School of Engineering
Brown University

EN175: Advanced Mechanics of Solids

Practice Final Examination

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use two double sided pages of reference notes, but no other materials may be consulted.
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1. (5 points) ______________

2. (5 points) ______________

3. (5 points) ______________

4. (6 points) ______________

5. (3 points) ______________

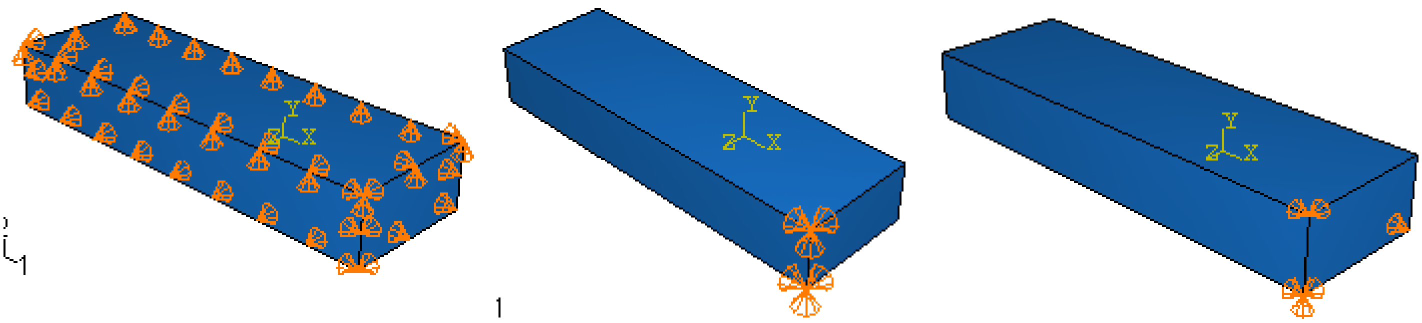
TOTAL (25 points)____________

1. Please answer the FEA related questions

1. What is the difference between a truss element and a solid element (please limit your answer to a sentence!)

2. What is the difference between the displacement fields in 6 noded and 3 noded triangular elements, and which are generally more accurate?

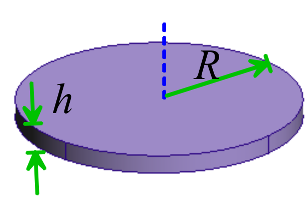
3. Which of the boundary conditions shown below properly constrain the solid for a static analysis?



(a) (b) (c)

4. A linear elastic FE calculation predicts a maximum Mises stress of 100MPa in a component. The solid is loaded only by prescribing tractions and displacements on its boundary. If the applied loads and prescribed displacements are all doubled, what will be the magnitude of the maximum Mises stress?

5. An FE calculation is conducted on a part. The solid is idealized as an elastic-perfectly plastic solid, with Youngs modulus 210 GPa, Poisson ratio $\nu = 0.3$. Its plastic properties are idealized with Mises yield surface with yield stress 500MPa. The solid is loaded only by prescribing tractions and displacements on its boundary. The analysis predicts a maximum von-Mises stress of 400MPa in the component. If the applied loads and prescribed displacements are all doubled, what will be the magnitude of the maximum Mises stress?



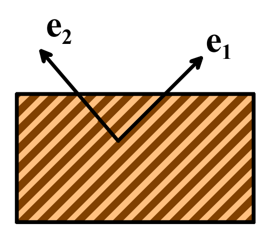
2. A thin circular plate with radius R has thickness h, Young's modulus E, Poisson's ratio ν and mass density ρ_0. The natural frequencies are calculated using FEA. Express the resulting relationship between the natural frequencies ω, geometry, and mass density in dimensionless form.

3. The figure shows a fiber reinforced composite laminate.

(i) When loaded in uniaxial tension parallel to the fibers, it fails at a stress of 500MPa.

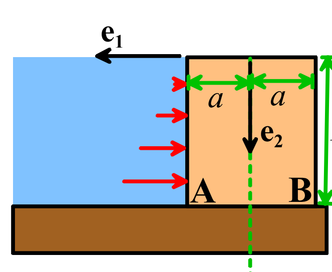
(ii) When loaded in uniaxial tension transverse to the fibers, it fails at a stress of 250 MPa.

