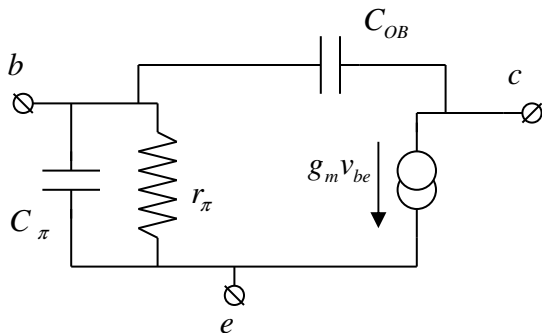


## Engineering 1620: High Frequency Effects in BJT Circuits – an Introduction Especially for the Friday before Spring Break

I prepared these notes a few years ago to give out around spring break because I would finish the material on the day before vacation break. As this material is only partially covered in Razavi, I wanted to summarize it for you. Unfortunately, this year I am finishing late in a rush. I hope it will also help you understand the calculations for Lab 5, a cascode amplifier for video signals. While there aren't any labs this year, I encourage you to look at the calculations for them, in this case for Lab 5.

Over recent classes, I developed a number of formulae first for the capacitances in the junctions of a transistor and then for their effect on the gain, input impedance and output impedance of a generalized common emitter amplifier. Let me begin by recounting the capacitances of a transistor. (A full derivation of these results is in an Appendix to this set of notes.)

The small-signal model in its simplest form has two capacitances: the base-emitter capacitance is commonly called  $C_\pi$  while the collector-base capacitance is variously  $C_{OB}$  or  $C_\mu$ . (The first nomenclature is common on data sheets while the latter is more common in papers. The CAD/SPICE usage is “cmu.” There is a slight but difficult to determine difference between the two when there is parasitic series resistance in the base. We do not observe this distinction.)

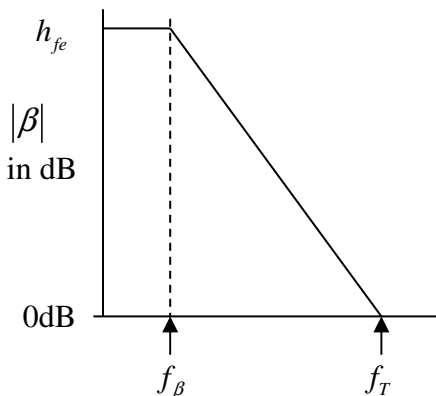


$C_{OB}$  is the capacitance of a reverse biased junction and often has the dependence on voltage of an abrupt-junction. For at least one value of collector-base voltage, it is straightforwardly shown on any datasheet. Often it is given as a graph against VCE.

The transconductance and the dynamic resistance of the base-emitter junction are needed first before one can tease the value of  $C_\pi$  out of the datasheet.

The Q-point determines the remaining model parameters in the usual way:  $g_m \approx \frac{qI_C}{kT}$  and

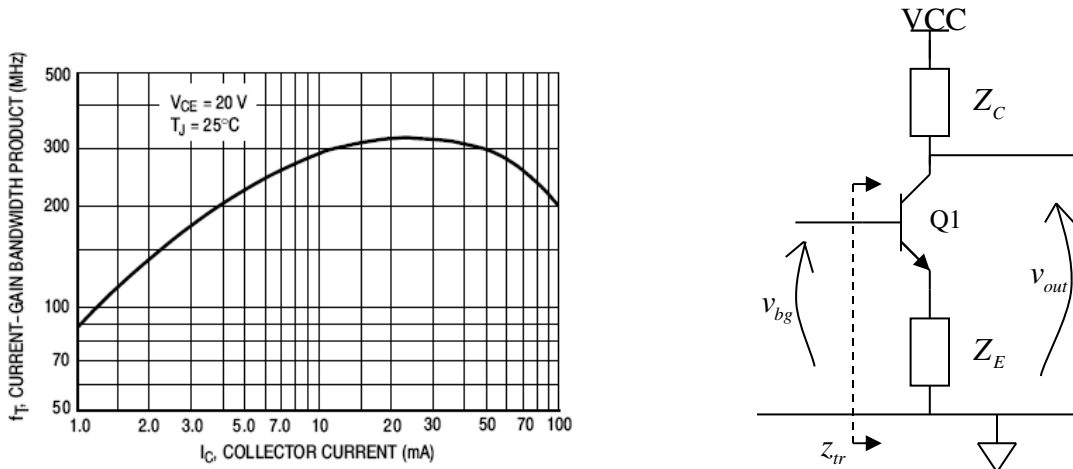
$$r_\pi = (1 + h_{fe})r_e \approx \frac{(1 + h_{fe})kT}{qI_E} \text{ where I am using } h_{fe} \text{ interchangeably with } \beta \text{ as the DC}$$



