



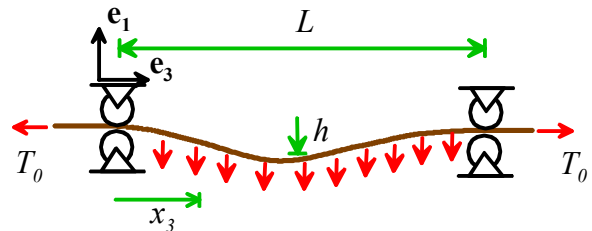
School of Engineering  
Brown University

EN1750: Advanced Mechanics of Solids

Homework 7: Strings, Beams, Membranes and Plates.

Due Friday Nov 15, 2019

1. An overhead transmission cable with mass density  $\rho$  and tensile strength  $\sigma_{UTS}$  has a uniform cross-sectional area  $A$ . IEEE standard 524 recommends that the tension in the cable should not exceed 10% of the strength of the cable.

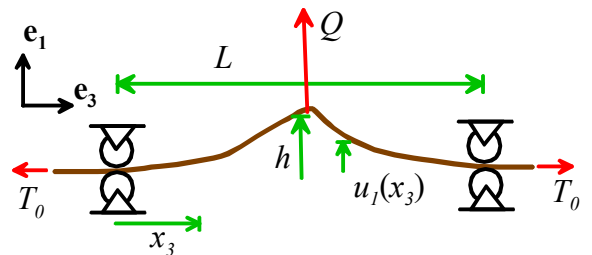


1.1 Find a formula for the sag  $h$  at mid-span of the cable, in terms of the cable length  $L$ ,  $\rho$  and  $\sigma_{UTS}$  (you can assume small deflections, and use the solution for the cable deflection calculated in class).

1.2 Find the maximum length of a [high conductivity copper](#) cable with mass density  $9 \text{ g/cm}^3$  and tensile strength  $170 \text{ MPa}$  if  $h/L$  is required to be less than  $0.1$

(For interest – this [accident report](#) is an excellent example of a well written engineering report and includes some relevant solid mechanics calculations related to cable deflections)

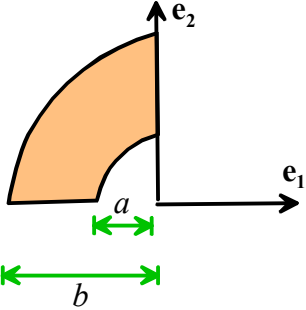
2. A cable with weight per unit length  $p$  is stretched by tension  $T_0$  and subjected to a transverse force  $Q$  at mid-span. Calculate the deflection  $h$  of the cable at mid-span. What value of  $Q$  (in term of  $p, L$ ) will make the displacement at mid-span zero? You can assume small deflections.



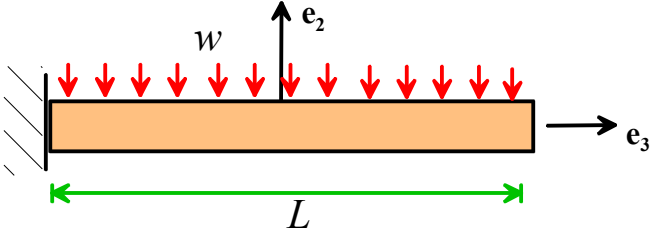
Some suggestions – it is easier to analyze just half the cable eg  $0 < x_3 < L/2$ , and treat the force  $Q$  as a boundary condition at  $L/2$ . You can prescribe the transverse force at  $L/2$  (but you have to think a bit about how to do this correctly). Alternatively, you could consider a small segment of the cable  $L/2 - dx_3 < x_3 < L/2 + dx_3$ , draw a free body diagram showing the forces acting on this small segment, and hence use a simple statics calculation to find an expression for the slope of the cable at  $L/2$ , which you can use as a boundary condition.

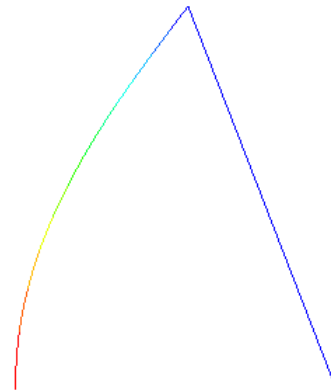
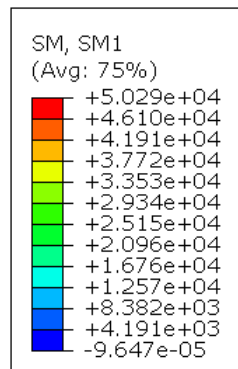
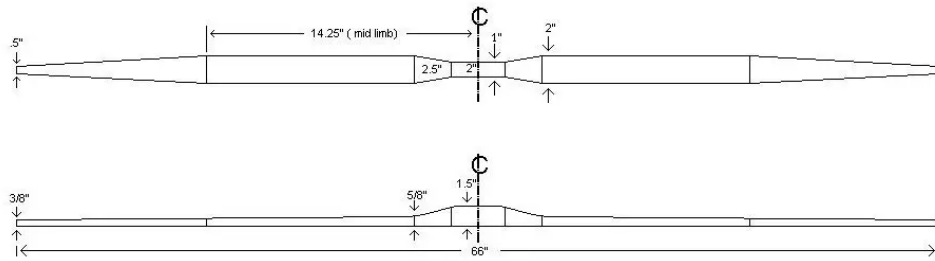
3. The figure shows the cross-section of a beam.

