



School of Engineering
Brown University

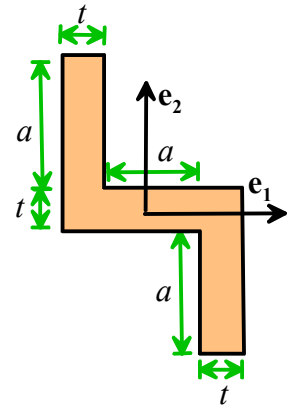
EN1750: Advanced Mechanics of Solids

Homework 7: Strings, Beams, Membranes and Plates.

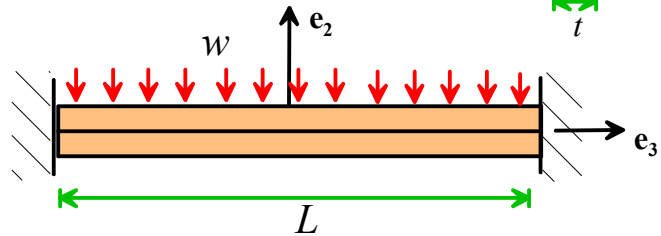
Due Friday Nov 16, 2018

1. The figure shows the cross-section of a beam.

1.1 Calculate the area moment of inertia tensor for the beam section shown (simplify the solution by assuming $t \ll a$) (you can use the matlab script from class).

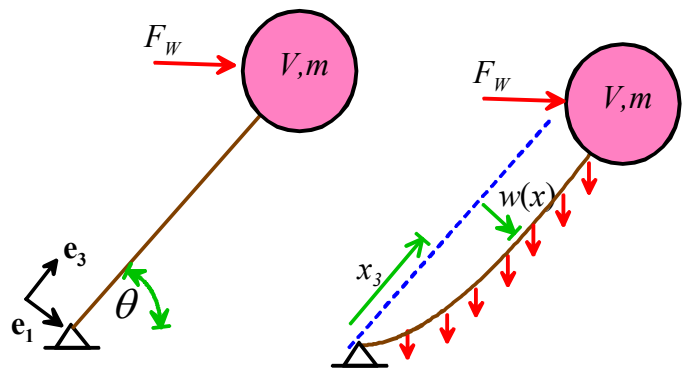


1.2 Suppose that a beam with length L and cross-section analyzed in 1.1 is subjected to a uniform load w acting in the (negative) e_2 direction. Calculate the deflection of the beam at mid-span (you can use the Matlab script from class; since this is a statically indeterminate problem it's hard to do the calculation by hand, but the boundary conditions are very simple and can all be fed into the matlab dsolve function.)



2. The figure shows a spherical balloon with volume V and mass m tethered by a cable with mass μ per unit length. The balloon is subjected to a lateral wind load F_W

2.1 For the limiting case of a massless cable ($\mu = 0$), the cable remains straight and has a constant internal tension T_3 . Find formulas for the angle θ and T_3 in terms of V, m, F_W and air density ρ . This is just a statics problem for the balloon; use Archimedes principle to find the buoyancy force on the balloon.



2.2 If the cable mass is significant the cable will deflect as shown in the figure. Assuming that the cable is in static equilibrium, write down

- The differential equation governing the variation of tension T_3 in the cable. Neglect wind force acting on the cable, but include gravity – this is the vertical force shown in the figure.
- The boundary condition for T_3 at $x_3 = L$

- The differential equation for the deflection w
- The boundary conditions for w at $x_3 = 0$ and $x_3 = L$

