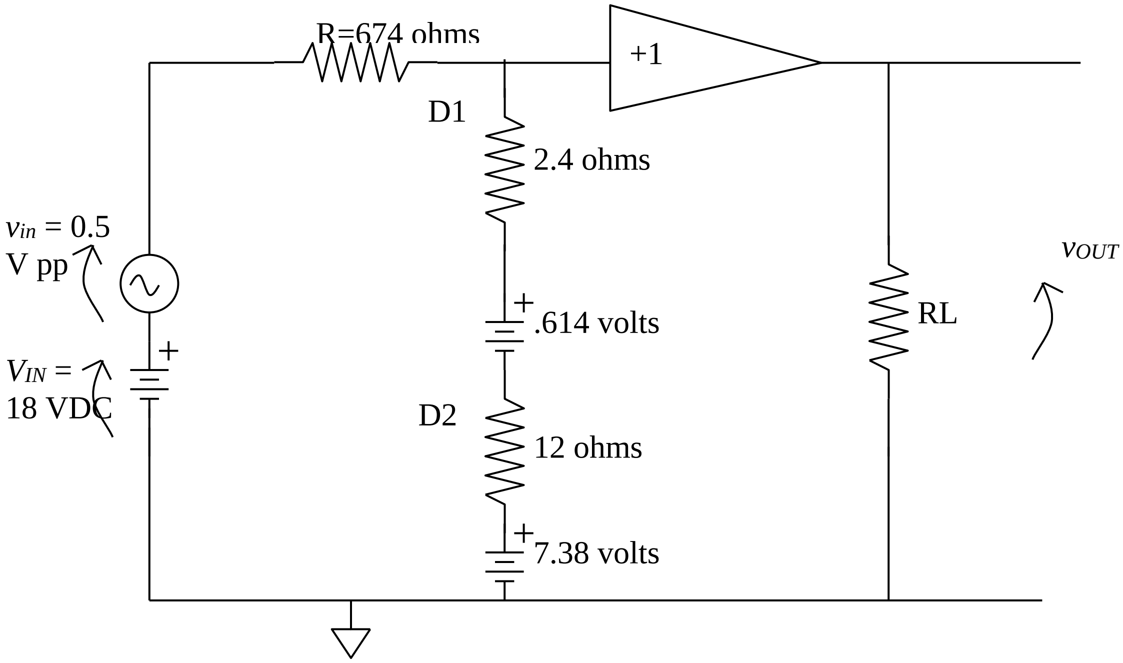
Engineering 1620 – Spring 2016
Answers to Homework 3: Diode Applications and Bipolar Transistors

1.) First one has to convert the problem to an appropriate linear model. The zener diode model voltage parameter is $V_Z = V_{ZT} - R_Z I_{ZT} = 7.5 - 12 \cdot 0.01 = 7.38$ volts. At 15ma, the forward biased diode has parameters $r_d = \frac{nkT}{qI_D} = \frac{1.4 \cdot 0.0256}{0.015} = 2.39$ ohms and $V_F = V_D - \frac{mkT}{q} = .65 - 1.4 \cdot 0.0256 = .614$ volts. The equivalent circuit is shown below and is the basis of our calculations.

1.1) The resistor shown in series with the two diodes is simply the value needed to get 15ma DC in the diode loop or $\frac{V_{IN} - V_F - V_Z}{0.015} = \frac{18 - .614 - 7.38}{0.015} = 674$ ohms.
1.2) The output voltage is V_{OUT} = .614 + 7.38 + 0.015*(2.4+12) = 8.21 volts and the ripple is $v_{out} = v_{in} \frac{2.4 + 12}{14.4 + 674} = 0.0105$ volts.

1.3) For a 10 degree C increase in temperature, the diode voltage decreases by 22 mv and the zener increases by 10*.027% of the DC voltage across the zener. That voltage is 7.38 + .015*12 = 7.56 volts. This implies a change of 10 * .00027*7.56 = 20.4 mv for a net change of -1.6 mV or -0.019 %.
1.4) The amplifier makes the output independent of the amount of current drawn by the load RL. The input of the amplifier draws no current and the current through the load, RL, is supplied by the power supply through the opamp.



