School of Engineering
Brown University

EN175: Advanced Mechanics of Solids

Final Examination
Monday Dec 16 2019

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use two pages of reference notes
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1. (10 points)	______________
2. (4 points)	______________
3. (7 points)	______________
4. (10 points)	______________
5. (12 points)	______________
6. (7 points)	______________
7. (10 points)	______________
TOTAL (60 points)	______________

1. Please answer the following general questions about FEA modeling briefly

1.1 What are the main differences between a small displacement/geometrically linear (NLGEOM OFF in ABAQUS) and a large displacement/geometrically nonlinear analysis (NLGEOM ON in ABAQUS)? (i.e. what approximations are made for NLGEOM OFF)

NLGEOM Off: small strain measures are used (eg the infinitesimal strain tensor) and deformation is neglected when solving the equations of equilibrium or linear momentum balance (i.e. spatial derivatives are taken with respect to undeformed coordinates)

NLGEOM On: large strain measures are used (eg the Lagrange strain tensor) and deformation is considered when solving the equations of equilibrium or linear momentum balance (i.e. spatial derivatives are taken with respect to deformed coordinates)

[2 POINTS]

1.2 Give three reasons why a nonlinear static simulation may not converge

Some possible reasons (there are many others):

(1) Boundary conditions do not constrain rigid body modes
(2) The user is attempting to apply a load that exceeds the maximum load that the component can support (eg a load that exceeds the buckling load, or collapse load on the structure) so that a static solution does not exist
(3) The FE mesh is poorly designed, leading to a poorly conditioned stiffness matrix
(4) The FE mesh has become excessively distorted by deformation in the component
(5) The user has requested a step or increment size that is too large
(6) The assembly may contain a part that has unconstrained rigid body modes because it is not in contact with its neighbors in the initial step
(7) A highly deformable surface has been selected as master surface in a master/slave contact pair (master surface should be the more rigid of the two contacting solids)

[2 POINTS]

1.3 Suggest a suitable choice of material model for each of the following applications:

(a) Calculate stresses near the contact between two gear teeth

Linear elasticity

(b) Model material removal in an orthogonal machining process

An elastic-plastic material model with damage, e.g. Johnson-Cook

(c) Model the rubber seal around a refrigerator door

A large strain elasticity model, eg neo-hookean or more complex model; or possibly a finite strain viscoelastic material model

[2 POINTS]

1.4 Explain briefly what is meant by a finite element interpolation.

The goal of FEA is to calculate displacements (in solid mechanics, or more generally other field variables) at a set of discrete points (nodes) in a component. The FE interpolations allow displacements at arbitrary positions between the discrete points to be calculated. They do this by sub-dividing the solid into discrete volume elements. The displacement in each element depends only on the nodes attached to the element, and does not depend on displacements at other nodes.

[2 POINTS]

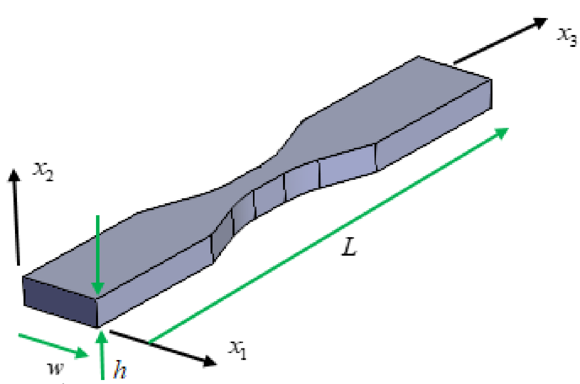
1.5 Explain the difference between a static and an explicit dynamic FEA simulation

A static analysis is calculating the displacements in a solid that is stationary, or deforming so slowly that its kinetic energy is negligible. It is solving the equations of static equilibrium. An explicit dynamic simulation is calculating time dependent displacements in a solid that is accelerating. It is solving the linear momentum conservation equation.

[2 POINTS]

2. The figure shows a tensile specimen. It is intended to be loaded in uniaxial tension parallel to the x_3 direction, with a constant engineering strain rate $\dot{E}_{33}$ applied to the specimen.

Specify boundary conditions that you would impose on the specimen if you were to conduct an FEA analysis of the tensile test.



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(Identify points or planes where BCs are to be applied by their coordinates or equations; eg 'apply $u_1 = 5$ on $x_2 = 0$' etc. If a boundary condition is a function of time specify the function.)