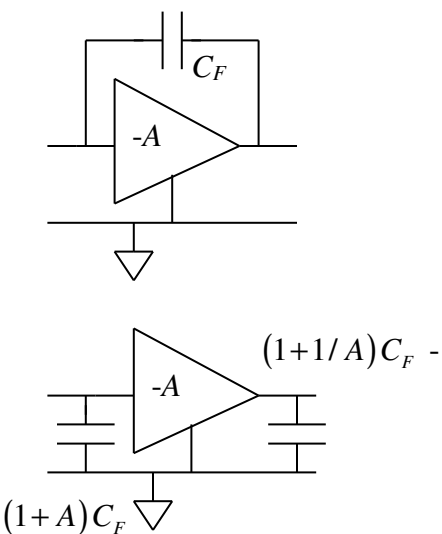
current gain of the transistor. In class, we showed that because of the base-emitter capacitance, the current gain becomes frequency dependent. For a grounded-collector circuit, the gain becomes

$$\beta(s) = \frac{i_c}{i_b} = \frac{h_{fe}}{1 + sr_\pi(C_\pi + C_\mu)}. \text{ This is a single pole}$$

function that looks like this sketch when  $|\beta|$  is graphed on a log-log Bode plot. Here we have defined two frequencies:  $f_\beta$  at which the gain is down  $-3\text{dB}$  from its DC value and  $f_T$  at which the magnitude of the gain is unity. If  $h_{fe}$  is large, then  $f_\beta \approx \frac{f_T}{1+h_{fe}}$ . The datasheet gives  $f_T$  as a function of the quiescent current because that frequency is not sensitive to the DC current gain. (It also helps in advertising the properties of a transistor to advertise the larger number!) The dependence of  $f_T$  on current comes from  $r_\pi$  (or equivalently  $r_e$ ) being inversely proportional to current. Part of the  $C_\pi$  capacitance is due to the depletion layer and is roughly independent of current. The other part of  $C_\pi$  is due to charge in transit from emitter to collector and when that is dominant,  $r_e C_\pi = \tau$ , the transit time. The typical curve for a 2N2222A device is shown below next to a generalized common emitter circuit.



The capacitance from collector to base complicates the simple calculation of gain and input impedance. To simplify the problem, we take advantage of the Miller effect (named after John W. Miller who published it in 1920). Miller's theorem points out the equivalence for input impedance, output impedance and gain of the two block diagrams below as long as the gain of the amplifier is known with the feedback capacitor in place. (The proof of the theorem simply equates the current in the feedback capacitor to the two currents through the capacitors of the second configuration.)



Because the feedback capacitor connects between output and input and the output voltage is often bigger than the input, the current in the capacitor is generally greater than it would be if the capacitor were across the input to ground. The theorem points out that this is equivalent to a larger, possibly frequency-dependent (because  $A = A(s)$ ) capacitor across the input and a marginally bigger one across the output. What often makes this theorem useful is that the low frequency gain is known and that gain is

constant enough to use over most of the useful frequency range of the amplifier.

We also developed a set of formulas for the input impedance and gain of the generalized common emitter amplifier shown at the top right. The idea was to use Miller's theorem to move the collector base capacitance to two places: the  $(1+1/A)C_{OB}$  component simply became part of  $Z_C$  and contributes to calculating the gain. The  $(1+A)C_{OB}$  element moved to the left and ended in parallel with the input. This transform left only  $C_\pi$  to complicate life. The input impedance with only that parasitic capacitance was:

$$z_{tr} = \frac{(1+h_{fe})(r_e + Z_E)}{1 + sr_\pi C_\pi} + \frac{sr_\pi C_\pi Z_E}{1 + sr_\pi C_\pi}$$

The first term is our old result for input impedance at low frequency but now there is a decrease in impedance with increasing frequency from an extra single pole at

$$f_\beta = \frac{1}{2\pi r_\pi C_\pi} = \frac{f_T}{1+h_{fe}},$$

a nice but not surprising result. The second term assures that even

when the base capacitance shorts the base-emitter junction, there will still be  $Z_E$  left as part of  $z_{tr}$ . This term is not important until roughly  $f_T$  and one would not usually try to use a device close to its maximum frequency limit. Notice that the first term has the form of a resistor  $R = (1+h_{fe})(r_e + Z_E)$  in parallel with a capacitor of value  $C = \frac{r_e C_\pi}{(r_e + Z_E)}$ .

