School of Engineering
Brown University

EN175: Advanced Mechanics of Solids

Final Examination
Monday Dec 16 2019

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use two pages of reference notes
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

____

1. (10 points) ______________

2. (4 points) ______________

3. (7 points) ______________

4. (10 points) ______________

5. (12 points) ______________

6. (7 points) ______________

7. (10 points) ______________

TOTAL (60 points) ____________

1. Please answer the following general questions about FEA modeling briefly

1.1 What are the main differences between a small displacement/geometrically linear (NLGEOM OFF in ABAQUS) and a large displacement/geometrically nonlinear analysis (NLGEOM ON in ABAQUS)? (i.e. what approximations are made for NLGEOM OFF that are not made for NLGEOM ON)

[2 POINTS]

1.2 Give three reasons why a nonlinear static simulation may not converge

[2 POINTS]

1.3 Suggest a suitable choice of material model for each of the following applications (just name them, there is no need to give equations):

(a) Calculate stresses near the contact between two gear teeth

(b) Model material removal in an orthogonal machining process

(c) Model the rubber seal around a refrigerator door

[2 POINTS]

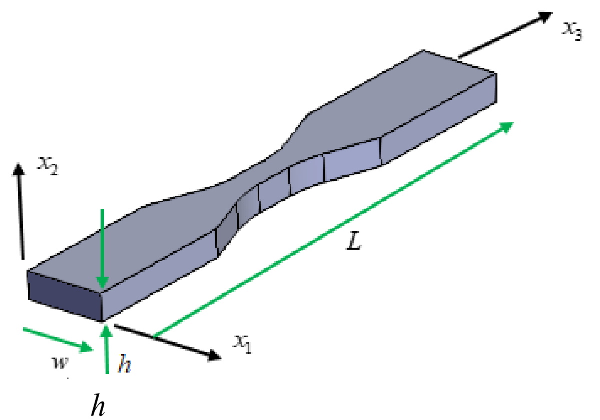
1.4 Explain briefly what is meant by a finite element interpolation.

[2 POINTS]

1.5 Explain the difference between a static and an explicit dynamic FEA simulation (explain the word ‘explicit’ as well as ‘dynamic’)

[2 POINTS]

2. The figure shows a tensile specimen. It is intended to be loaded in uniaxial tension parallel to the x_3 direction, with a constant engineering strain rate $\dot{E}_{33}$ applied to the specimen.



Specify boundary conditions that you would impose on the specimen if you were to conduct an FEA analysis of the tensile test.

(Identify points or planes where BCs are to be applied by their coordinates or equations; eg 'apply $u_1 = 5$ on $x_2 = 0$' etc. If a boundary condition is a function of time specify the function.)

[4 POINTS]

3. Consider the following displacement field

$$u_1 = \frac{x_1}{x_1^2 + x_2^2} \qquad u_2 = \frac{x_2}{x_1^2 + x_2^2} \qquad u_3 = 0$$

3.1 Calculate the nonzero components of the infinitesimal strain tensor

[3 POINTS]

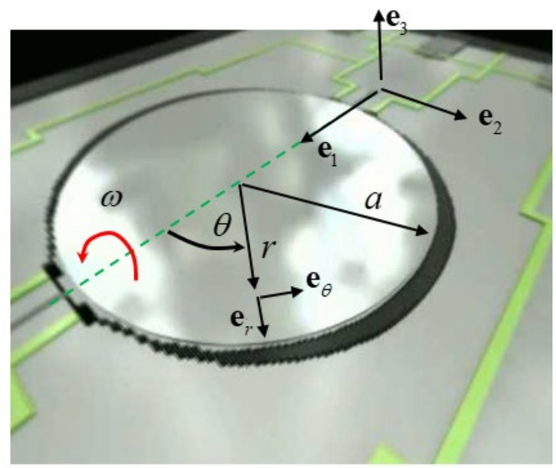
3.2 Show that the material is incompressible

[1 POINT]

3.3 Calculate the principal strains (as functions of x_1, x_2)

[3 POINTS]

4. The figure shows a MEMS mirror, which is a thin flat disk with radius a that spins at constant angular speed ω about the $\mathbf{e}_1$ axis. The mirror is made from a material with mass density ρ, Young's modulus E and Poisson's ratio ν



4.1 Write down the acceleration vector of a material particle at position (r,θ) in the disk, expressing your answer in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis shown, which rotates with the disk (this is an engn40 problem - note that every point in the disk is in circular motion about the axis of rotation and use the circular motion formulas)

[1 POINT]

4.2 The (steady state) stress state in the disk can be shown to be

$$\sigma_{rr} = C(a^2 - r^2)(8\sin^2\theta + \nu - 1)$$

$$\sigma_{\theta\theta} = C(a^2 - r^2)(8\sin^2\theta + \nu - 1) + 2C\left(4(a^2 - r^2)\cos 2\theta + r^2(1-\nu)\right)$$

$$\sigma_{r\theta} = 4C\sin 2\theta(a^2 - r^2)$$

where $C = \rho\omega^2/16$. (All other stress components are zero).

