2) Analyzing deformation of solids with FEA

Focus: High level Overview: Derive eqs later

FEA is a general procedure for solving PDEs

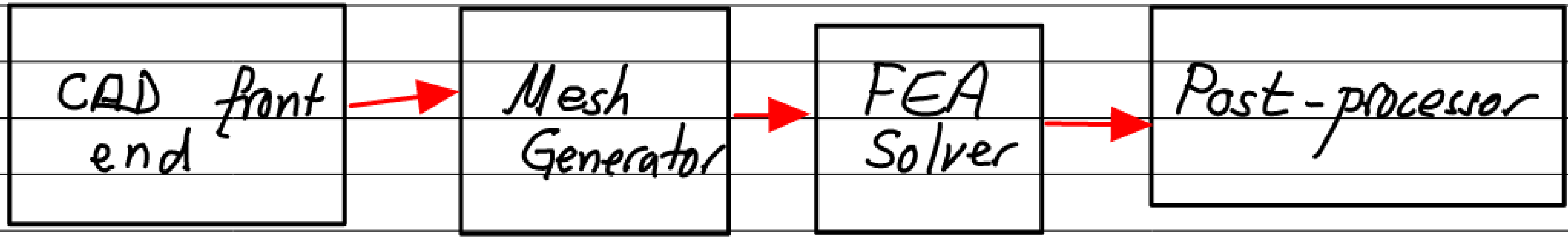
In Solids, we solve :

(1) Strain - Displacement eq

(2) stress - strain law

(3) Equilibrium or $\underline{F} = m\underline{a}$

General structure of FEA



Specifying an FEA problem

(1) Geometry - Mesh

(2) Material Model

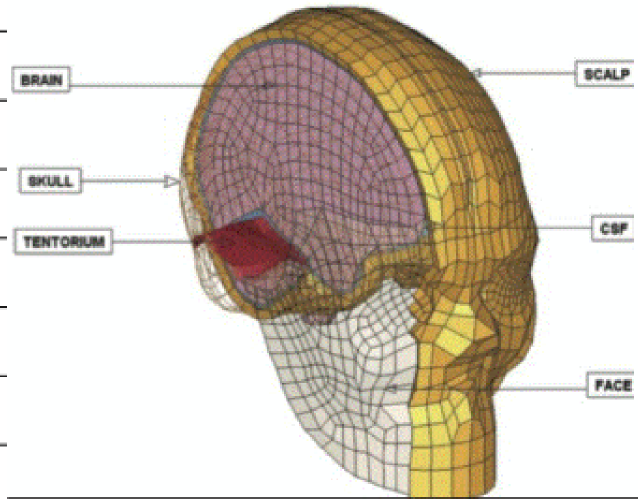
(3) Specify Loading : (Contacts)