The resistance value is the low-frequency result we derived a couple of weeks ago. The capacitance is proportional to but generally smaller than  $C_\pi$ . We will use this result in an analysis example.

The gain formula has a similar form:  $G = -\frac{\alpha Z'_C}{(r_e + Z_E)} \cdot \frac{1}{1 + sr_e Z_E C_\pi / (r_e + Z_E)}$  where  $Z'_C$  is

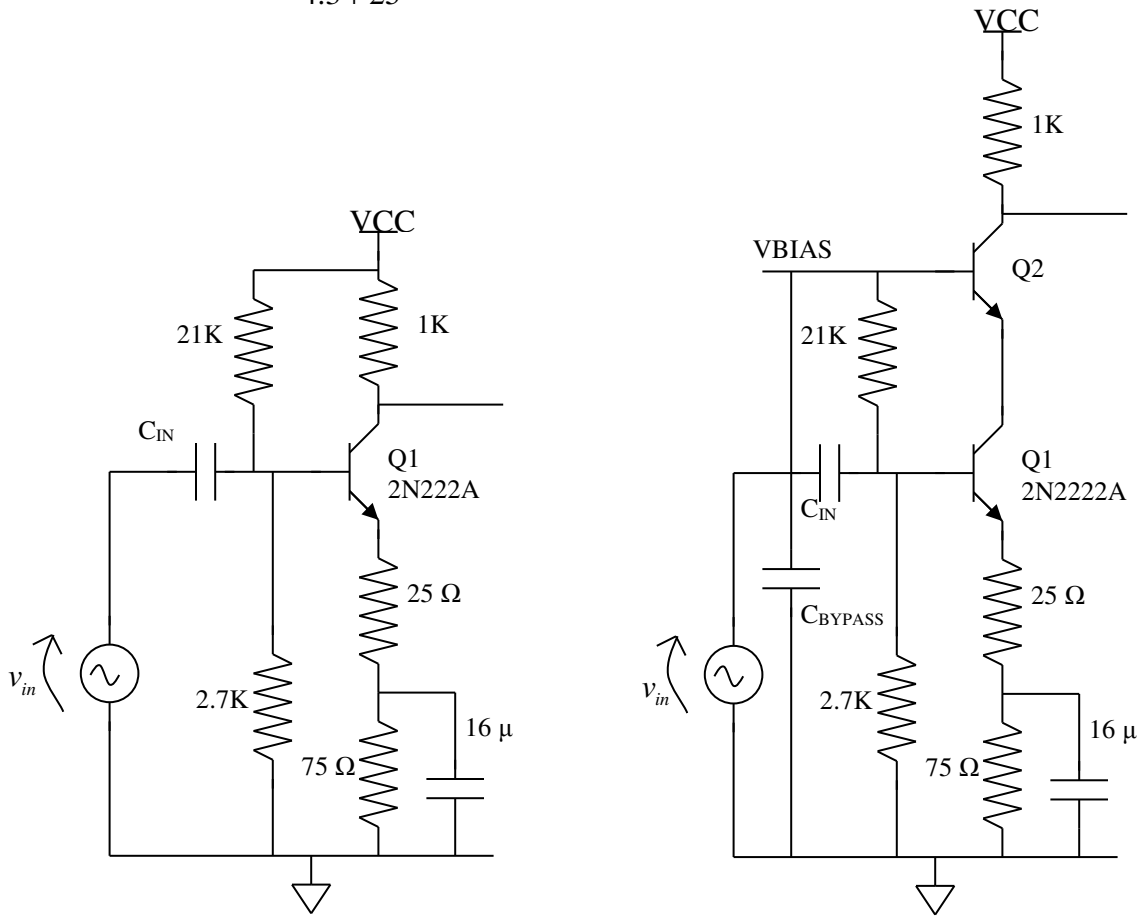
$Z_C$  in parallel with Miller's second capacitor  $C_\mu(1+1/A)$ . [I am playing fast and loose with exact results here. Actually as  $A$  becomes frequency dependent one has to be careful to include the effect of that change on the input capacitance. We will see this more clearly in MOS circuits later.] The first factor is the low frequency gain and the second is a new pole generally somewhat above  $f_T$ . In other words,  $C_\pi$  does not have a big effect on the Gain except through a decrease in input impedance that causes loading of the input signal source.

