Open and Closed-Loop Frequency Response of Typical Operational Amplifiers

All amplifiers must have finite gain and a frequency response that decreases with increasing frequency beyond some maximum value. This is an obvious practical matter and is also a result of the mathematics of Laplace and Fourier transforms. The latter require that $H(s) < \infty \forall s$ and $\int_{-\infty}^{+\infty} |H(s)|^2 \, df < \infty$. In designing an operational amplifier, you choose $H(s)$ to optimize the device for as wide a bandwidth as possible with given resources but also to make it as easy as possible for the customer to use.

A single pole transfer function is what meets these criteria:

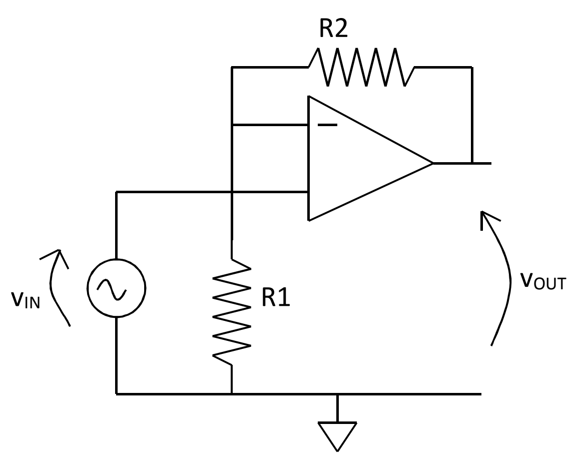
- It is the lowest order of transfer function that can be realized. Hence its gain rolls off with frequency as slowly as possible.
- It has less than 90 degrees phase shift for all usable frequencies, so it is easier to use feedback without causing self-oscillation.

With a single pole frequency dependence, the terminal relations of an opamp become:

$$v_{OUT} = \frac{A_0}{1 + s\tau_D}(v_+ - v_-)$$

Consider the simplest, positive gain circuit as shown here. The inverting input is simply

$v_- = \frac{R1 v_{OUT}}{R1 + R2} = \frac{v_{OUT}}{G_0}$ where G_0 will turn out to be the DC gain.



Substitute this relation into the amplifier transfer function to get: $v_{OUT}\left(1 + s\tau_D + \frac{A_0}{G_0}\right) = A_0 v_{IN}$. With a little more manipulation, this becomes:

$$H(s) = \frac{v_{OUT}}{v_{IN}} = \frac{A_0}{1 + s\tau_D + \frac{A_0}{G_0}} = \frac{G_0}{\frac{G_0}{A_0} + 1 + s\tau_D \frac{G_0}{A_0}} \approx \frac{G_0}{1 + s\tau' G_0}$$

where τ' is $\frac{\tau_D}{A_0}$ and $f_{GBW} = \frac{1}{2\pi\tau'} = \frac{A_0}{2\pi\tau_D}$. The cutoff frequency of the circuit with gain is

$$f_{-3dB} = \frac{1}{2\pi\tau' G_0} = \frac{f_{GBW}}{G_0} \quad \text{or} \quad f_{GBW} = G_0 f_{-3dB}$$