Show that the stress state satisfies the equations of linear momentum balance (use cylindrical polar coordinates)

[4 POINTS]

4.3 Show that the stress state satisfies boundary conditions at the edge of the disk (*r*=*a*)

[1 POINT]

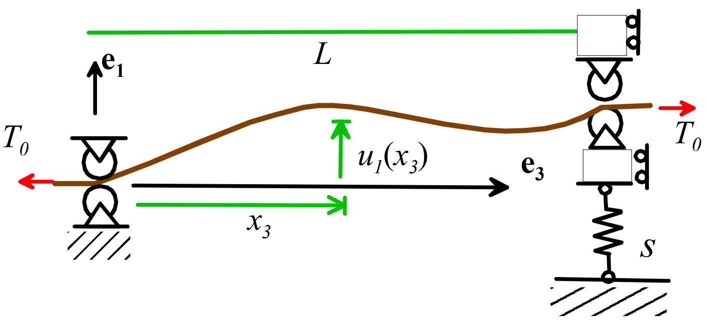
4.4 What other equations must be satisfied for the stress state to be a correct solution? (you don't need to show that the conditions are satisfied. You can just name the equations, you don't need to write them out)

[2 POINTS]

4.5 Suppose that the mirror is made from Si, which fails by fracture if the maximum principal stress exceeds a critical value σ_0 . Find a formula for the maximum admissible angular speed ω . You can assume that the critically stressed location is at $\theta = 0$, $r=0$.

[2 POINTS]

5. A cable with mass m per unit length is stretched by a tension T_0. The end at $x_3 = 0$ is pinned, while the support at $x_3 = L$ can move vertically, and is held in place by a spring with stiffness s The goal of this problem is to calculate the natural frequencies of vibration.



5.1 Write down the boundary condition for the transverse displacement u_1 of the cable at $x_3 = 0$.

[1 POINT]

5.2 Show that the boundary condition at $x_3 = L$ is

$$T_0 \frac{du_1}{dx_3} + s u_1 = 0$$

(assume small deflections, and that $u_1(L) = 0$ when the system is in its static equilibrium configuration).

[2 POINTS]

5.3 State the equation of motion for the cable (you don't need to derive it), and show that the standing wave solution $u_1 = \sin \omega t (A \sin kx_3 + B \cos kx_3)$ satisfies the equation. Give the relation between wave number *k,* natural frequency ω and wave speed *c*.

[3 POINTS]

5.4 Show that the natural frequencies are given by

$$\omega_n = \frac{\beta_n}{L}\sqrt{\frac{T_0}{m}}$$

where β_n are the roots of the equation

$$\beta_n \cos \beta_n + \frac{Ls}{T_0} \sin \beta_n = 0$$

[4 POINTS]

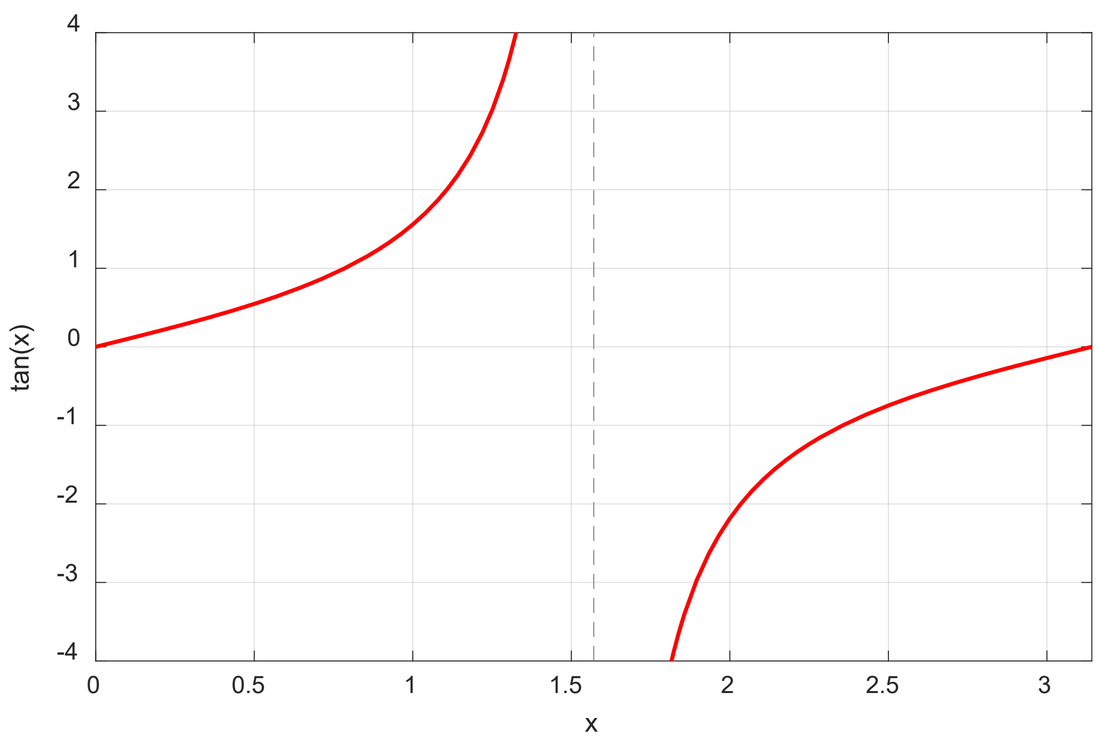
5.5 Find formulas (in terms of T_0, L, m) for the lowest natural frequency of the system for