Now let us look at an example of these effects. The circuit on the left below is one we used as a low frequency example some time ago. It has a quiescent point around 6 mA, a current gain about  $h_{fe} = 120$  typical, and therefore  $r_e = 4.2$  ohms and  $r_\pi = 500$  ohms.

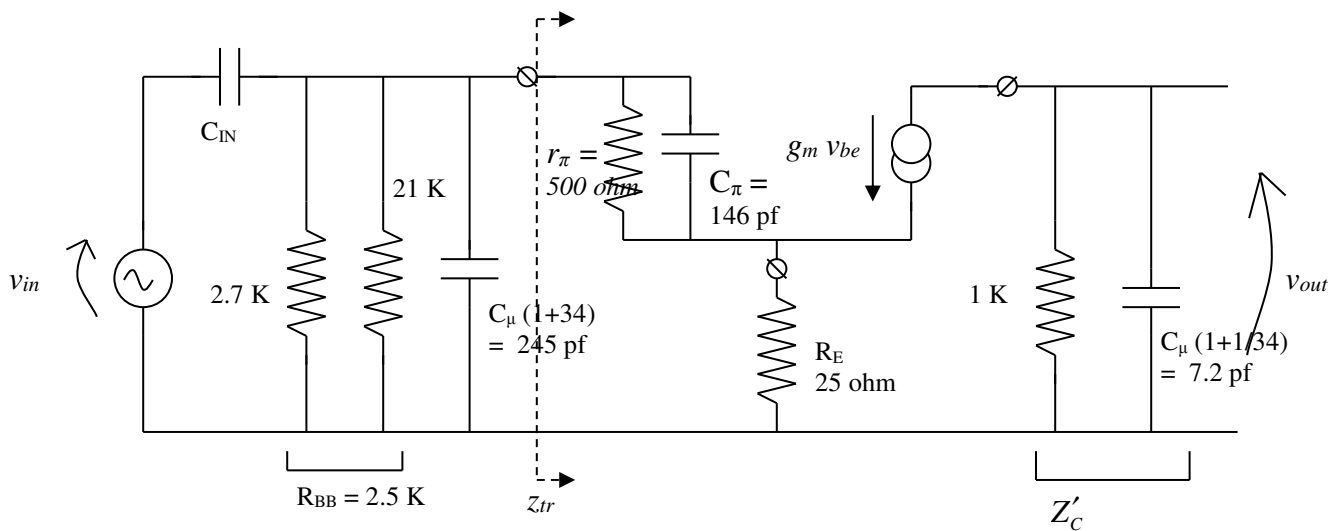
From the graph above,  $f_T \approx 250$  MHz. The datasheet value of  $C_{OB}$  is 7 pf. From this,

$$C_\pi = \frac{1}{2\pi f_T r_e} - C_{OB} = \frac{1}{6.28 * 4.2 * 2.5 \cdot 10^{+8}} - 7 \cdot 10^{-12} = 146 \text{ pf.}$$

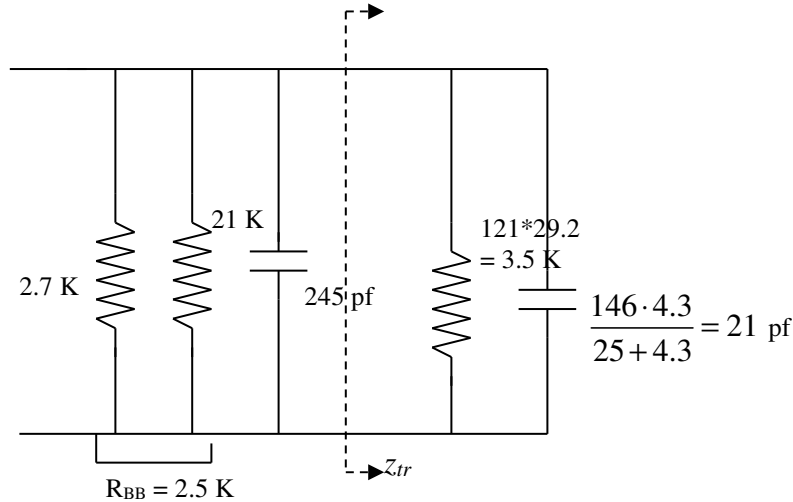
The midband gain is  $\frac{-.99 \cdot 1 \cdot 10^3}{4.3 + 25} = -34$ .