Apply, eg (other choices are possible):

- $u_1 = 0 \qquad x_3 = 0$
- $u_2 = u_1 = 0 \qquad x_1 = x_2 = x_3 = 0$ (i.e. at one point – to constrain rigid rotation)
- $u_2 = 0 \qquad x_1 = w \qquad x_2 = x_3 = 0$ (i.e. at one point – to constrain rigid rotation)
- $u_3 = \dot{E}_{33} L t \qquad x_3 = L$

[4 POINTS]

3. Consider the following displacement field

$$u_1 = \frac{x_1}{x_1^2 + x_2^2} \qquad u_2 = \frac{x_2}{x_1^2 + x_2^2} \qquad u_3 = 0$$

3.1 Calculate the nonzero components of the infinitesimal strain tensor

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = \frac{1}{x_1^2 + x_2^2} - \frac{2x_1^2}{(x_1^2 + x_2^2)^2} = \frac{x_2^2 - x_1^2}{(x_1^2 + x_2^2)^2}$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = \frac{1}{x_1^2 + x_2^2} - \frac{2x_2^2}{(x_1^2 + x_2^2)^2} = \frac{x_1^2 - x_2^2}{(x_1^2 + x_2^2)^2}$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) = -\frac{2x_1 x_2}{(x_1^2 + x_2^2)^2}$$

[3 POINTS]

3.2 Show that the material is incompressible

For an incompressible material $\varepsilon_{11} + \varepsilon_{22} = 0$, which is clearly satisfied.

[1 POINT]

3.3 Calculate the principal strains (as functions of x_1, x_2)

The principal strains are the eigenvalues of the strain tensor

$$\det\left(\frac{1}{\left(x_1^2 + x_2^2\right)^2}\begin{bmatrix} x_2^2 - x_1^2 & 2x_1x_2 \\ 2x_1x_2 & -(x_2^2 - x_1^2) \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\Rightarrow -\left(x_2^2 - x_1^2 - \left(x_1^2 + x_2^2\right)^2 \lambda^2\right)\left(x_2^2 - x_1^2 + \left(x_1^2 + x_2^2\right)^2 \lambda^2\right) - 4x_1^2 x_2^2 = 0$$

$$\Rightarrow \left(x_1^2 + x_2^2\right)^4 \lambda^4 - \left(x_2^2 - x_1^2\right)^2 + 4x_1^2 x_2^2 = 0$$

$$\Rightarrow \left(x_1^2 + x_2^2\right)^4 \lambda^4 - \left(x_2^2 + x_1^2\right)^2 = 0$$

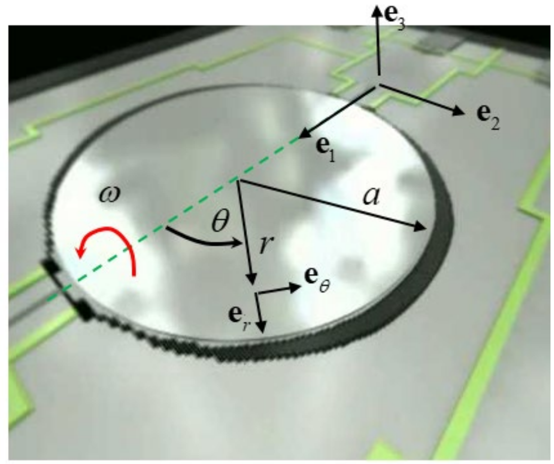
$$\Rightarrow \lambda = \pm \frac{1}{\sqrt{x_2^2 + x_1^2}}$$

Hence $\varepsilon_1 = \dfrac{1}{\sqrt{x_2^2 + x_1^2}} \qquad \varepsilon_2 = -\dfrac{1}{\sqrt{x_2^2 + x_1^2}}$

(As a check note that $\varepsilon_1 + \varepsilon_2 = 0$)

[3 POINTS]

4. The figure shows a MEMS mirror, which is a thin flat disk with radius a that spins at constant angular speed ω about the $\mathbf{e}_1$ axis. The mirror is made from a material with mass density ρ, Young's modulus E and Poisson's ratio ν



4.1 Find the acceleration vector of a material particle at position (r,θ) in the disk, expressing your answer in the $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis shown, which rotates with the disk (this is an engn40 problem - note that every point in the disk is in circular motion about the axis of rotation and use the circular motion formulas)

$$\mathbf{a} = -\omega^2 r \sin\theta(\mathbf{e}_r \sin\theta + \mathbf{e}_\theta \cos\theta)$$

