EN1750: Advanced Mechanics of Solids

Example problems on plasticity and failure.

Ungraded

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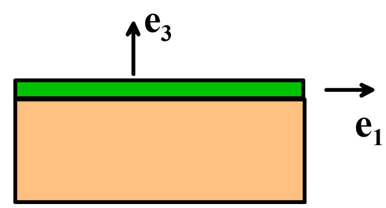
1. An isotropic, elastic-perfectly plastic thin film with Young's Modulus E, Poisson's ratio ν, yield stress in uniaxial tension Y and thermal expansion coefficient α is bonded to a stiff substrate. It is stress free at some initial temperature and then heated. The substrate prevents the film from stretching in its own plane, so that $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{12} = 0$, while the surface is traction free, so that the film deforms in a state of plane stress. Calculate the critical temperature change ΔT_y that will cause the film to yield, using (a) the Von Mises yield criterion and (b) the Tresca yield criterion.

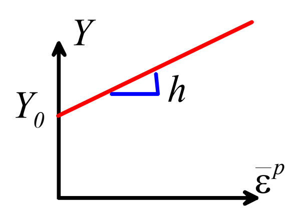


2. Assume that the thin film described in the preceding problem shows so little strain hardening behavior that it can be idealized as an elastic-perfectly plastic solid, with uniaxial tensile yield stress Y. Suppose the film is stress free at some initial temperature, and then heated to a temperature $\beta \Delta T_y$, where ΔT_y is the yield temperature calculated in the preceding problem, and $\beta > 1$.

2.1 Find the stress in the film at this temperature

2.2 The film is then cooled back to its original temperature. Find the stress in the film after cooling.

3. Suppose that the thin film described in the preceding problem is made from an elastic, isotropically hardening plastic material with a Mises yield surface, and yield stress-v-plastic strain as shown in the figure. The film is initially stress free, and then heated to a temperature $\beta \Delta T_y$, where ΔT_y is the yield temperature calculated in problem 1, and $\beta > 1$.



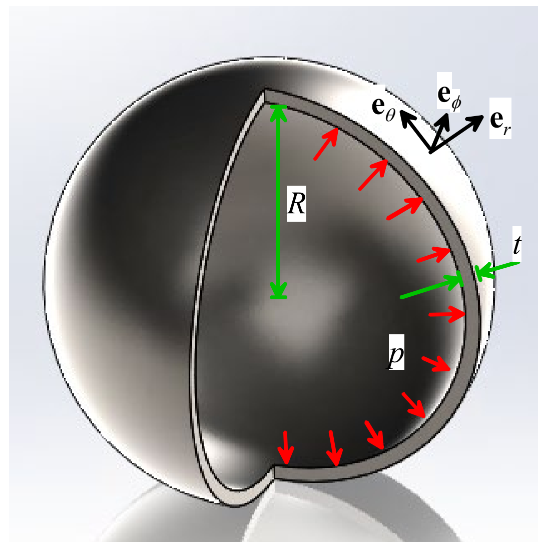
3.1 Find a formula for the stress in the film at this temperature. To do this you will need to assume the film remains in a state of plane biaxial stress $\sigma_{11} = \sigma_{22} = \sigma$ with all other stresses zero, and then

- (i) Find formulas for the hydrostatic stress, deviatoric stress and Von Mises stress in terms of the unknown stress
- (ii) Write down an expression for the total strain rate (elastic, plastic and thermal) in terms of the stress rate, using the expressions from class
- (iii) Use the condition that the total strain rate is zero to relate the stress rate to the rate of change of temperature
- (iv) Integrate the result of (iii) with time (with initial condition $\sigma = -Y$ when $\Delta T = \Delta T_y$) to find the stress.

3.2 The film is then cooled back to its original temperature. Find the stress in the film after cooling.

3.3 The film is cooled further by a temperature change $\Delta T < 0$. Calculate the critical value of ΔT that will cause the film to reach yield again.

4. A thin-walled sphere with radius R and wall thickness t is made from an elastic-plastic material with Youngs modulus E , Poissons ratio ν and a linear hardening relation $Y = Y_0 + h\varepsilon_e$. The sphere is subjected to monotonically increasing internal pressure p (with $dp/dt > 0$), which generates a stress state (in spherical-polar coordinates) $\sigma_{rr} \approx 0,\ \sigma_{\phi\phi} = \sigma_{\theta\theta} = pR/(2t)$ (note that these are principal stresses)



4.1 Find a formula for the Von-Mises stress in the sphere wall, in terms of p,R and t. Hence, calculate the pressure that will first cause yield in the sphere wall.