Find the position of the centroid, and calculate the area moments of inertia of the cross section. Simplify your answer by assuming that  $b=a+t$ , with  $t \ll a$  (i.e. take the Taylor expansion to first order in  $t$ ). You can do the calculation in MATLAB – cut and paste a screenshot into your HW solution if you do this.



4. Suppose that a cantilever beam with length  $L$  and cross-section analyzed in problem 3 is subjected to a uniform load  $w$  acting in the (negative)  $e_2$  direction. Calculate the deflection of the beam at its tip (you can use the Matlab script from class, or do the calculation by hand)





5. In this problem you will explore the beam elements available in ABAQUS by setting up a simple finite element simulation to predict the static force-v-draw curve for an English longbow. Dimensions of a simple home-made bow (from [this reference](#)) are shown in the figure.

5.1 Because of symmetry, we only need to model half the bow. Create a 3D deformable part with wire base feature, with one end at the origin and the other at  $y = 830\text{mm}$  (the undeformed bow is straight; it gets bent when the string is attached). Note that this procedure will compute *half* the total draw force (there will be an additional force from the bottom half of the bow that has not been modeled).

5.2 Use a Young's modulus for Yew wood of 6.5 GPa (convert to  $\text{N}/\text{mm}^2$  since lengths are in mm), and make the cross-section a 40mmx12mm rectangle (it is possible to create tapered cross-sections in ABAQUS/CAE but you have to define the taper of every element in the beam separately, which is extremely tedious to do by hand and best done with a python script). Don't forget to assign a beam section orientation – make sure the normal to the flat side of the bow is parallel to the x direction.

5.3 Create a second part for the bow string. Make one end of the string coincident with the tip of the bow, and make the string 40mm shorter than the bow. Give the bow string a modulus of 110 GPa and a circular cross section with radius 0.72mm.


5.4 Create two steps, both with 1 sec duration – these will be used to string the bow, and then draw it. Make sure NLGEOM is selected for both steps. Make the initial and maximum increment size 0.1s for both steps. In the step module for the first step, open the Field Output Request manager and add requests to plot the

- ▼  Forces/Reactions
  - RF, Reaction forces and moments
  - RT, Reaction forces
  - RM, Reaction moments
  - CF, Concentrated forces and moments
  - SF, Section forces and moments

section forces/moments. Also, (i) create a new Set (use Tools > Set > Create... name the set drawpoint, Select Geometry, and select the bottom end of the bowstring); then (ii) create a history output request to request displacements/velocity/acceleration and forces/reactions for this set (this will be used to plot the force-v-draw curve).

5.5 To tie the bowstring to the bow, select Constraint > Create ..., select a tie constraint; in the menu below the viewport select Node Region, then select the end points of the string and bow as the master and slave (it doesn't matter which is which). If you need to, you can right click a part under the 'instance' branch of the model tree to hide it. In the 'edit constraint' menu uncheck the 'Tie rotational DOFs if applicable' option

5.6 In the boundary condition module, create a BC for the 'initial' step that makes all the degrees of freedom (U and UR) to zero for the node on the bow that lies at the origin. This BC will be applied throughout the analysis by default.

5.7 We need to define boundary conditions for the two load steps. The first step will be used to string the bow. To do this we need to apply a force to bend the bow, and pull the string down so its bottom end lies at  $y=0$ . Apply a horizontal force (acting in the positive x direction) to the bow at its tip with magnitude 20 N (if your bow is vertical, the force will be horizontal). Click the  button next to the drop down menu to define an amplitude for this force; check the 'tabular' radio button, and enter a table that starts at 1 at time  $t=0$  and ends at 0 at time  $t=1$ . Then select Amp-1 in the dropdown menu. This will bend the bow so it can be pulled down by the bowstring, but will remove the force by the end of the step. Next, create a BC that will apply a displacement to the free end of the bowstring that will pull it down to be level with the base of the bow (this will be  $U2=-40\text{mm}$ ). A standard 'Ramp' amplitude will work. Finally open both the Load and BC managers and deactivate the two BCs created for step 1 in step 2.

5.8 To draw the bow, (i) create a BC that will fix  $U2$  at the bottom of the bowstring to remain constant in the second step. To do this open the BC manager, and make sure the BC you defined at the end of the string in Step 1 is deactivated in Step 2. Then create a new BC, select the bottom end of the bowstring, select 'Fixed at Current Position' in the Edit BC window, and check the box for  $U2$ . Next, create a second BC that will apply a constant horizontal velocity ( $V1$ ) of 200 mm/sec to the bottom end of the bowstring. This will draw the bow by 200mm by the end of the step.

5.9 In the mesh module seed the bow and string with 10mm/20mm mesh size, respectively, and mesh both parts with the default element type.

5.10 Submit and run the job. If all works correctly you should see a deformed shape and bending moment distribution that looks something like the picture on the previous page.

5.11 Create and hand in a plot of the force-v-draw curve for the bow. Work out how to correct the draw force to double the reaction force. For a rough comparison, you can find measured force-v-draw curves for a few bows on [this page](#) – but those bows are made from different materials and have a different cross-section to the one analyzed here (which seems to be a rather light bow).