By employing Miller's theorem, we can draw the small signal model as:



For  $Z_{in}$  we have (neglecting  $C_{IN}$  because it provides only a low-frequency, high-pass cut-off):



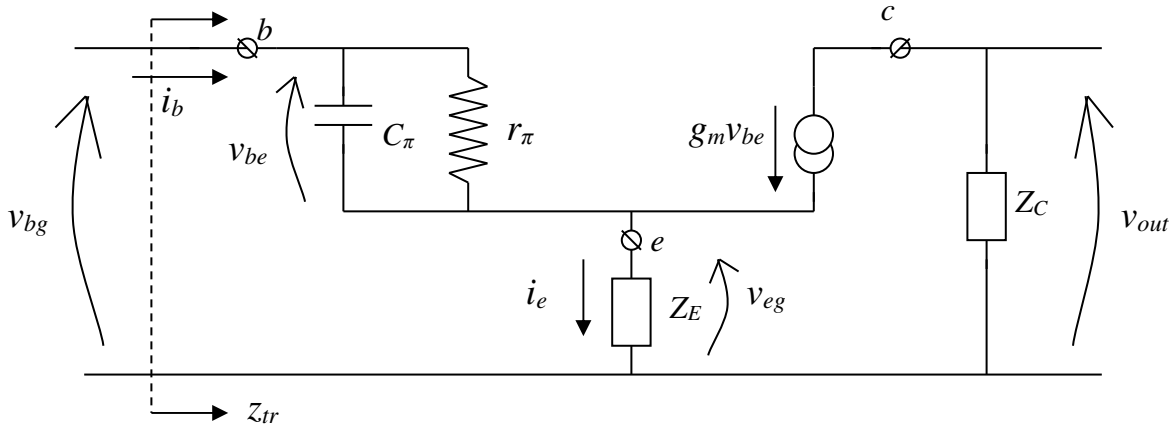
The resulting system is 1.42 K in parallel with 266 pf. The input impedance is dominated by the Miller capacitance (245 pf of 266 pf total) and its magnitude will start to decrease with single pole behavior at 421 KHz. By contrast the effect of  $C_{\mu}$  on the gain is to introduce a high-frequency low-pass pole at about 23 MHz from the 1K resistor and 7 pf capacitance in parallel. [If there is appreciable source loading at this frequency, it is even possible that this second pole will be even higher in frequency but that is a later topic.]

An amplifier that has constant gain to some high frequency but has so low an input impedance as to load the signal source well below its gain cutoff frequency is a poor design because one cannot use the gain for the full range of the amplifier's potential usefulness. The circuit on the right above is called a cascode amplifier and it attempts to solve this problem with a second transistor. The tandem arrangement of a common emitter stage, Q1, with a common base stage, Q2, is called a cascode connection. (And no, this is not a spelling error.) The voltage gain of the common emitter Q1 is very low, fractional in this case, because  $Z_C$  for that stage is the input impedance of the common base stage, Q2. (That input impedance is  $z_{in2} = r_{e2} \parallel C_{\pi2}$ . The capacitance of Q2 is not important at a few

megahertz so the voltage gain of Q1 is  $G = -\frac{\alpha r_e}{(r_e + Z_E)} \cong -\frac{0.99 \cdot 4.2}{4.2 + 25} = -.15$ ) There is no

longer a direct capacitance between input and output. The output load no longer affects the input impedance. For that reason, cascode circuits are sometimes said to be 'unilateral.' This time the input impedance is the same 1.42 K resistive part but the capacitor is only 21+7 = 28 pf and the capacitive part becomes a factor in loading the input only above 4.5 MHz. That is a full order of magnitude improvement in pole placement.

