



School of Engineering
Brown University

EN1750: Advanced Mechanics of Solids

Homework 1: Introduction to finite element analysis

Due Friday Sept 20, 2019

1. Please answer the following general questions about finite element analysis

- a. Choose a suitable material model from the list below for each of the following applications:
- (i) Calculating the natural frequencies of vibration of a wind-turbine blade
 - (ii) Calculating the energy absorbed by a crash-rail in a vehicle
 - (iii) Calculating the deformation of the eye induced by a noncontact 'air puff' tonometer
 - (iv) Calculating the contact pressure between an elastomeric aircraft door seal and the doorframe

Material models: (a) Linear elastic; (b) Hyperelastic; (c) Viscoelastic; (d) Elastic-plastic

- (i) -> (a) (deformations and stresses are expected to be small; natural frequencies are also not usually a well-defined quantity for solids made of nonlinear materials)
- (ii) -> (d); crash-rails are designed to dissipate energy through permanent deformation
- (iii) -> (c) most tissue has a time-dependent elastic response
- (iv) -> (b); door seals experience large deformations and a suitable large strain elasticity model would be needed

[2 POINTS]

- b. Two linear elasto-static analyses are conducted of the same part, with the same positions and numbers for the nodes, but in one simulation the displacement field is interpolated using linear elements; in the other, quadratic elements are used. Which analysis is likely to give the more accurate solution, and why?

Quadratic elements are likely to be slightly more accurate as they will give a closer approximation to the smooth displacement fields we expect to see in a linear elastic solution

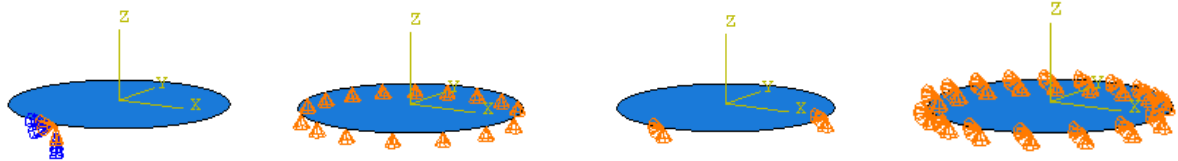
[1 POINT]

- c. What is the 'default' boundary condition at the exterior of a finite element mesh (i.e. if no boundary conditions are entered for a node/element face, what loading or displacements will be applied in the simulation?)

The boundary is free of traction (no forces are applied to the boundary)

[1 POINT]

d. The circular plate shown in the figure is meshed with plate elements for a static analysis (the plate will be loaded by pressure and in-plane traction applied to its surface). Indicate which of the analyses will fail and which will succeed



$$U1=U2=U3=0$$

$$UR1=UR2=UR3=0$$

(a)

$$U3=0$$

(b)

$$U1=U2=U3=0$$

(c)

$$U1=U2=U3=0$$

(d)

(a) is OK; (b) has three unconstrained rigid body modes and will fail; (c) has one unconstrained rigid body mode (rotation about the line connecting the two pinned points) and will fail; (d) is OK.

[2 POINTS]

e. If a dynamic simulation is conducted of the plates described in problem (d) which analyses will fail and which will succeed?

The analyses will all work; $\mathbf{F}=\mathbf{ma}$ always has a solution regardless of boundary conditions.

[1 POINT]

f. Explain what is meant by (i) ‘Newton iterations’ in a static nonlinear finite element finite element simulation. List three reasons why Newton iterations may not converge.

The discrete approximations to the nonlinear PDEs (either geometric or material nonlinearity) that govern the deformation of a solid have the form $\mathbf{R}(\mathbf{u}) = \mathbf{f}$ where \mathbf{u} is a (large) vector of unknown displacements; \mathbf{R} is a (same size) vector of generalized forces on each node; and \mathbf{f} is a set of external (or internal) forces acting on the nodes. These are a set of nonlinear equations, which are solved by Newton-Raphson iteration.

If the analysis includes contact between two surfaces, additional iterations may be required to determine the area of contact. These look similar to Newton iterations.