2.) To find the optimal collector current, one has to begin by computing the impedance of the shaker. The reactance of the coil at 60 Hz is 22.6 ohms. The phase angle is $\varphi = \tan^{-1}\left(\frac{22.6}{10}\right) = 66.1$ deg. The magnitude of the impedance is $\sqrt{10^2 + 22.6^2} = 24.7$ ohms. Then the optimal collector current is

$$I_{Copt} = \frac{V_{CC} - V_{CESAT}}{R_{DC} + (1-\gamma)|Z_{AC}|} = \frac{40 - 1.3}{11.5 + .9 \cdot 26.2} = 1.1 \text{amperes.}$$

We next want to select values for the two biasing resistors. The complicating factor is that h_{FE} varies from 750 to 20,000 and that is a 27:1 range. To maintain fairly tight control of the quiescent current, R_{BB} will probably have to be low, much lower than the minimum of $(1 + h_{FE}) R_E = 751 \cdot 1.5 = 1125$ ohms. I tried using a lower resistor R1 = 220 ohms (a standard resistance about 20 % of 1125) and R_{BB} will be a little lower than that. By using the nominal conditions (h_{FE} = 4000, V_{BE} = 1.35) and KCL at the base node, this gives R2 = 2.66 K. With these values, V_{BB} = 3.06 volts and R_{BB} = 203 ohms. The range of base-emitter voltage is the temperature change times twice the temperature coefficient of one emitter-base junction or 70*2*.0022= .308 volts. Then one tests the two cases: 1.) for h_{FE} = 750, V_{BE} = 1.504 volts (1.35 + .154) one gets I_C = .88 Amps which is 80 % of the nominal value – just enough! 2.) for h_{FE} = 20000, V_{BE} = 1.20 volts (1.35 – .154) one gets I_C = 1.23 amps or 112 % of nominal - a comfortable margin. There are certainly other solutions near these values.

For the input capacitor calculation, we need the input impedance of R_{BB} ||Ztr at nominal conditions or 2660||220||(4000*1.53) = 197 ohms. One DB loss is a factor of .89 and the ratio of 60 Hz to cutoff must be 1.95 for a single pole circuit like this one. ($0.89 = \sqrt{\frac{(60/f_{3DB})^2}{1+(60/f_{3DB})^2}}$) That places the 3 DB cutoff at 30.5 Hz and

$$C_{IN} = \frac{1}{2\pi R_{in} f_c} = \frac{1}{2\pi 196 \cdot 30.5} = 26.6 \mu f.$$

3.1) The first stage is common base CB and the second common collector CC.

3.2) With no base current in Q1, the voltage across the 3 K biasing resistor will be $3 \cdot 12/13 = 2.77$ volts. With a base current of 0.01 mA, this voltage difference decreases to 2.75 volts. (The effect of base current might been neglected altogether given this value.) So RE1 = (2.75-0.7)/5e-3 = 410 ohms.

3.3) The output impedance of the Q1 stage is 1 K and that is the source impedance for the CC stage. The output impedance of that stage is RE2 || (1K/101 + re2) = 18 ohms. Very likely RE2 will have little effect so re2 = 8 ohms and IE2 = 3.21 mA. The base current of Q2 = .032 mA and the emitter voltage is (5 - .032-0.7) for RE2 = 4.27/3.21 = 1.33 K.

3.4) The midband gain is .99*(1K || 101*2.7K)/(180 + re1) = 4.81 (13.6 DB).

3.5) The midband input impedance is re1 + 180 = 206 ohms.
$c_{IN} = \frac{1}{2\pi f R_{in}} = \frac{1}{2\pi 1\cdot 10^4 \cdot 206}$ which is 0.077 ufd.
3.6) The input capacitor prevents the signal source from changing the bias conditions of the amplifier itself.