## Appendix: The Effect of $C_\pi$ on Input Impedance and Gain of a BJT CE Circuit



### Basic Equations:

$$\text{KCL at the emitter terminal, } e: 0 = i_b + g_m v_{be} - \frac{v_{eg}}{Z_E}$$

$$\text{Ohm's law across base-emitter: } v_{be} = \frac{r_\pi i_b}{1 + sC_\pi r_\pi}$$

$$\text{KVL from input across base-emitter and emitter to ground: } v_{bg} = v_{be} + v_{eg}$$

$$\text{Definition of } z_{tr}: z_{tr} \equiv \frac{v_{bg}}{i_b}$$

Connection between hybrid-pi transconductance model and current controlled h-model:  $g_m r_\pi = \beta_0$  where  $\beta_0$  is the low-frequency current gain.

### Steps to solve:

$$\frac{v_{eg}}{Z_E} = i_b \left( 1 + \frac{g_m r_\pi}{1 + sC_\pi r_\pi} \right) = i_b \left( \frac{1 + \beta_0 + sC_\pi r_\pi}{1 + sC_\pi r_\pi} \right)$$

$$v_{eg} = Z_E \left( \frac{1 + \beta_0 + sC_\pi r_\pi}{1 + sC_\pi r_\pi} \right) i_b \quad \text{and} \quad v_{be} = \frac{r_\pi i_b}{1 + sC_\pi r_\pi}$$

$$v_{bg} = \left( \frac{r_\pi + Z_E (1 + \beta_0 + sC_\pi r_\pi)}{1 + sC_\pi r_\pi} \right) i_b \quad (\text{Used: } r_\pi = (1 + \beta_0) r_e)$$

Input impedance is:

$$z_{ir} = \frac{(1 + \beta_0)(r_e + Z_E) + sC_\pi r_\pi Z_E}{1 + sC_\pi r_\pi}$$

$$z_{ir} = \frac{(1 + \beta_0)(r_e + Z_E)}{1 + sC_\pi r_\pi} + \frac{sC_\pi r_\pi Z_E}{1 + sC_\pi r_\pi}$$

Gain is the ratio:

$$H(s) = \frac{v_{out}}{v_{bg}} = \frac{-Z_C i_C}{v_{bg}} = \frac{-Z_C g_m v_{be}}{v_{bg}} = \frac{-Z_C g_m r_\pi i_b}{v_{bg} (1 + sC_\pi r_\pi)} = \frac{-Z_C g_m r_\pi i_b}{v_{bg} (1 + sC_\pi r_\pi)} = \frac{-Z_C \beta_0}{z_{ir} (1 + sC_\pi r_\pi)}$$

$$H(s) = \frac{-Z_C \beta_0}{(1 + \beta_0)(r_e + Z_E) + sC_\pi r_\pi Z_E} = \frac{-Z_C \beta_0}{(1 + \beta_0)(r_e + Z_E)} \cdot \frac{1}{1 + sC_\pi r_\pi Z_E / (r_e + Z_E)}$$

### Summary:

The input impedance is lowered by  $C_\pi$  beginning at the beta cutoff frequency. It is also asymptotic to  $Z_E$  for frequencies above  $f_T$ . In this equation the first term is the low frequency impedance with a new pole at the beta cutoff frequency. The second term takes care of the behavior that makes  $Z_E$  the impedance at very high frequency.

$$z_{ir} = \frac{(1 + \beta_0)(r_e + Z_E)}{1 + sC_\pi r_\pi} + \frac{sC_\pi r_\pi Z_E}{1 + sC_\pi r_\pi}$$

Notice that the first term has the form of a resistance in parallel with a capacitance. As you did in the first lab, we can manipulate that into the form of a resistance with an equivalent capacitance in parallel by multiplying and dividing the frequency in the denominator by the resistance from the numerator.

$$z_{tr} = \frac{(1 + \beta_0)(r_e + Z_E)}{1 + sC'R'} \quad \text{if } C' = \frac{C_\pi r_e}{(r_e + Z_E)} \text{ and } R' = (1 + \beta_0)(r_e + Z_E)$$

The gain is little affected except there is a new pole at some frequency above  $f_T$ . The first factor is the low frequency gain and the second is a new high frequency pole.

$$H(s) = \frac{-Z_C \beta_0}{(1 + \beta_0)(r_e + Z_E)} \cdot \frac{1}{1 + sC_\pi r_e Z_E / (r_e + Z_E)}$$