[1 POINT]

4.2 The (steady state) stress state in the disk can be shown to be

$$\sigma_{rr} = C(a^2 - r^2)(8\sin^2\theta + \nu - 1)$$
$$\sigma_{\theta\theta} = C(a^2 - r^2)(8\sin^2\theta + \nu - 1) + 2C\left(4(a^2 - r^2)\cos 2\theta + r^2(1-\nu)\right)$$
$$\sigma_{r\theta} = 4C\sin 2\theta(a^2 - r^2)$$

where $C = \rho\omega^2/16$. (All other stress components are zero).

Show that the stress state satisfies the equations of linear momentum balance (use cylindrical polar coordinates)

The linear momentum balance equations are

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} = \rho a_r$$
$$\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial\sigma_{r\theta}}{\partial r} + 2\frac{\sigma_{r\theta}}{r} = \rho a_\theta$$

where

$$\rho a_r = -\rho\omega^2 r\sin^2\theta = -16Cr\sin^2\theta$$
$$\rho a_\theta = -\omega^2\rho r\sin\theta\cos\theta = -16Cr\sin\theta\cos\theta$$

Substituting the given stress field:

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{1}{r}(\sigma_{rr}-\sigma_{\theta\theta})+\frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta}$$
$$=-2rC\left(8\sin^2\theta+\nu-1\right)-8\frac{C}{r}(a^2-r^2)\cos 2\theta-2Cr(1-\nu)+8\frac{C}{r}(a^2-r^2)\cos 2\theta$$
$$=-16Cr\sin^2\theta$$
$$\frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta}+\frac{\partial \sigma_{r\theta}}{\partial r}+2\frac{\sigma_{r\theta}}{r}$$
$$=\frac{8C}{r}(a^2-r^2)2\sin\theta\cos\theta-\frac{16C}{r}(a^2-r^2)\sin 2\theta-8Cr\sin 2\theta+\frac{8C}{r}(a^2-r^2)\sin 2\theta$$
$$=-16Cr\sin\theta\cos\theta$$

[4 POINTS]

4.3 Show that the stress state satisfies boundary conditions at the edge of the disk (*r=a*)

The boundary conditions are $\sigma_{rr}=\sigma_{r\theta}=0$ This is clearly satisfied

[1 POINT]

4.4 What other conditions must be satisfied for the stress state to be a correct solution? (you don't need to show that the conditions are satisfied)

The stress state must be related to the strains by the elastic stress-strain relations, and the strain field must be compatible.

[2 POINTS]

4.5 Suppose that the mirror is made from Si, which fails by fracture if the maximum principal stress exceeds a critical value σ_0 . Find a formula for the maximum admissible angular speed ω . You can assume that the critically stressed location is at $\theta=0$, *r=0*.

At *r=0* , $\theta=0$ we have

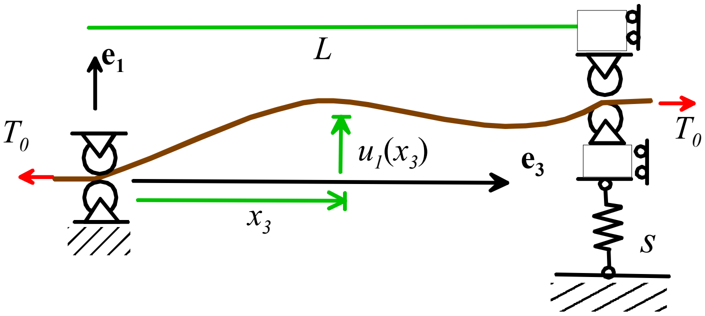
$$\sigma_{rr}=Ca^2(\nu-1)$$
$$\sigma_{\theta\theta}=Ca^2(7+\nu)$$
$$\sigma_{r\theta}=0$$

Since the stress is diagonal $\sigma_{rr},\sigma_{\theta\theta}$ are the principal stresses. At the critical angular speed

$$\rho\omega^2a^2(7+\nu)/16=\sigma_0 \Rightarrow \omega=\frac{4}{a}\sqrt{\frac{\sigma_0}{\rho(7+\nu)}}$$

[2 POINTS]

5. A cable with mass m per unit length is stretched by a tension T_0 . The end at $x_3 = 0$ is pinned, while the support at $x_3 = L$ can move vertically, and is held in place by a spring with stiffness s The goal of this problem is to calculate the natural frequencies of vibration.