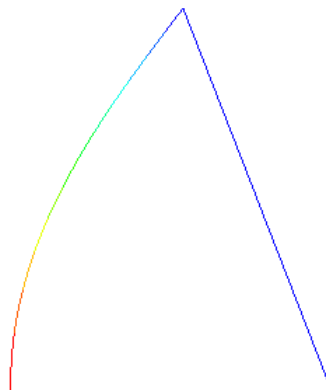
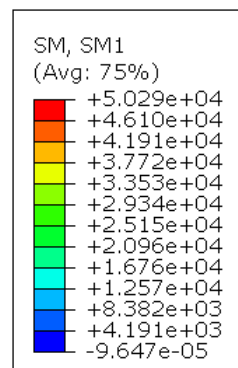
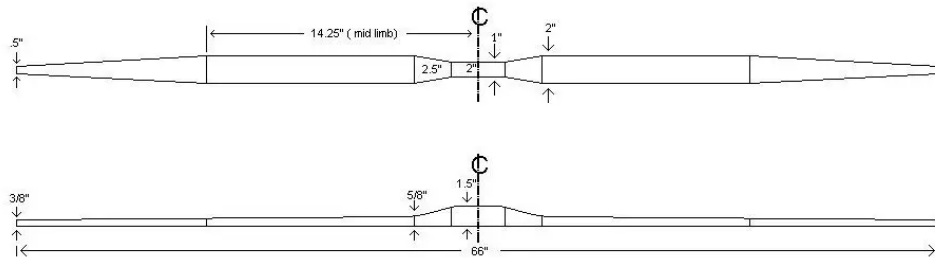
2.3 Hence, solve the equation system in the preceding section to show that the cable deflection w is

$$w = \frac{b}{c} \left\{ -x_3 + \frac{T_0}{c} \log \frac{T_0 - c(L - x_3)}{T_0 - cL} \right\}$$

$$c = \mu g \sin \theta \quad b = \mu g \cos \theta \quad T_0 = \sqrt{F_W^2 + (\rho g V - mg)^2}$$

Your solution should follow these steps:

- Find a formula for the axial tension T_3 as a function of x_3 by integrating the first equation from (2.1) and using the boundary condition.
- Substitute your formula for T_3 into your differential equation for w , and integrate it once to find dw/dx_3 (this is easy to do by hand). Use the boundary condition for w at $x_3 = L$ to eliminate the constant of integration.
- Integrate again (a symbolic manipulation program will help here; do an indefinite integral and solve for the constant of integration from the boundary condition for w at $x_3 = 0$)



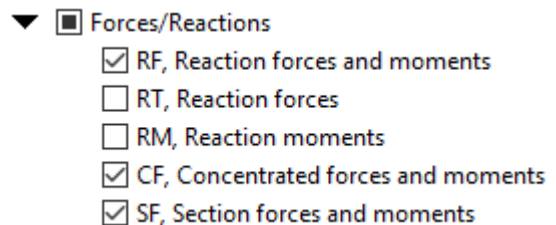
3. In this problem you will explore the beam elements available in ABAQUS by setting up a simple finite element simulation to predict the static force-v-draw curve for an English longbow. Dimensions of a simple home-made bow (from [this reference](#)) are shown in the figure.

3.1 Because of symmetry, we only need to model half the bow. Create a 3D deformable part with wire base feature, with one end at the origin and the other at $y=830\text{mm}$ (the undeformed bow is straight; it gets bent when the string is attached). Note that this procedure will compute *half* the total draw force (there will be an additional force from the bottom half of the bow that has not been modeled).

3.2 Use a Young's modulus for Yew wood of 6.5 GPa (convert to N/mm^2 since lengths are in mm), and make the cross-section a $40\text{mm}\times 12\text{mm}$ rectangle (it is possible to create tapered cross-sections in ABAQUS/CAE but you have to define the taper of every element in the beam separately, which is extremely tedious to do by hand and best done with a python script). Use Assign> Beam section orientation> to specify the orientation of the cross-section of the beam – make the normal to the flat side of the bow parallel to the x direction. You have to enter a unit vector specifying the orientation of one of the local axes for the x-section; the data you enter will depend on the orientation of the rectangle in your x-sect (i.e. whether the base or the height is the longer dimension).


