



School of Engineering
Brown University

EN1750: Advanced Mechanics of Solids

Homework 4: Internal forces and equations of motion.

Due Friday Oct 11, 2019

1. For the stress tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} -400 & 300 & 0 \\ 300 & 400 & 0 \\ 0 & 0 & 600 \end{bmatrix}$$

please calculate

1.1 The hydrostatic stress

1.2 The principal stresses and their directions

1.3 The tractions acting on a plane with normal $\mathbf{n} = (\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3) / \sqrt{3}$

2. The “Christensen” failure criterion is used to predict the strength of a material that fails by either yielding under compressive loads, or brittle fracture under tensile loads. It predicts that a material will fail if the principal stresses $(\sigma_1, \sigma_2, \sigma_3)$ satisfy

$$\left(\frac{1}{T} - \frac{1}{C} \right) (\sigma_1 + \sigma_2 + \sigma_3) + \frac{1}{2TC} \left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 \right\} \geq 1$$

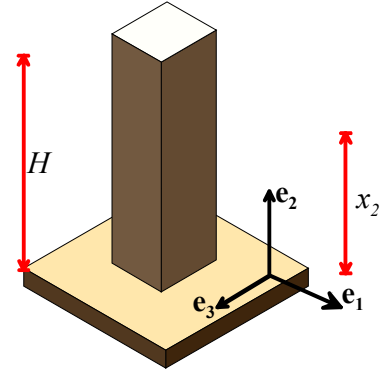
Where T and C are the magnitudes of stress that will cause failure in uniaxial tension and compression, respectively.

[This paper](#) reports tensile and compressive failure stresses $T = 6.9 \text{ MPa}$ and $C = 94 \text{ MPa}$ for Carrara Marble.

2.1 What is the failure stress of marble subjected to hydrostatic tension (i.e. a stress state $\sigma_{ij} = \sigma_0 \delta_{ij}$ - the stress matrix has equal diagonal components and no shear stresses)?

2.2 What is the failure stress of the marble subjected to a shear stress (eg in a torsion test)?

3. A prismatic concrete column of mass density ρ supports its own weight, as shown below. (Assume that the solid is subjected to a uniform gravitational body force of magnitude g per unit mass).



3.1 Show that the stress distribution

$$\sigma_{22} = -\rho g(H - x_2)$$

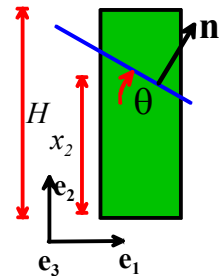
satisfies the equations of static equilibrium

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} = 0$$

and also satisfies the boundary conditions $\mathbf{n} \cdot \boldsymbol{\sigma} = \mathbf{0}$ on all free boundaries.

Assume small deformations, so the difference between the position of a material particle in the deformed and undeformed solid can be neglected.

3.2 Find a formula for the traction vector acting on a plane with normal $\mathbf{n} = \sin \theta \mathbf{e}_1 + \cos \theta \mathbf{e}_2$ at a height x_2



3.3 Find the normal and tangential tractions acting on the plane with normal $\mathbf{n} = \sin \theta \mathbf{e}_1 + \cos \theta \mathbf{e}_2$ at a height x_2

3.4 Suppose that the concrete contains a large number of randomly oriented microcracks. A crack which lies at an angle θ to the horizontal will propagate if

$$|\mathbf{T}_t| + \mu T_n > \tau$$

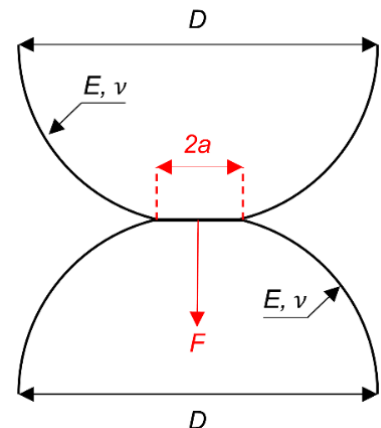
where μ is the friction coefficient between the faces of the crack and τ is a critical shear stress that is related to the size of the microcracks and the fracture toughness of the concrete.