5.1 Write down the boundary condition for the transverse displacement u_1 of the cable at $x_3 = 0$.

The left hand end is pinned so $u_1 = 0$

[1 POINT]

5.2 Show that the boundary condition at $x_3 = L$ is

$$T_0 \frac{du_1}{dx_3} + s u_1 = 0$$

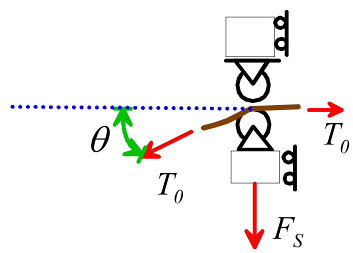
(assume small deflections, and that $u_1(L) = 0$ when the system is in its static equilibrium configuration).

One way to show this is to draw a FBD for the support at the right – summing forces in the vertical direction gives

$$-T_0 \sin\theta - F_s = 0$$

Noting that $\sin\theta \approx \theta = \dfrac{du_1}{dx_3}$ $\quad F_s = s u_1(L)$

then gives the stated answer



[2 POINTS]

5.3 State the equation of motion for the cable (you don't need to derive it), and show that the standing wave solution $u_1 = \sin\omega t(A\sin kx_3 + B\cos kx_3)$ satisfies the equation. Give the relation between wave number *k,* natural frequency ω and wave speed *c*.

The governing equation is the wave equation

$$\frac{\partial^2 u_1}{\partial x_3^2} = \frac{1}{c^2}\frac{\partial^2 u_1}{\partial t^2}$$

Where $c = \sqrt{\frac{T_0}{m}}$

Substituting the given solution gives

$$-k^2 \sin\omega t(A\sin kx + B\cos kx) = -\frac{\omega^2}{c^2}\sin\omega t(A\sin kx + B\cos kx)$$

So if we pick

$$k = \frac{\omega}{c}$$

the equation is satisfied

[3 POINTS]

5.4 Show that the natural frequencies are given by

$$\omega_n = \frac{\beta_n}{L}\sqrt{\frac{T_0}{m}}$$

where β_n are the roots of the equation

$$\beta_n \cos\beta_n + \frac{Ls}{T_0}\sin\beta_n = 0$$

The boundary conditions can be expressed as

$$\begin{bmatrix} 0 & 1 \\ kT_0\cos kL + s\sin kL & -kT_0\sin kL + s\cos kL \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The determinant of the matrix must vanish for a nontrivial solution, which gives

$$T_0 k\cos kL + s\sin kL = 0$$

Set $kL = \beta$

$$\frac{T_0\beta}{L}\cos\beta + s\sin\beta = 0 \Rightarrow \beta\cos\beta + \frac{sL}{T_0}\sin\beta = 0$$

Finally using the dispersion relation gives

$$\frac{\omega L}{c} = \beta \Rightarrow \omega = \beta\frac{c}{L} = \frac{\beta}{L}\sqrt{\frac{T_0}{m}}$$

[4 POINTS]