Iterations may fail to converge because

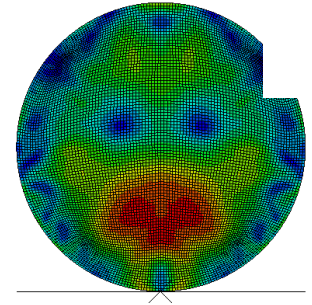
(i) Governing equations are ill conditioned – eg because of improper boundary conditions; severely distorted elements; or an attempt to analyze deformation of an incompressible material using elements that are not designed for this purpose

(ii) The contact iterations may be unable to satisfy constraint equations at contacts

(iii) A static solution may not exist – eg because of material failure or buckling.

[2 POINTS]

- g. The figure shows a finite element simulation of an elastic cylinder with radius R , mass density ρ , Young's modulus E and Poisson's ratio ν colliding with a frictionless rigid surface. The goal of the simulation is to calculate the contact time T as a function of material properties and geometry. Re-write the relationship $T = f(E, \rho, \nu, R)$ in dimensionless form.



The variables have units

$$T - s$$

$$R - m$$

$$E - N / m^2 = (kgm / s^2) / m^2 = kg / (ms^2)$$

$$\rho - kg / m^3$$

$$\nu - \text{dimensionless}$$

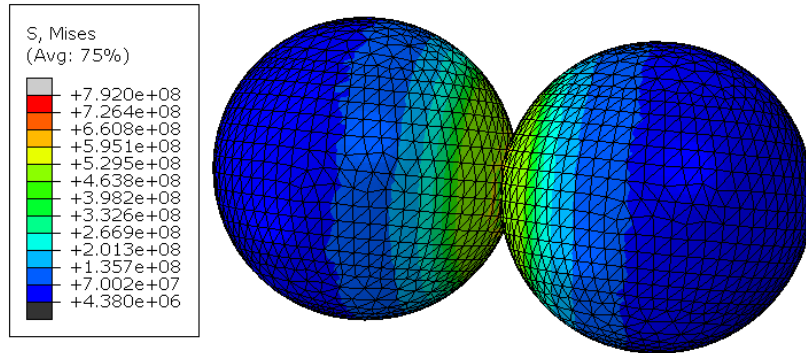
We see that $E / (\rho R^2)$ has units of s^{-2} , so that

$$\frac{T}{R} \sqrt{\frac{E}{\rho}} = f(\nu)$$

(many equivalent relations are possible too, of course)

[3 POINTS]


2. Set up ABAQUS to conduct an explicit dynamic calculation of the impact of two identical spheres, as shown below.




Here's a summary of the model parameters:

1. Sphere radius 0.1m
2. Mass density $\rho = 1000 \text{ kg} / \text{m}^3$
3. Young's modulus $E = 210 \text{GNm}^{-2}$, Poisson ratio $\nu = 0.3$
4. Analysis time period 0.0007sec
5. First, run an analysis with perfectly elastic spheres. Then repeat the calculation for elastic-plastic spheres, with yield stress $\sigma_Y = 10000 \text{MNm}^{-2}$ $\sigma_Y = 5000 \text{MNm}^{-2}$ $\sigma_Y = 1000 \text{MNm}^{-2}$, $\sigma_Y = 500 \text{MNm}^{-2}$, and again with $\sigma_Y = 50 \text{MNm}^{-2}$ (you could automate this with a python script if you like, but its probably not worth the effort for just 5 simulations)
6. Contact formulation – hard contact, with no friction
7. Give one sphere an initial velocity of $v_0 = 100 \text{m/s}$ towards the other sphere.