Assume that $\mu = 1$. Find the orientation of the microcrack that is most likely to propagate. Hence, find an expression for the maximum possible height of the column.

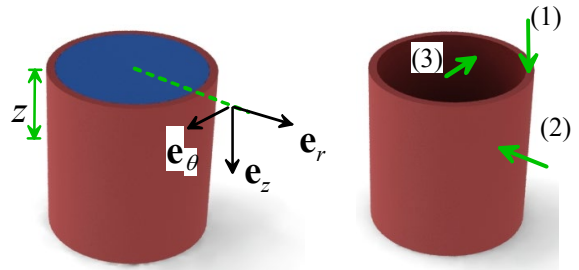
4. The contact pressure distribution acting between two axially symmetric elastic solids has the form

$$p(x, y) = p_0 \sqrt{1 - r^2 / a^2} \quad r = \sqrt{x^2 + y^2}$$

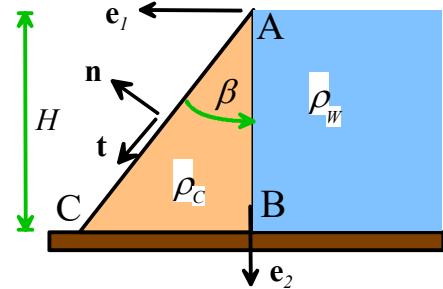
Where a is the radius of the contact patch, and p_0 is the maximum contact pressure. Find an expression for the force acting on the contacting solids in terms of p_0, a .



5. The figure shows a tumbler with mass density ρ_T filled with fluid with mass density ρ . Using the cylindrical-polar basis shown, write down the boundary conditions for stress components on the three surfaces marked (1), (2), (3) in the figure.



6. The figure shows a simple design for a dam. The wedge-shaped dam is made from a material with weight density ρ_C . It is loaded on its vertical face by fluid pressure $p = -\rho_W x_2$, where ρ_W is the weight density of the fluid.



6.1 Write down formulas for unit vectors \mathbf{t} , \mathbf{n} tangent and normal to the back face of the dam, in terms of β (you will need this information for part 6.2 and 6.3)

6.2 Write down the boundary conditions on the two faces AB, AC of the dam, in terms of the 2D stress components $\sigma_{11}, \sigma_{22}, \sigma_{12}$.

6.2 Consider the stress state

$$\sigma_{11} = -\rho_W x_2$$

$$\sigma_{22} = \rho_C (x_1 \cot(\beta) - x_2) - \rho_W \cot^2 \beta (2x_1 \cot(\beta) - x_2)$$

$$\sigma_{12} = -\rho_W x_1 \cot^2 \beta$$

Show that the stress state satisfies equilibrium in the dam (inside the wedge) and also satisfies the boundary conditions on the faces AB and AC. Note that on face AC $x_1 = x_2 \tan \beta$.

6.3 Suppose that the dam is made from concrete, which cannot safely withstand tensile stress. Assuming that the maximum principal tensile stress occurs at B, find a formula for the minimum allowable value for the angle β , in terms of ρ_C, ρ_W

6.4 Assume that the concrete fails by crushing if the minimum principal stress reaches $\sigma_3 = -\sigma_c$. Assuming that the minimum principal stress occurs at point C, and that β has the minimum value calculated in 7.3, find an expression for the maximum height of the dam, in terms of ρ_C, ρ_W . You may find the trig identity $1 + \tan^2 \beta = 1 / \cos^2 \beta$ helpful.

7. Importing CAD data into ABAQUS. Work through the tutorial on this topic. Then, use Solidworks to create a simple part, import it into ABAQUS (try both a step or iges import and a .inp file import. Its best to avoid the STL file import), and run a simulation. Keep everything simple, or you will run into problems. Try to make your part differ from those submitted by your classmates!

Please upload your solidworks model and cae file to canvas as a solution to this problem.

[10 POINTS]