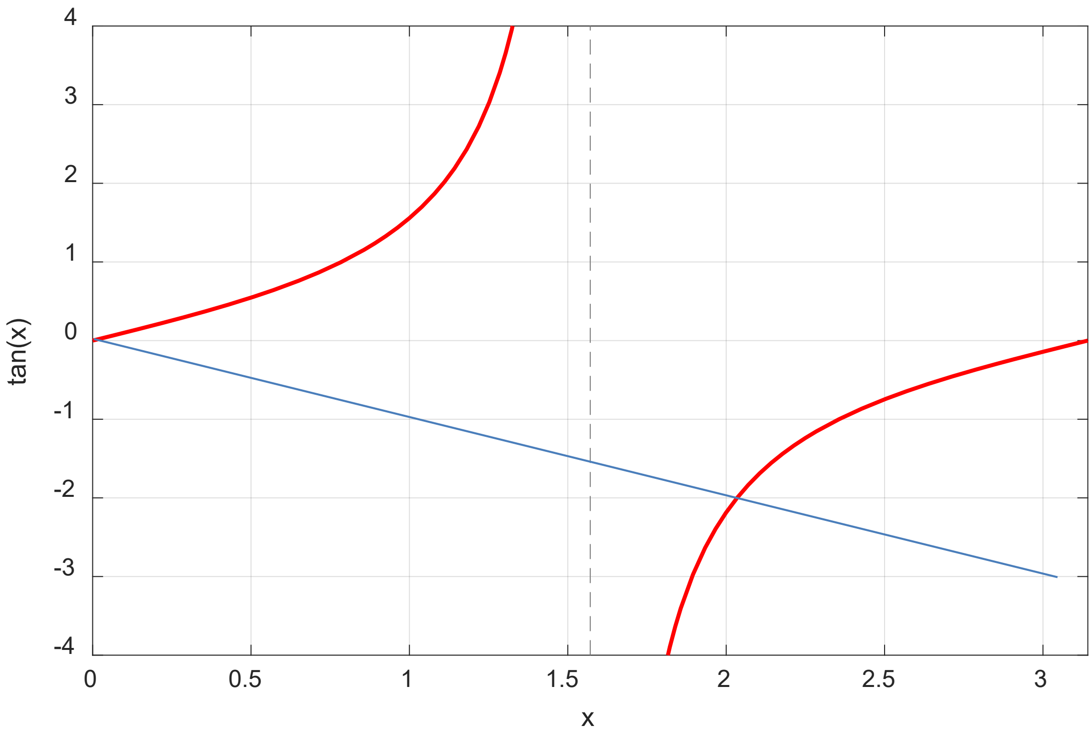
5.5 Find formulas (in terms of T_0, L, m) for the lowest natural frequency of the system for

(a) $\frac{Ls}{T_0} = 0$

(b) $\frac{Ls}{T_0} \to \infty$

(c) $\frac{Ls}{T_0} = 1$

The plot of $\tan(x)$ provided in the figure may be helpful.

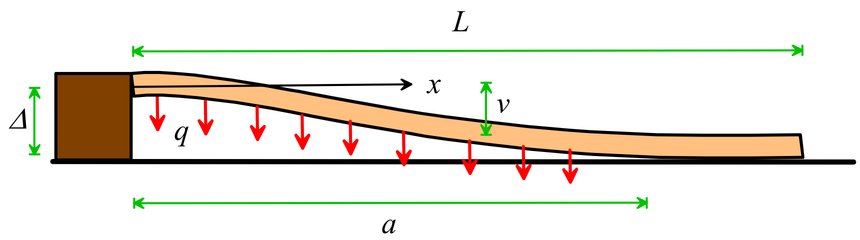


For $\frac{Ls}{T_0} = 0$ we require $\cos\beta_n = 0 \Rightarrow \beta_1 = \pi/2 \Rightarrow \omega_1 = \frac{\pi}{2L}\sqrt{\frac{T_0}{m}}$

For $\frac{Ls}{T_0} \to \infty$ we require $\sin\beta_n = 0 \Rightarrow \beta_1 = \pi \Rightarrow \omega_1 = \frac{\pi}{L}\sqrt{\frac{T_0}{m}}$

For $\frac{Ls}{T_0} = 1$ we require $\tan\beta_1 = -\beta_1 \Rightarrow \beta_1 \approx 2.05 \Rightarrow \omega_1 = \frac{2.03}{L}\sqrt{\frac{T_0}{m}}$ (using the graph)

[2 POINTS]



6 The figure shows a MEMS cantilever beam with length L, Young's modulus E and area moment of inertia I. When straight, the beam is a height Δ above a surface. A constant attractive force per unit length q acts between the surface and the beam. As a result, the beam is bent, and a portion $a<x<L$ of the beam comes into contact with the surface, as indicated in the figure. The goal of this problem is to estimate the length a of the cantilever that is *not* in contact.

6.1 Show that

$$v = \begin{cases} \Delta(1-\cos(2\pi x/a))/2 & 0<x<a \\ \Delta & a<x<L \end{cases}$$

is a kinematically admissible (downward) deflection of the beam

The boundary conditions are

$$v = dv/dx = 0 \qquad x = 0$$
$$v = \Delta \qquad dv/dx = 0 \qquad x > a$$

The stated displacement satisfies all these. The displacement is also continuous, has continuous slope, and is twice differentiable.

