EN175: Advanced Mechanics of Solids

Final Examination
Monday Dec 17 2018

School of Engineering
Brown University

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use two pages of reference notes
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1-5. (11 points) _____________

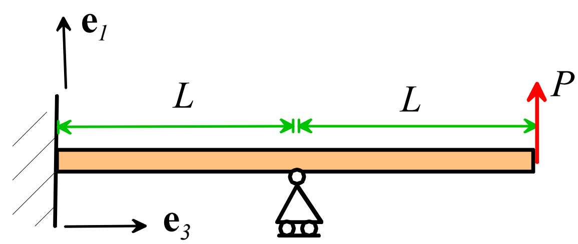
6. (16 points) _____________

7. (6 points) _____________

8. (12 points) _____________

TOTAL (45 points)_____________

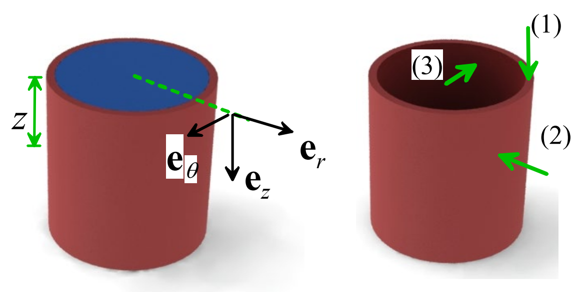
1. The figure shows a beam subjected to constraints and forces. Indicate whether the following functions are kinematically admissible displacement fields for calculating the beam's potential energy



	Admissible?	
(a) $u_1 = x_3(L - x_3)$	Y	N
(b) $u_1 = x_3^3(L - x_3)^2$	Y	N
(c) $u_1 = x_3^2(L - x_3)(2L - x_3)$	Y	N
(d) $u_1 = x_3^2(2L - x_3)$	Y	N

[2 POINTS]

2. The figure shows a tumbler with mass density ρ_T filled with fluid with mass density ρ . Using the cylindrical-polar basis shown, write down the boundary conditions for stress components on the three surfaces marked (1), (2), (3) in the figure.



(1) (Top surface, $z = 0, \quad a < r < a + t$)

(2) (Outer surface $r=a+t$)

(3) (Inner surface $r=a$)

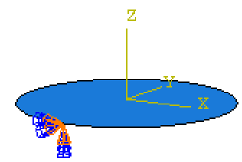
[3 POINTS]

3. The circular plate shown in the figure is meshed with plate elements for a static analysis. Indicate whether the boundary conditions shown properly constrain the solid.

Properly constrained?

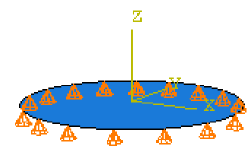
Y N

U1=U2=U3=0
UR1=UR2=UR3=0



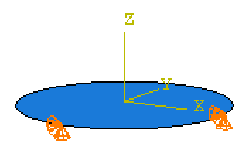
Y N

U3=0



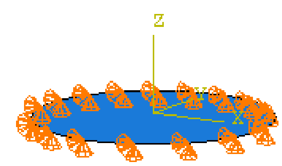
Y N

U1=U2=U3=0



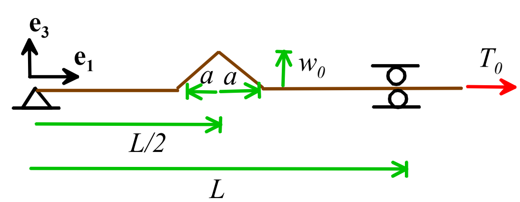
Y N

U1=U2=U3=0



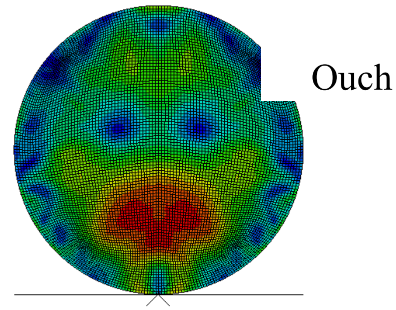
[2 POINTS]

4. The string shown in the figure has mass per unit length m and is stretched by an axial tension T_0 . It is prevented from moving in a vertical direction at its ends $x_3 = 0, x_3 = L$. At time *t=0* it is released from rest with the displacement distribution shown in the figure. Use the space below to draw the shape of the string at time $t = L\sqrt{m/T_0}$



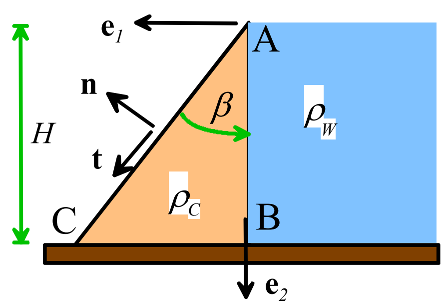
[2 POINTS]

5. The figure shows a finite element simulation of an elastic cylinder with radius R, mass density ρ, Young's modulus E and Poisson's ratio ν colliding with a frictionless rigid surface. The goal of the simulation is to calculate the contact time T as a function of material properties and geometry. Re-write the relationship $T = f(E, \rho, \nu, R)$ in dimensionless form.



[2 POINTS]

6. The figure shows a simple design for a dam. The wedge-shaped dam is made from a material with weight density ρ_C. It is loaded on its vertical face AB by fluid pressure $p = -\rho_W x_2$, where ρ_W is the weight density of the fluid.