4.2 Find the hydrostatic and deviatoric stresses in the sphere wall.

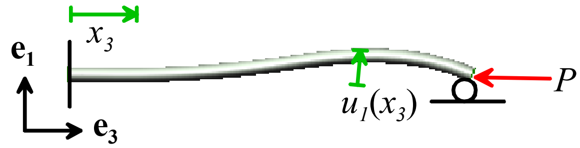
4.3 Hence, find a formula for the Von Mises plastic strain rate $d\varepsilon_e/dt$ in the sphere wall, in terms of $dp/dt, h, R, t$

4.4 Hence, find a formula for the total strain rates $d\varepsilon_{rr}/dt, d\varepsilon_{\theta\theta}/dt$ (include both elastic and plastic strain rates, and give solutions for pressure both below and above yield) in the shell.

4.5 Find the total hoop strains $\varepsilon_{\theta\theta}, \varepsilon_{\phi\phi}$ when the pressure reaches a value $p = 4tY_0/R$

4.6 Find a formula for the change in radius of the sphere when the pressure reaches a value $p = 4tY_0/R$

5. The figure shows a beam that is clamped at one end and pinned at the other. The beam has area moments of inertia $I_{22} = I_{11} = I, \; I_{12} = 0$. Calculate the buckling load (use the buckling mode that gives the lowest load).

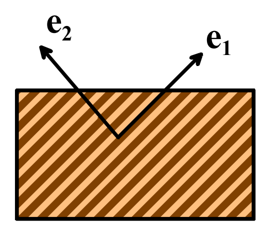


6. The figure shows a fiber reinforced composite laminate.

(i) When loaded in uniaxial tension parallel to the fibers, it fails at a stress of 500MPa.

(ii) When loaded in uniaxial tension transverse to the fibers, it fails at a stress of 250 MPa.

(iii) When loaded at 45 degrees to the fibers, it fails at a stress of 223.6 MPa

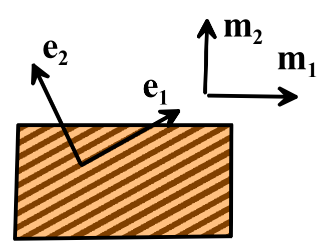


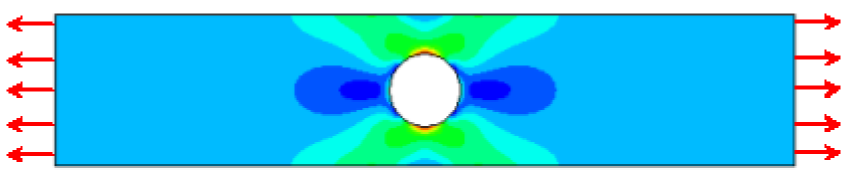
Failure in the laminate is to be predicted using the Tsai-Hill criterion

$$\left(\frac{\sigma_{11}}{\sigma_{TS1}}\right)^2 + \left(\frac{\sigma_{22}}{\sigma_{TS2}}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{\sigma_{TS1}^2} + \frac{\sigma_{12}^2}{\sigma_{SS}^2} = 1$$

6.1 Use the measurements to calculate values for the parameters $\sigma_{TS1}, \sigma_{TS2}, \sigma_{SS}$.

6.2 The laminate is then loaded in uniaxial tension at 30 degrees to the fibers. Calculate the expected failure stress under this loading, assuming that the material can be characterized using the Tsai-Hill failure criterion.





7. A specimen of steel has a yield stress of 500MPa. Under fully reversed cyclic loading at a stress amplitude of 200 MPa it is found to fail after 10^4 cycles, while at a stress amplitude of 100MPa it fails after 10^5 cycles. This material is to be used to fabricate a plate, with thickness *h*, containing circular holes with radius $a<<h$. The plate will be subjected to constant amplitude fully reversed cyclic uniaxial stress far from the holes, and must have a life of at least 10^5 cycles. What is the maximum stress amplitude (far from the hole) that the plate can withstand?

8. A spherical pressure vessel with internal radius a and external radius b=1.5a is repeatedly pressurized from zero internal pressure to a maximum value p. The sphere has yield stress Y, ultimate tensile strength σ_{UTS} and its fatigue behavior (under fully reversed uniaxial tension) can be characterized by Basquin's law $\sigma_a N^b = C$. You can assume that the elastic stresses in the vessel are given by

$$\sigma_{rr} = -p\frac{a^3(b^3 - r^3)}{r^3(b^3 - a^3)} \qquad \sigma_{\theta\theta} = p\frac{a^3(b^3 + 2r^3)}{2r^3(b^3 - a^3)}$$

8.1 Find an expression for the fatigue life of the vessel in terms of p, and relevant geometric and material properties. Assume that the effects of mean stress can be approximated using Goodman's rule. Assume that $p/Y < 2(1 - a^3/b^3)/3$