[2 POINTS]

6.2 Find a formula for the potential energy of the beam, in terms of Δ, a, E, I, L, q (be sure to include the contribution to the potential energy from the section $a<x<L$. You can assume that this section is also subjected to the force per unit length q). You might find the integral

$$\int_0^a \cos^2\left(2\pi x/a\right)dx = a/2$$

helpful.

We have that

$$\Pi = \int_0^L \frac{1}{2}EI\left(\frac{d^2v}{dx^2}\right)^2 dx - \int_0^L qv(x)dx$$
$$= \int_0^a \frac{1}{2}EI\left(\frac{\Delta}{2}\frac{4\pi^2}{a^2}\cos\frac{2\pi x}{a}\right)^2 dx - \int_0^a q\frac{\Delta}{2}\left(1-\cos\frac{2\pi x}{a}\right)dx - \int_a^L \Delta q dx$$
$$= \frac{\Delta^2 EI\pi^4}{a^3} - \frac{q\Delta a}{2} - \Delta q(L-a) = \frac{\Delta^2 EI\pi^4}{a^3} + \frac{q\Delta a}{2} - \Delta qL$$

[3 POINTS]

6.3 Hence, find a formula for a

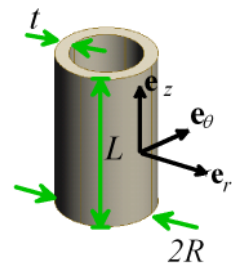
We want to minimize Π so

$$\frac{\partial \Pi}{\partial a} = -3\frac{\Delta^2 EI\pi^4}{a^4} + \frac{q\Delta}{2} = 0$$

$$\Rightarrow a = \pi\left(\frac{6\Delta EI}{q}\right)^{1/4}$$

[2 POINTS]

7. A cylindrical, thin-walled pressure vessel with close ends, initial radius R, length L and wall thickness $t<<R$ is subjected to internal pressure p. The vessel is made from an isotropic elastic-plastic solid with Young's modulus E, Poisson's ratio ν, and its yield stress varies with accumulated plastic strain ε_e as $Y = Y_0 + h\varepsilon_e$.



Recall that the stresses in a thin-walled pressurized tube are related to the internal pressure by $\sigma_{zz} = pR/(2t)$, $\sigma_{\theta\theta} = pR/t$ $\quad \sigma_{rr} \approx 0$

7.1 Calculate the Von-Mises stress in the tube

$$\sigma_e = \sqrt{\frac{1}{2}\left\{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right\}} = \frac{pR}{t}\sqrt{\frac{1}{2}\left\{(1-\frac{1}{2})^2+(1-0)^2+(\frac{1}{2}-0)^2\right\}} = \frac{\sqrt{3}pR}{2t}$$

[1 POINT]

7.2 Hence, find the critical value of internal pressure required to initiate yield in the solid (use the Von-Mises criterion)

Use the Von-Mises yield criterion

$$\sigma_e = \sqrt{\frac{1}{2}\left\{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right\}} = \frac{pR}{t}\sqrt{\frac{1}{2}\left\{(1-\frac{1}{2})^2+(1-0)^2+(\frac{1}{2}-0)^2\right\}} = \frac{\sqrt{3}pR}{2t} = Y_0$$

$$\Rightarrow p = \frac{2Y_0 t}{\sqrt{3}R}$$

[1 POINT]

7.3 Find a formula for the strain increment $d\varepsilon_{rr}, d\varepsilon_{\theta\theta}, d\varepsilon_{zz}$ resulting from an increment in pressure dp (neglect changes in the tube geometry)

Below yield the strains are elastic

$$\frac{d\varepsilon_{ij}}{dt} = \frac{d\varepsilon_{ij}^e}{dt} + \frac{d\varepsilon_{ij}^p}{dt} + \frac{d\varepsilon_{ij}^T}{dt} = \frac{1+\nu}{E}\left(\frac{d\sigma_{ij}}{dt} - \frac{\nu}{1+\nu}\frac{d\sigma_{kk}}{dt}\delta_{ij}\right) \qquad p < \frac{2Y_0 t}{\sqrt{3}R}$$

$$\Rightarrow \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \end{bmatrix} = \frac{1+\nu}{E}\frac{R}{2t}\frac{d}{dt}\begin{bmatrix} 0 \\ 2p \\ p \end{bmatrix} - \frac{\nu}{E}\frac{R}{2t}\frac{d}{dt}\begin{bmatrix} p \\ p \\ p \end{bmatrix} = \frac{R}{2Et}\begin{bmatrix} -\nu dp/dt \\ (2-\nu)dp/dt \\ dp/dt \end{bmatrix} \qquad p < \frac{2Y_0 t}{\sqrt{3}R}$$

(or can use matrix formulas from class directly).