3.3 Create a second part for the bow string. Make one end of the string coincident with the tip of the bow, and make the string 40mm shorter than the bow. Give the bow string a modulus of 110 GPa and a circular cross section with radius 0.72mm .

3.4 Create two steps, both with 1 sec duration – these will be used to string the bow, and then draw it. Make sure NLGEOM is selected for both steps. Make the initial and maximum increment size 0.1s for both steps. In the step module for the first step, open the Field Output Request manager and add requests to plot the section forces/moments. Also, (i) create a new Set (use Tools > Set > Create... name the set drawpoint, Select Geometry, and select the bottom end of the bowstring); then (ii) create a history output request to request displacements/velocity/acceleration and forces/reactions for this set (this will be used to plot the force-v-draw curve).



3.5 To tie the bowstring to the bow, select Constraint > Create ..., select a tie constraint; in the menu below the viewport select Node Region, then select the end points of the string and bow as the master and slave (it doesn't matter which is which). If you need to, you can right click a part under the 'instance' branch of the model tree to hide it. In the 'edit constraint' menu uncheck the 'Tie rotational DOFs if applicable' option. This will allow the string to rotate freely relative to the end of the bow.

3.6 In the boundary condition module, create a BC for the 'initial' step that makes all the degrees of freedom (U and UR) to zero for the node on the bow that lies at the origin. This BC will be applied throughout the analysis by default.

3.7 We need to define boundary conditions for the two load steps. The first step will be used to string the bow. To do this we need to apply a force to bend the bow, and pull the string down so its bottom end lies at $y=0$. Apply a horizontal force (acting in the positive x direction) to the bow at its tip with magnitude 20 N (if your bow is vertical, the force will be horizontal). Click the  button next to the drop down menu to define an amplitude for this force; check the 'tabular' radio button, and enter a table that starts at 1 at time $t=0$ and ends at 0 at time $t=1$. Then select Amp-1 in the dropdown menu. This will bend the bow so it can be pulled down by the bowstring, but will remove the force by the end of the step. Next, create a BC that will apply a displacement to the free end of the bowstring that will pull it down to be level with the base of the bow (this will be $U2=-40\text{mm}$). A standard 'Ramp' amplitude will work. Finally open both the Load and BC managers and deactivate the two BCs created for step 1 in step 2.

3.8 To draw the bow, (i) create a BC that will fix U2 at the bottom of the bowstring to remain constant in the second step. To do this open the BC manager, and make sure the BC you defined at the end of the string in Step 1 is deactivated in Step 2. Then create a new BC, select the bottom end of the bowstring, select 'Fixed at Current Position' in the Edit BC window, and check the box for U2. Next, create a second BC that will apply a constant horizontal velocity (V1) of 200 mm/sec to the bottom end of the bowstring. This will draw the bow by 200mm by the end of the step.

3.9 In the mesh module seed the bow and string with 10mm/20mm mesh size, respectively, and mesh both parts with the default element type.

3.10 Submit and run the job. If all works correctly you should see a deformed shape and bending moment distribution that looks something like the picture on the previous page. If you would like to visualize the stress distribution in the bow and see the cross-section, you can go to View> ODB Display Options and then check the box showing Render Beam Profiles. Then you can go back to Result > Field Output and select BEAM_STRESS and select the stress component you want. You may also need to go back to the Step module, and use Output Request > Field Output Requests > to select additional stress components if they are not available in the menu (see step 3.4). Of course if you do this you have to re-run the job... As discussed in class, the shear stresses S12 and S13 will be nonzero in shear flexible beams, but will be zero in the Euler-Bernoulli version (which appears as the 'cubic formulation' in the element type in ABAQUS).

3.11 Create and hand in a plot of the force-v-draw curve for the bow. (Be sure to remember to double the value that comes out of ABAQUS, because you only modeled half the bow). For a rough comparison, you can find measured force-v-draw curves for a few bows on [this page](#) – but those bows are made from different materials and have a different cross-section to the one analyzed here (which seems to be a rather light bow).