Follow these steps to set up the analysis

1. In the part module use Part> Create to create a 3D deformable body; use Solid/Revolution for the base feature. Enter 2 for the Approximate Size, then draw a closed semicircle with radius 0.1m in the sketch window. Add a construction line  down the y axis. Right click the window; select Cancel Procedure; then select Done. Then select the construction line to define the axis of revolution. Enter 360 degrees for the revolve angle
2. In the Property module, select Material > Create; then under the General tab enter the density; and under the Mechanical> Elastic tab enter Young's modulus and Poissons ratio. Later you will use Material>Manager to edit these properties and add a 'plastic' definition. To define a material with a constant yield stress you just enter the yield stress and 0 in the two columns of the table.
3. In the Assembly module you can create the two spheres by making two instances of the same part. When you create the second instance, check the button that says 'auto offset from other instances.' This will automatically place the two spheres with their centers on the x axis, a small distance apart.
4. In the Step module create an Explicit Dynamic step, with time period 0.0007s. You can use default options for everything else. Also use Output>Field Output Requests > Manager and edit F-output-1. In the Frequency menu change the Interval from 20 to 100.

5. In the Interaction module (i) Select Property>Create from the Interaction menu (ii) Select Contact in the menu and hit Continue (iii) In the next menu select the Normal Contact option under the Mechanical menu, and hit OK to accept the default model. (iv) Select the Tangential contact option and accept frictionless; (v) Then go to the Interaction>Create... (vi) Select Explicit (general contact) and hit continue (vii) Hit OK to accept the default settings.
6. You use the 'Load' module to assign an initial velocity to one of the spheres (this is a bit weird). Go to the Load module, select Predefined Field>Create. **Select Initial in the menu next to step near the top of the window**, then check the Mechanical radio button just below it. Hit OK. Select the leftmost sphere by **dragging a box around the entire sphere**. Don't click on it – if you do this it will only assign velocities to a piece of the sphere, not the whole thing. This causes serious problems in the analysis later. Hit 'Done'. Finally enter 100 m/s for V1 in the appropriate box and hit OK.
7. In the Mesh module make sure the 'Part' radio button is checked above the viewport. Select Mesh > Controls, select Tet elements and press OK. The part should change color. Then select Mesh> Element type, and assign quadratic tet elements to the part (this should be the default, but you never know). Then seed the part with a mesh size of 0.01m and mesh it.
8. Create a job and submit the analysis. You can use Job>Monitor to track the progress of the computation – the 'Step Time' column will tell you how far along the simulation has got.
9. In the Visualization module open the odb file. You can select  to plot the deformed shape and Mises stress contours, and Animate to play a movie of the collision.

Please hand in a copy of graphs showing the variation of the total kinetic energy of the system as a function of time, for each analysis case. You can plot the graph by going to the Result>History output menu, and selecting the ALLKE variable, then pressing Plot. The easiest way to print them is to do a screen dump (alt+print screen) and paste the graphs into a word document.

Finally, please answer the following questions:

1. Suppose that the main objective of the analysis is to compute the restitution coefficient of the spheres, defined as $e = |\mathbf{v}_{A1} - \mathbf{v}_{B1}| / |\mathbf{v}_{A0} - \mathbf{v}_{B0}|$ where $\mathbf{v}_{A0}, \mathbf{v}_{A1}$ denote the initial and final velocities of sphere A, and the same convention is used for sphere B. List all the material and geometric parameters that appear in the problem (include the initial condition).

We have that

$$e = f(R, \rho, E, \nu, \sigma_y, v_0)$$

[1 POINT]

2. Express the functional relationship governing the restitution coefficient in dimensionless form. Show that for a perfectly elastic material, the restitution coefficient must be a function of a single dimensionless group (and one additional dimensionless variable). For an elastic-plastic material, you should find that the restitution coefficient is a function of two groups and a variable.

e is already dimensionless. Note that E / ρ σ_Y / ρ both have units of m^2s^{-2} . There is no way to combine the variables to include R in a dimensionless group. So for an elastic material we could write

$$e = g\left(\frac{\rho v_0^2}{E}, \nu\right)$$

For an elastic-plastic material we could write

$$e = g\left(\frac{\rho v_0^2}{E}, \frac{\rho v_0^2}{\sigma_Y}, \nu\right)$$

There are other possibilities too – eg

$$e = g\left(\frac{\rho v_0^2}{E}, \frac{E}{\sigma_Y}, \nu\right)$$

Anything with two independent dimensionless groups should get credit

[1 POINT]

3. If the sphere radius is doubled, what happens to the restitution coefficient? (DON'T DO ANY FEA TO ANSWER THIS!)

There should be no change, since R does not appear in the dimensionless group.