4.) The starting point for the problem is to see if the transistor were operating in such a way that it was not in saturation, what the quiescent current would be. We try to apply the simple formula for a circuit with three-resistor biasing, namely
$I_C = \frac{h_{FE}\left(V_{BB} - V_{BE}\right)}{R_{BB} + \left(1 + h_{FE}\right) R_E}$. To estimate V_{BE}, we estimate first the collector current as the expected voltage across R3 divided by the value of R3 or (9-5)/33 = 0.12 mA. Then $V_{BE} = \frac{kT}{q}\ln\left(1.2\cdot 10^{-4} / 1.2\cdot 10^{-14}\right) = 0.59$ volts. For this circuit:
$V_{BB} = 9\frac{270}{270+330} = 4.05$ volts; $R_{BB} = \frac{270\cdot 330}{270+330} = 149$ Kohm. Evaluating the formula gives: $I_C = \frac{120\left(4.05 - .58\right)}{149 + 121\cdot 3.3} = .71$ mA. This is obviously ridiculous because the collector voltage would have to be 9 - .71*33 = -14.3 volts. That is not physically possible so the transistor must be in saturation.

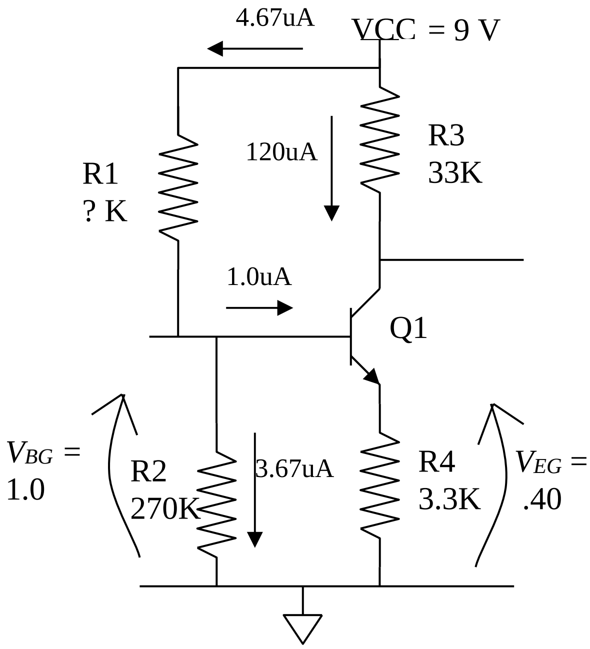
Under that condition the collector-emitter voltage will be $V_{CE} = V_{CESAT} = 0.2$ volts and the current gain will be small.

To find the real conditions of the circuit in saturation, just do KCL at the emitter using the emitter voltage, V_{EG}, as the unknown. Then $I_E = I_C + I_B$ and

$$\frac{V_{EG}}{3.3} = \frac{9 - 0.2 - V_{EG}}{33} + \frac{4.05 - 0.59 - V_{EG}}{148}$$

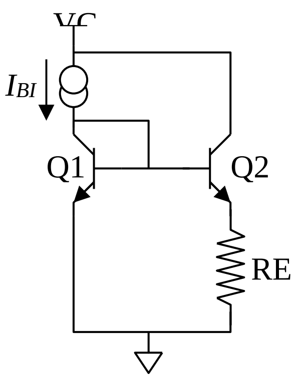
for which the solution is V_{EG} = 0.861 volts; I_C = .260 mA; I_B = .021 mA; $h_{FE} = 12$; V_{CG} = 1.06 volts; and V_{BG} = 1.46 volts. I am willing to accept a simpler solution - neglect the base current and calculate I_C based simply on $V_{CE} = V_{CESAT} = 0.2$ volts.