(a) $\dfrac{Ls}{T_0} = 0$

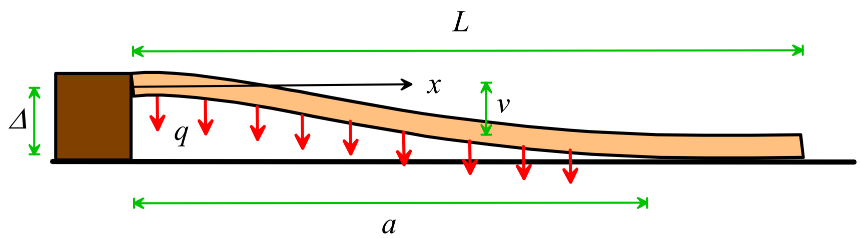
(b) $\dfrac{Ls}{T_0} \to \infty$

(c) $\dfrac{Ls}{T_0} = 1$

The plot of $\tan(x)$ provided in the figure may be helpful.



[2 POINTS]



6 The figure shows a MEMS cantilever beam with length L, Young's modulus E and area moment of inertia I. When straight, the beam is a height Δ above a surface. A constant attractive force per unit length q acts between the surface and the beam. As a result, the beam is bent, and a portion $a<x<L$ of the beam comes into contact with the surface, as indicated in the figure. The goal of this problem is to estimate the length a of the cantilever that is *not* in contact.

6.1 Show that

$$v = \begin{cases} \Delta(1-\cos(2\pi x/a))/2 & 0<x<a \\ \Delta & a<x<L \end{cases}$$

is a kinematically admissible (downward) deflection of the beam

[2 POINTS]

6.2 Find a formula for the potential energy of the beam, in terms of Δ, a, E, I, L, q (be sure to include the contribution to the potential energy from the section $a<x<L$. You can assume that this section is also subjected to the force per unit length q). You might find the integral

$$\int_0^a \cos^2\left(2\pi x/a\right)dx = a/2$$

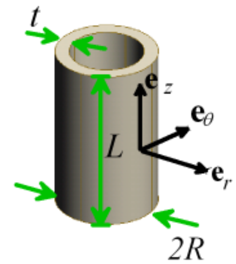
helpful.

[3 POINTS]

6.3 Hence, find a formula for a

[2 POINTS]

7. A cylindrical, thin-walled pressure vessel with close ends, initial radius *R,* length *L* and wall thickness *t*<<*R* is subjected to internal pressure *p*. The vessel is made from an isotropic elastic-plastic solid with Young's modulus *E*, Poisson's ratio ν, and its yield stress varies with accumulated plastic strain ε_e as $Y = Y_0 + h\varepsilon_e$.



Recall that the stresses in a thin-walled pressurized tube are related to the internal pressure by $\sigma_{zz} = pR/(2t)$, $\sigma_{\theta\theta} = pR/t$ $\quad \sigma_{rr} \approx 0$

7.1 Calculate the Von-Mises stress in the tube

[1 POINT]

7.2 Hence, find the critical value of internal pressure required to initiate yield in the solid (use the Von-Mises criterion)

[1 POINT]

7.3 Find a formula for the strain increment $d\varepsilon_{rr}, d\varepsilon_{\theta\theta}, d\varepsilon_{zz}$ resulting from an increment in pressure dp (neglect changes in the tube geometry)

[4 POINTS]

7.4 Suppose that the pressure is increased 10% above the initial yield value. Find a formula for the change in radius, length and wall thickness of the vessel. Assume small strains.

[4 POINTS]