6.1 Write down formulas for unit vectors **t, n** tangent and normal to the back face of the dam, in terms of β (you will need this information for part 6.2 and 6.3)

[2 POINTS]

6.2 Write down the boundary conditions on the two faces AB, AC of the dam, in terms of the 2D stress components $\sigma_{11}, \sigma_{22}, \sigma_{12}$.

[2 POINTS]

6.2 Consider the stress state

$$\sigma_{11} = -\rho_W x_2$$
$$\sigma_{22} = \rho_C \left(x_1 \cot(\beta) - x_2 \right) - \rho_W \cot^2 \beta (2x_1 \cot(\beta) - x_2)$$
$$\sigma_{12} = -\rho_W x_1 \cot^2 \beta$$

Show that the stress state satisfies equilibrium in the dam (inside the wedge) and also satisfies the boundary conditions on the faces AB and AC. Note that on face AC $x_1 = x_2 \tan \beta$.

[6 POINTS]

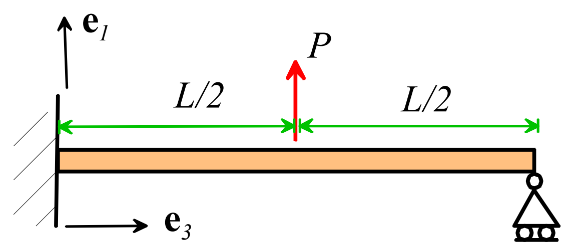
6.3 Suppose that the dam is made from concrete, which cannot safely withstand tensile stress. Assuming that the maximum principal tensile stress occurs at B, find a formula for the minimum allowable value for the angle β , in terms of ρ_c , ρ_W

[2 POINTS]

6.4 Assume that the concrete fails by crushing if the minimum principal stress reaches $\sigma_3 = -\sigma_c$. Assuming that the minimum principal stress occurs at point C, and that β has the minimum value calculated in 7.3, find an expression for the maximum height of the dam, in terms of ρ_C , ρ_W . You may find the trig identity $1 + \tan^2 \beta = 1 / \cos^2 \beta$ helpful.

[4 POINTS]

7. The figure shows a beam that is clamped at $x_3 = 0$ and pinned at $x_3 = L$. It is subjected to a point force P at mid-span $x_3 = L/2$.



7.1 Show that $\hat{v} = Cx_3^2(L - x_3)$ is a kinematically admissible deflection for the beam

[2 POINTS]

7.2 Hence, find a formula for the potential energy of the beam, in terms of E, I, C, P, L . You can assume that the potential energy of a beam is

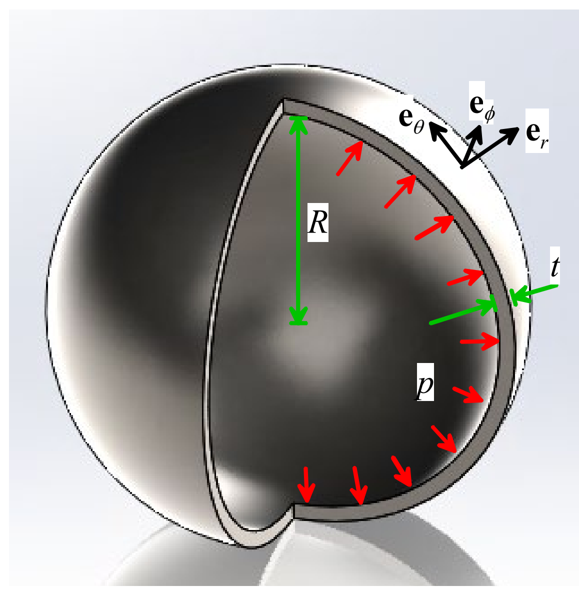
$$\Pi = \int_0^L \frac{1}{2} EI \left(\frac{d^2 v}{dx^2} \right)^2 dx - \int_0^L q(x) v(x) dx$$

[2 POINTS]

7.3 Hence, use the Rayleigh-Ritz method to estimate the deflection of the beam at $x_3 = L/2$.

[2 POINTS]

8. A thin-walled sphere with radius R and wall thickness t is made from an elastic-plastic material with Young's modulus E , Poissons ratio ν and a linear hardening relation $Y = Y_0 + h\varepsilon_e$. The sphere is subjected to monotonically increasing internal pressure p (with $dp/dt > 0$) , which generates a stress state (in spherical-polar coordinates) $\sigma_{rr} \approx 0,\ \sigma_{\phi\phi} = \sigma_{\theta\theta} = pR/(2t)$ (note that these are principal stresses)



8.1 Find a formula for the Von-Mises stress in the sphere wall, in terms of p,R and t. Hence, calculate the pressure that will first cause yield in the sphere wall.

[2 POINTS]

8.2 Find the hydrostatic and deviatoric stresses in the sphere wall.

[2 POINTS]

8.3 Hence, find a formula for the Von Mises plastic strain rate $d\varepsilon_e / dt$ in the sphere wall, in terms of $dp / dt, h, R, t$

[2 POINTS]

8.4 Hence, find a formula for the total strain rates $d\varepsilon_{rr} / dt, d\varepsilon_{\theta\theta} / dt$ (include both elastic and plastic strain rates, and give solutions for pressure both below and above yield) in the shell.

[2 POINTS]

8.5 Find the total hoop strains $\varepsilon_{\theta\theta}, \varepsilon_{\phi\phi}$ when the pressure reaches a value $p = 4tY_0 / R$

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[2 POINTS]

8.6 Find a formula for the change in radius of the sphere when the pressure reaches a value $p = 4tY_0 / R$

[2 POINTS]