The problem is that there is too much base current and V_{BB} must be reduced or R4 increased or R3 reduced. You are told to make a change without lowering the input impedance or changing gain or output impedance. The only remaining choice is to raise the value of R1. The easy way to select a new value for R1 is to use KCL at the base terminal. The desired collector current is $I_C = \frac{9-5}{33} = .121$ ma. The emitter current is to be 122 uA for an emitter voltage of 0.4 volts. The base node voltage is just about 1.0 volts and the base current 1 uA. Current through R2 is 3.67 uA and through R1 4.67 uA. R1 is then 1.72 Megohms. With the new value of R1, V_{BB} = 1.23 volts and R_{BB} = 233K.



For a current gain of 80, $I_C = \frac{80(1.23-.59)}{233+81\cdot 3.3} = 0.102\text{ uA}$. Likewise for a gain of 160, I_C = 134 uA.

5.) Here is the circuit for this problem again. It should be obvious that the base-emitter voltage of Q2, V_{BE2}, is smaller than that of Q1 because of the voltage drop across resistor RE. For that reason, its collector and base currents will be less than those of Q1 and so we can very likely neglect at least the base current of Q1.



5.1) Applying KCL at the collector of Q1 implies: $I_{BI} = I_{B1} + I_{C1} = (1+\beta)I_{B1}$. Solve for the base current of Q1 and calculate the collector current from that: $I_{C1} = \frac{\beta I_{BI}}{1+\beta} = \alpha I_{BI}$

5.2) The same equation for collector current holds for each transistor, namely:

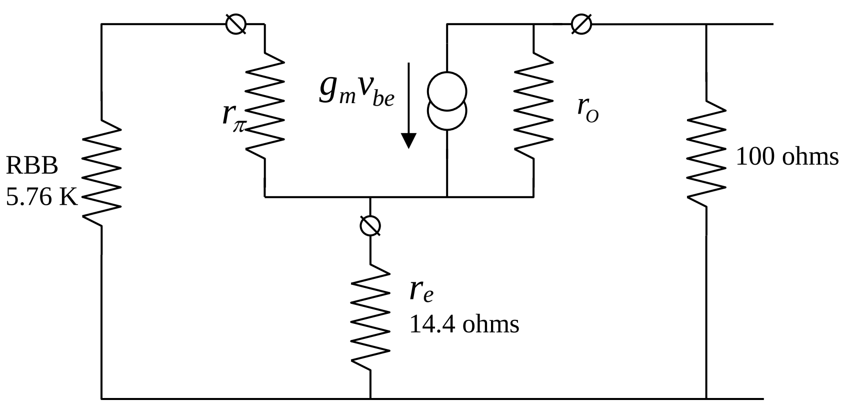
$i_{C1} = I_{SCE}\exp\left(\frac{qv_{BE1}}{nkT}\right)$ and $i_{C2} = I_{SCE}\exp\left(\frac{qv_{BE2}}{nkT}\right)$. Apply a Kirchoff voltage loop around the base to emitter of Q1, up across RE, and across emitter to base of Q2 back to the starting point to get: $v_{BE1} = v_{BE2} + i_{C2}R_E$. Use this result to substitute for v_{BE2} in the exponential equation to get: $i_{C2} = I_{SCE}\exp\left(\frac{q(v_{BE1} - i_{C2}R_E)}{nkT}\right) = i_{C1}\exp\left(\frac{-qi_{C2}R_E}{nkT}\right)$. Now use the result of part 1 to eliminate Q1 and arrive at a transcendental equation with i_{C2} as the only unknown: $i_{C2} = \alpha I_{BI} \exp\left(\frac{-qi_{C2}R_E}{nkT}\right)$. An alternate form of this would be:

$i_{C2} = \frac{nkT}{qR_E}\ln\left(\frac{\alpha I_{BI}}{i_{C2}}\right)$.

5.3) For a current gain of 100, alpha is .99 and the ratio $\frac{i_{C2}}{\alpha I_{BI}} = \frac{.05}{.99} = .0505$. The natural log of 0.0505 is –2.986 so $R_E = \frac{nkT}{qi_{C2}}\ln\left(\frac{\alpha I_{BI}}{i_{C2}}\right) = \frac{1.03 \cdot .0256 \cdot 2.986}{5.05 \cdot 10^{-5}} = 1560$ ohms.