(iii) When loaded at 45 degrees to the fibers, it fails at a stress of 223.6 MPa



The laminate is then loaded in uniaxial tension at 30 degrees to the fibers.
Calculate the expected failure stress under this loading, assuming that the material can be characterized using the Tsai-Hill failure criterion.

4. A rectangular dam is subjected to pressure $p(x_2) = \rho_w x_2$ on one face, where ρ_w is the weight density of water. The dam is made



from concrete, with weight density ρ_c (and is therefore subjected to a body force $\rho_c \mathbf{e_2}$ per unit volume). The goal is to calculate formulas for *a* and *L* to avoid failure.

4.1 Write down the boundary conditions on all four sides of the dam.

4.2 Consider the following approximate state of stress in the dam

$$\sigma_{22} = \frac{\rho_w x_2^3 x_1}{4a^3} + \frac{\rho_w x_2 x_1}{20a^3}\left(-10x_1^2 + 6a^2\right) - \rho_c x_2$$

$$\sigma_{11} = -\frac{\rho_w x_2}{2} + \frac{\rho_w x_2 x_1}{4a^3}\left(x_1^2 - 3a^2\right)$$

$$\sigma_{12} = \frac{3\rho_w x_2^2}{8a^3}(a^2 - x_1^2) - \frac{\rho_w}{8a^3}\left(a^4 - x_1^4\right) + \frac{3\rho_w}{20a}(a^2 - x_1^2)$$

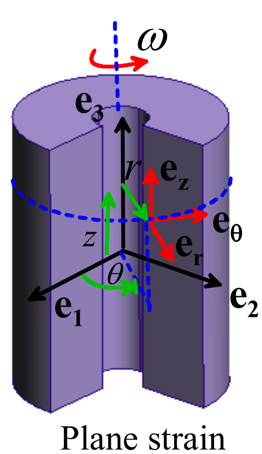
Show that (i) The stress state satisfies the equilibrium equations (ii) the stress state exactly satisfies boundary conditions on the sides $x_1 = \pm a$, (iii) The stress does not satisfy the boundary condition on $x_2 = 0$ exactly.

4.3 Show, however, that the resultant *force* acting on $x_2 = 0$ is zero, so by Saint Venant's principle the stress state will be accurate away from the top of the dam.

4.4 The concrete cannot withstand any tension. Assuming that the greatest principal tensile stress is located at point A $(x_1 = a, x_2 = L)$, find an expression for the minimum width of the dam, in terms of L, ρ_c, ρ_w .

4.5 The concrete fails by crushing when the minimum principal stress reaches $\sigma_{1\min} = -\sigma_c$. Assuming the greatest principal compressive stress is located at point B, $(x_1 = -a, x_2 = L)$ find an expression for the maximum height of the dam.

5. The figure shows a long hollow cylindrical shaft with inner radius a and outer radius b, which spins with angular speed ω about its axis. Assume that the disk is made from an elastic-



Plane strain

perfectly plastic material with yield stress Y and density ρ The goal of this problem is to calculate the critical angular speed that will cause the cylinder to collapse.

5.1 Using the cylindrical-polar basis shown, list any stress or strain components that must be zero. Assume plane strain deformation.

5.2 Write down the boundary conditions that the stress field must satisfy at $r=a$ and $r=b$

5.3 Write down the linear momentum balance equation in terms of the stress components, the angular velocity and the disk's density.

5.4 Using the plastic flow rule, show that $\sigma_{zz} = (\sigma_{rr} + \sigma_{\theta\theta})/2$ if the cylinder deforms plastically under plane strain conditions

5.5 Using Von-Mises yield criterion, show that the radial and hoop stress must satisfy $\left|\sigma_{\theta\theta} - \sigma_{rr}\right| = 2Y/\sqrt{3}$

5.6 Hence, show that the radial stress must satisfy the equation

$$\frac{d\sigma_{rr}}{dr} = -\rho r \omega^2 + \frac{2}{\sqrt{3}}\frac{Y}{r}$$

5.7 Hence, calculate the critical angular speed that will cause plastic collapse.