School of Engineering
Brown University

EN175: Advanced Mechanics of Solids

Practice Midterm Examination

NAME: ______________________________

General Instructions

- No collaboration of any kind is permitted on this examination.
- Two double sided pages of reference notes are permitted.
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1. (8 points) ______________

2. (5 points) ______________

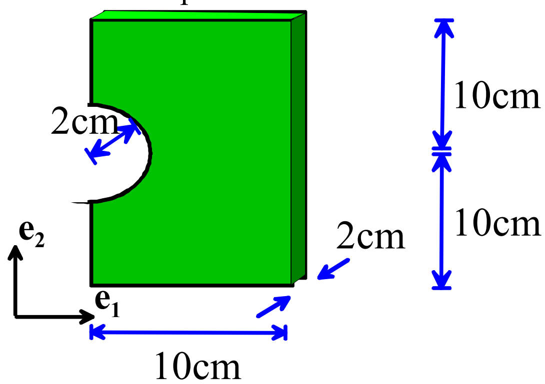
3. (6 points) ______________

4. (3 points) ______________

5. (4 points) ______________

TOTAL (25 points) ____________

1. You are asked to conduct a finite element analysis of an edge notched compact tension specimen, illustrated in the picture below. The specimen is made from steel, and will be loaded in tension by clamping the top and bottom edges of the specimen in the grips of a tensile testing machine. You are to determine the load that will initiate plastic deformation in the solid.



1.1 Write out the field equations that determine the solution (e.g. the equations of stress equilibrium, and other governing equations)

1.2 Draw a sketch showing the region of the specimen that you will model (if you plan to model the whole thing, sketch the entire specimen). Using your sketch, state the boundary conditions that you will apply.

1.3 Select an appropriate element type for your computation (you don't need to specify the ABAQUS name for the element, simply state, e.g. that you will use axisymmetric constant strain triangles)

1.4 Explain briefly how you would determine the load required to initiate plastic deformation from your finite element solution.

3. A solid with volume V is subjected to a displacement field (which corresponds to a homogeneous deformation) given by

$$u_1 = \lambda_1 x_1 + a_1 \qquad u_2 = \lambda_2 x_2 + a_2 \qquad u_2 = \lambda_3 x_3$$

Calculate formulas for

(3.1) The components of the deformation gradient tensor

(3.2) The volume of the solid after deformation

(3.3) The components of the Lagrange strain tensor

(3.4) The components of the infinitesimal strain tensor

(3.5) Using the Lagrange strain tensor, calculate the length after deformation of a line element that has unit length and direction $\mathbf{m} = (3\mathbf{e}_1 + 4\mathbf{e}_3)/5$ the undeformed solid

4. The matrix below gives components of a stress tensor (in MPa) in a basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$

$$\begin{bmatrix} 300 & -100 & 0 \\ -100 & 300 & 0 \\ 0 & 0 & 300 \end{bmatrix}$$

Calculate

(4.1) The hydrostatic stress

(4.2) The deviatoric stress tensor

(4.3) The principal stresses and the components of the principal stress directions in the $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ basis

(4.4) The Von-Mises stress

(4.5) The internal traction vector acting on an internal surface with normal $\mathbf{n} = (\mathbf{e}_1 + \mathbf{e}_2)/\sqrt{2}$

(4.6) The normal and tangential components of traction acting on an internal surface with normal $\mathbf{n} = (\mathbf{e}_1 + \mathbf{e}_2)/\sqrt{2}$

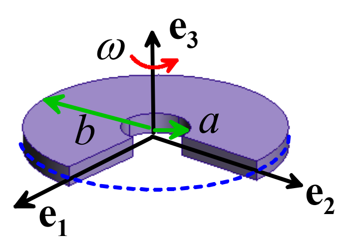
5. State whether the stress field given below satisfies the equations of static equilibrium

$$\sigma_{11} = A\left(x_1^2 x_2 - 2x_2^3/3\right) \qquad \sigma_{22} = A(x_2^3/3 - c^2 x_2 + 2c^3/3) \qquad \sigma_{12} = \sigma_{21} = A(c^2 - x_2^2)x_1$$

(with all other components zero).

6. The stress field in a *solid* disk with outer radius b, mass density ρ_0 and Poisson's ratio ν that spins with angular speed ω is given by

$$\sigma_{rr} = (3+\nu)\frac{\rho_0\omega^2}{8}\left\{b^2 - r^2\right\} \qquad \sigma_{\theta\theta} = \frac{\rho_0\omega^2}{8}\left\{(3+\nu)b^2 - (3\nu+1)r^2\right\}$$



The stress field in a thin disk with radius b with a hole of radius a in its center, subjected to pressure p_a on the interior of the hole, is given by

$$\sigma_{rr} = -p_a \frac{\left(b^2 - r^2\right)a^2}{\left(b^2 - a^2\right)r^2} \qquad \sigma_{\theta\theta} = p_a \frac{\left(b^2 + r^2\right)a^2}{\left(b^2 - a^2\right)r^2}$$

Use the principle of superposition to determine the stress state in a spinning disk with a (traction free) hole with radius a at its center.