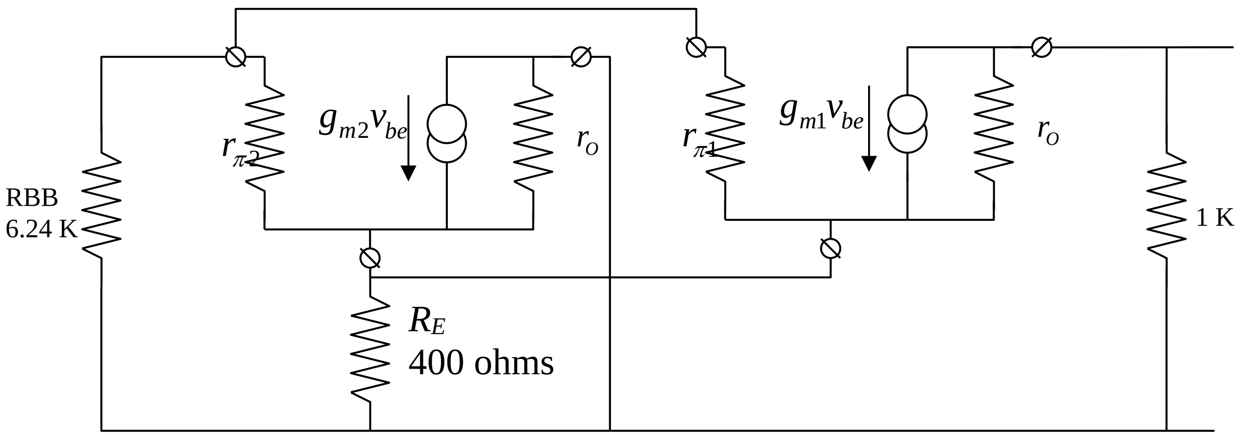
6.1) The two transistors have the same base-emitter voltages because they carry the same emitter currents. The Thevenin equivalent circuit for the two biasing resistors is a 1.6 volt source and 5.76 K resistor. From KVL using the equation for IC in terms of IS, VT and VBE we have $V_{BB} - R_{BB}I_B - 2V_T \ln\left(\frac{\beta I_B}{I_S}\right) = 0$. You can solve this several ways: take a guess (the numbers were dummied up to give a nice round answer), iterate, or use a standard tool such as MATLAB, Mathematica, Excel, etc. I used Excel and got I_B = 1.79e-5 and IC = 1.79 mA. VBE was 0.75 volts.

6.2) The lower transistor is diode connected so its small signal model is just a resistor with value VT/IE or 14.4 ohms. The small signal model of the upper transistor has gm = 69.6 mA/volt and $r_\pi = 1.45 \cdot 10^3$ ohms. In the circuit, the output resistance of the transistor from the early effect is infinite.



7.a) Neither transistor is saturated so the total emitter current of both transistors is equivalent to a single transistor with IS = 7.5e-16. The Thevenin equivalent of the bias network is 1.2 volts and 6.24 K. This can be solved the same way as the last problem but just a little guessing gives a good answer. A total collector current of 1 mA distributed as .667 mA through Q1 and .333 through Q2 gives a good balance in the circuit. VBE = 0.718 volts and the Kirchoff loop equation is balanced within 0.016 volt in 1.2 volts.

7.b) The small signal model has different values of transconductance and base-emitter resistance for the two devices. In the circuit I have swapped the order of Q1 and Q2 to make the circuit easier to read. Q1 has $g_{m1} = 26$ mA/volt and $r_{\pi 1} = 3876$ ohms. For Q2, the transconductance is half that and the resistance is twice that.



8) The base-emitter voltage is 0.741 volts for 1.0 mA collector current. The base current is 10 uA and the voltage at the node where RB ties to the other two resistors is 2.5 – 1e3*1.01e-3 = 1.49 volts. Ohm's law gives RB = (1.49 - .741)/1e-5 = 74.9 K.