If yield is exceeded, we can use the general formula

$$\frac{d\varepsilon_{ij}}{dt} = \frac{d\varepsilon_{ij}^e}{dt} + \frac{d\varepsilon_{ij}^p}{dt} + \frac{d\varepsilon_{ij}^T}{dt} = \frac{1+\nu}{E}\left(\frac{d\sigma_{ij}}{dt} - \frac{\nu}{1+\nu}\frac{d\sigma_{kk}}{dt}\delta_{ij}\right) + \frac{3}{2}\frac{\left\langle S_{kl}\frac{d\sigma_{kl}}{dt}\right\rangle}{h\sigma_e}\frac{3}{2}\frac{S_{ij}}{\sigma_e} + \frac{d\Delta T}{dt}\alpha\delta_{ij}$$

The hydrostatic stress is $\frac{1}{3}\frac{pR}{2t}(2+1)=\frac{pR}{2t}$

The deviatoric stress is $S_{rr}=-\frac{pR}{2t} \qquad S_{zz}=0 \qquad S_{\theta\theta}=\frac{pR}{2t}$

The Von Mises stress is $\sigma_e=\frac{\sqrt{3}pR}{2t}$

Therefore

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\end{bmatrix}=\frac{1+\nu}{E}\frac{R}{2t}\frac{d}{dt}\begin{bmatrix}0\\ 2p\\ p\end{bmatrix}-\frac{\nu}{E}\frac{R}{2t}\frac{d}{dt}\begin{bmatrix}p\\ p\\ p\end{bmatrix}+\frac{3}{2}\frac{\langle (pR/2t)d(pR/t)/dt\rangle}{h\left[\sqrt{3}pR/(2t)\right]^2}\frac{3}{2}\frac{pR}{2t}\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}$$

$$=\frac{R}{2Et}\begin{bmatrix}-\nu dp/dt\\ (2-\nu)dp/dt\\ dp/dt\end{bmatrix}+\frac{3R}{4t}\frac{1}{h}\frac{dp}{dt}\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}$$

(or use matrix formulas from class directly)

[4 POINTS]

7.4 Suppose that the pressure is increased 10% above the initial yield value. Find a formula for the change in radius, length and wall thickness of the vessel. Assume small strains.

We need to do this calculation in two steps.

Before the tube reaches yield, it deforms elastically. The strains at the point of yield are

$$\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\end{bmatrix}=\frac{R}{2Et}\begin{bmatrix}-\nu\\ (2-\nu)\\ 1\end{bmatrix}\frac{2Y_0t}{\sqrt{3}R}=\frac{Y_0}{\sqrt{3}E}\begin{bmatrix}-\nu\\ (2-\nu)\\ 1\end{bmatrix}$$

We now integrate the results of 7.3 to find the total strain after the tube exceeds yield

$$\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\end{bmatrix}=\frac{R}{2Et}\begin{bmatrix}-\nu\\ (2-\nu)\\ 1\end{bmatrix}\frac{2(1.1)Y_0t}{\sqrt{3}R}+\frac{3R}{4t}\frac{1}{h}\frac{2(0.1)Y_0t}{\sqrt{3}R}\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}=\frac{1.1Y_0}{\sqrt{3}E}\begin{bmatrix}-\nu\\ (2-\nu)\\ 1\end{bmatrix}+\frac{\sqrt{3}}{2}\frac{(0.1)Y_0}{h}\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}$$

We can use the strains to calculate the changes in thickness, radius and length:

$$\varepsilon_{rr}=\Delta t/t \qquad \varepsilon_{\theta\theta}=\Delta R/R \qquad \varepsilon_{zz}=\Delta L/L$$

Therefore

$$\begin{bmatrix}\Delta t\\ \Delta R\\ \Delta L\end{bmatrix}==\frac{1.1Y_0}{\sqrt{3}E}\begin{bmatrix}-\nu t\\ (2-\nu)R\\ L\end{bmatrix}+\frac{\sqrt{3}}{2}\frac{(0.1)Y_0}{h}\begin{bmatrix}-t\\ R\\ 0\end{bmatrix}$$

[4 POINTS]