per decade**. Midband gain is 2X **or 6 dB.**

3.3) This is a little harder. By a KVL loop, $v_{in} = v_g + 1\cdot 10^3 \cdot 1\cdot 10^{-2} v_g = 11 v_g$ Then the current source is $\frac{0.01\cdot v_{in}}{11} = 0.909\cdot v_{in}$ mA. Through 20 K this is a low frequency gain of 18.18 **or 25.2 dB.** This is a low pass filter with cutoff from the 20 K resistor and 100 pF capacitor at a cutoff frequency of 79.5 KHz. Above cutoff the **slope is -20 dB per decade**.

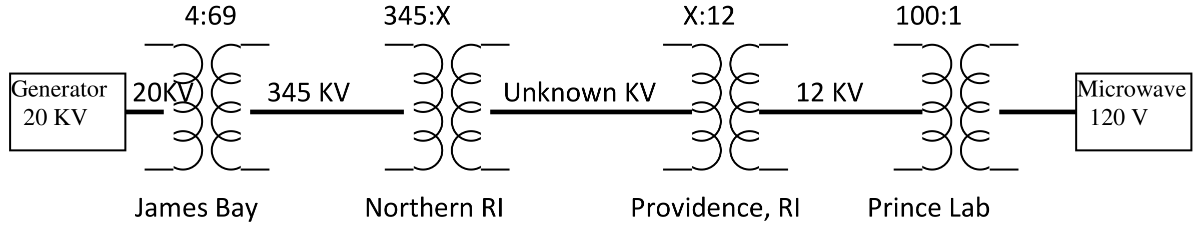


4.1) To make up for the 7% loss, 645 watts has to leave the hydroelectric plant on a 345 KV line or a current of 1.87 mA. Over 3000 Km (2 wires!) the resistive loss is 15 watts and the resistance must be no more than 4.29 megohms. Solving for A_C with the usual resistance formula $A_c = \frac{\rho l}{R}$ gives a cross section of $1.95\cdot 10^{-4}$ cm^2**. The edge of a square of that size is 0.014 cm (0.0055 inches).**

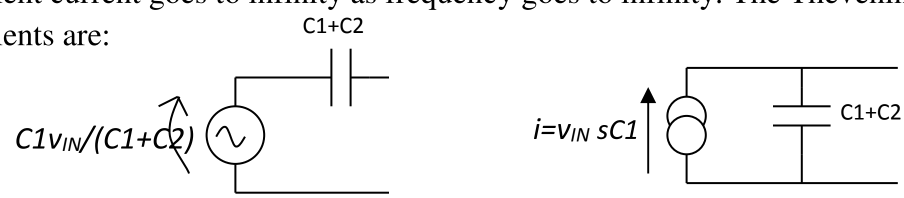
4.2) The volume of the wire is the cross section times its length or 58,800 cc. At 2.7 grams per cc this is 159 Kg or 350 pounds and $ 271. **The capital investment is $ 1356.** The return on investment must be $ 203.44. Dividing by the number of five minute intervals in a 365 day year gives the **cost of this line to Brown as 0.193 cents**. The microwave oven drew 0.6 KW for 1/12 hour at **a cost of 0.60 cents. The fraction is 33 %.**

4.3) Not counting the transformer at Albany, there has to be **at least four transformers** in the chain, one step-up between the generator and the 345 KV line (**turns ratio: 4:69**), one in northern RI of unknown ratio since we do not know the voltage used from there to Providence (**turns ratio: 345:X** where X is the unknown line voltage in KV), one at Franklin Square (**turns ratio: X:12**), and one in Prince Lab to go down from 12 KV to 120 volts (**turns ratio: 100:1**).