[1 POINT]

4. Show that the kinetic energy lost during impact can be expressed in dimensionless form as

$$\frac{\Delta K}{R^3 \rho v_0^2} = f\left(\frac{\rho v_0^2}{\sigma_Y}, \frac{\rho v_0^2}{E}, \nu\right)$$

Note that the second term is very small for any practical application (including our simulation), so in interpreting data we need only to focus on behavior in the limit $\rho v_0^2 / E \rightarrow 0$.

We already know that

$$\Delta K = f(R, \rho, E, \nu, \sigma_Y, v_0)$$

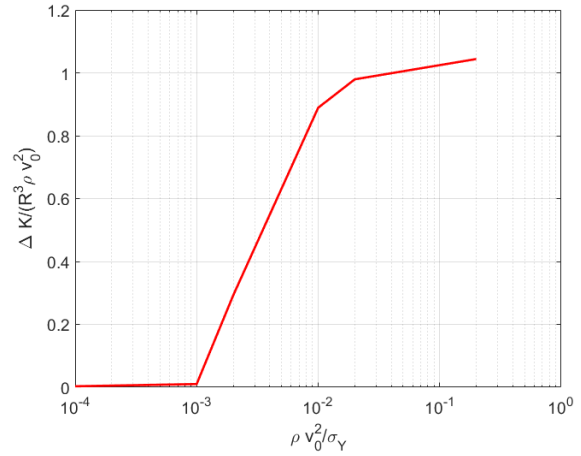
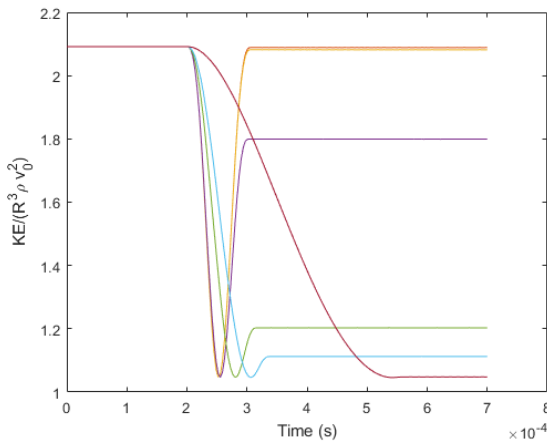
The quantity $R^3 \rho v_0^2$ is proportional to the initial KE of the sphere and can be used to normalize the KE. The remaining groups will work out just like those for e , so

$$\frac{\Delta K}{R^3 \rho v_0^2} = f\left(\frac{\rho v_0^2}{\sigma_Y}, \frac{\rho v_0^2}{E}, \nu\right)$$

[1 POINT]

5. Use your plots of KE as a function of time to determine the change in KE for each analysis case. Hence, plot a graph showing $\Delta K / R^3 \rho v_0^2$ as a function of $\rho v_0^2 / \sigma_Y$ (you can use MATLAB or any other plotting tool to do this, you don't need to use ABAQUS).

The figure below shows the KE –v- time plots for the various cases (there's an extra one), and



[5 POINTS]

6. What is the critical value of $\rho v_0^2 / \sigma_Y$ where *no* energy is lost? (you may find it helpful to plot $\Delta K / R^3 \rho v_0^2$ against $\log(\rho v_0^2 / \sigma_Y)$ to see this more clearly). If no energy is lost, the impact is perfectly elastic.

From the figure it appears that no energy is lost if $\rho v_0^2 / \sigma_Y < 10^{-3}$

[1 POINT]

7. The usual assumption in classical mechanics is that restitution coefficient is a material property. Comment briefly on this assumption in light of your simulation results.

The restitution coefficient is constant under two conditions (i) $\rho v_0^2 / \sigma_Y$ is small (so the restitution coefficient is close to 1) and $\rho v_0^2 / \sigma_Y$ is large (so restitution coefficient is close to 0).

[2 POINTS]