4.4)

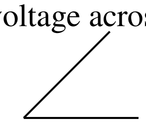


5.) Set the voltage to zero and it follows immediately that the Thevenin equivalent source impedance is the sum of the two capacitances C1 + C2. **The open circuit transfer function is** $H(s) = \frac{sC1}{sC1+sC2} = \frac{C1}{C1+C2}$ **and is independent of frequency from DC to daylight**. For most people the idea that a capacitor network can pass DC is very odd but is correct in this case. The slightest resistor load will, of course, convert this into a high pass filter in keeping with the idea that there should not be signal at DC. In practice, there are circuits with so little resistive load that their time constants are measured in at least tens of years. (The ability to manufacture such a near-perfect capacitor pair is the basis for almost all flash memory.) The upper frequency limit is more problematic as the input impedance decreases with increasing frequency and at high enough frequency, the source will no longer be ideal. This is reflected in the way that the Norton equivalent current goes to infinity as frequency goes to infinity. The Thevenin and Norton equivalents are:



6.1) No current flows in the secondary because the polarity of the secondary voltage is opposite to the direction of the diode. When the switch closes, the dot end of the primary is positive wrt to the non-dotted end. That means the dotted end of the secondary if positive and that would try to force current in the non-conducting direction of the diode.

6.2) The I-V relation for an inductor is $v = L_P \frac{di}{dt}$. When the switch closes the voltage across the inductor is 325 volts and the current rises from 0 linearly at a rate $\frac{di}{dt} = \frac{325}{L_P}$.



6.3) At 1 microsecond, $i = \frac{3.25 \cdot 10^{-4}}{L_P}$ and the stored energy is $U = \frac{1}{2} L_P i^2 = \frac{5.28 \cdot 10^{-8}}{L_P}$.

6.4) The only energy that is being continuously dissipated and that must be made up from the mains power is the 5 V * 1 A = 1 watt (1 joule per second) going into the load resistor. In 2.5 microseconds that is 2.5e-6 joules. Equating this to the result of part 6.3 gives $L_P = 21mH$.

6.5) The secondary current is a triangle with baseline of 1 usec, the same duration as the switch closure. In that time the average power flow in the 2.5 usec period is $2.5 \cdot 1 \cdot 10^{-6} i_{PEAK} = 5 \cdot 2.5 \cdot 10^{-6} i_{LOAD}$. Since the load current is 1 ampere, the secondary peak current must be 5 amperes.

6.6) The turns ratio is just the ratio of the primary to secondary voltages or N_P:N_S = 325:5 = 65:1. To have 2 turns on the secondary would require 130 turns on the primary.

7..1) The linear part of the circuit consists of the two sources, RG, and C. It is **a high-pass filter for v_{in} to v_{GS}.** The cutoff **frequency is** $\frac{1}{2\pi RGC} = 10.6$ **Hz**.

7.2) Since both of the signal frequencies are at least an order of magnitude larger than the high-pass cutoff frequency, **I assume that they pass through the filter unattenuated, then** $v_{GS} = 0.7 + 0.25\sin(\omega_1 t) + 0.25\sin(\omega_2 t)$

7.3) Use the identities: $\sin^2(\omega t) = 0.5(1 - \cos(2\omega t))$ and $\sin(\omega_1 t)\sin(\omega_2 t) = 0.5(\cos((\omega_1 - \omega_2)t) - \cos((\omega_1 + \omega_2)t))$ to reduce the product terms to simple sinewaves. I am interested in the frequency components of the drain current so I will write the arguments of the sine and cosine functions as the frequency of the signal rather than the $\omega_n t$ form. I will also leave the constant K_N as a premultiplier and do the conversion to numerical values of the peak current in part 7.4.

$$i_D = K_N \begin{bmatrix} 0.49 + 0.350\sin(1KHz) + 0.350\sin(1.5KHz) + 0.03125 - 0.03125\cos(2KHz) + 0.03125 \\ -0.03125\cos(3KHz) + 0.0625\cos(500Hz) - 0.0625\cos(2.5KHz) \end{bmatrix}$$

(Note: the sum of the three DC terms is 0.5525. The number of significant digits is probably more than is reasonable.)

7.4) The peak current for both linear signal **terms is 175 µA.**

7.5) The peak currents at each component of the drain current require multiplying the appropriate term in brackets by the K_N constant. Thus for **components other than the 1 and 1.5 KHz terms, we have: 31.25 µA at 500 Hz and 2.5 KHz; and 15.6 µA at 2 KHz and 3 KHz.** The fundamental at 1 KHz is 175 µA. So the **second harmonic distortion is 8.9 %.** This is over an order of magnitude **worse than what is marginally acceptable** in a consumer audio device**. Not a good amplifier.**