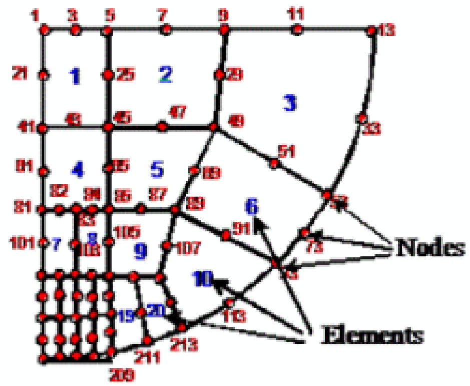
(4) Analysis technique

2.1 FEA mesh

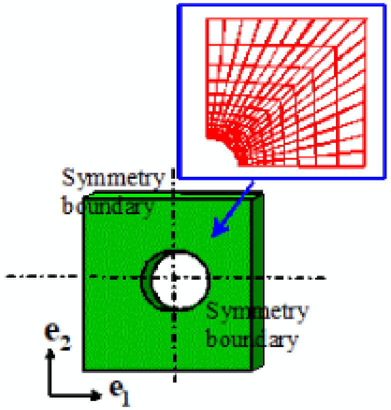
3D: Sub-divide solid into cells

2D: Sub-divide area

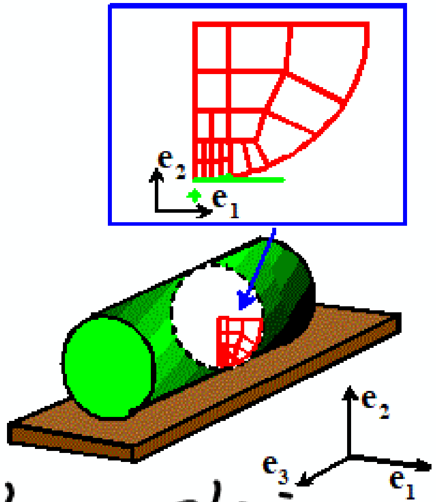




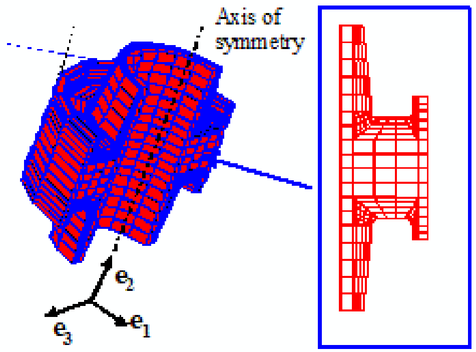
Types of 2D mesh



Plane Stres: Solid is thin sheet loaded in-plane

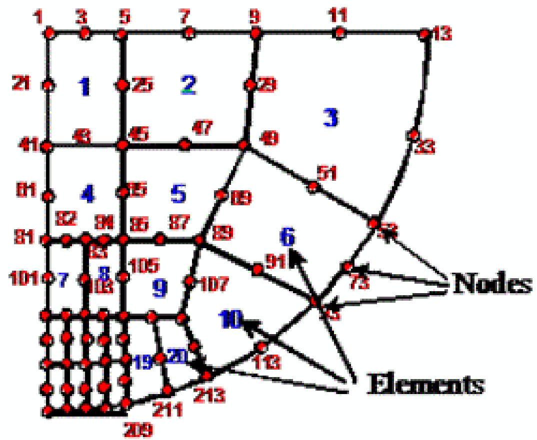


Plane strain
Solid is long cylinder
No axial stretch



Axisymmetry

Features of a mesh

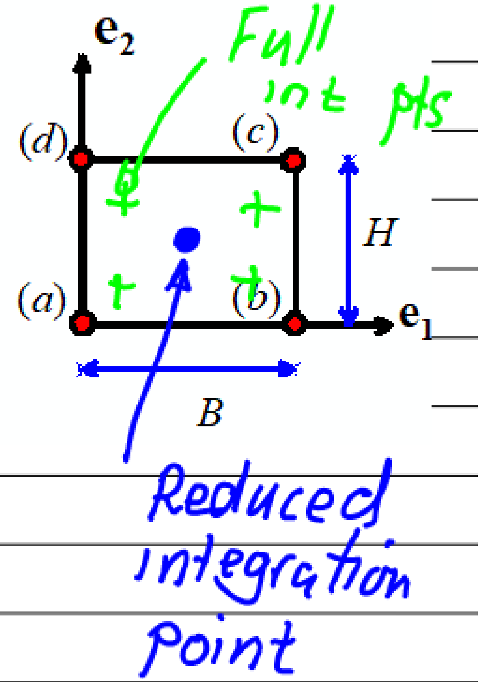


Nodes : Discrete points in solid

Displacement vector will be calculated at each node

(sometimes also rotations)

Elements : Sub divide solid



Properties: (1) Connectivity (list of nodes on element)

(2) Interpolation scheme : specifies displacement inside element

example: for 2D rectangle : $\xi = X_1/B \quad \eta = X_2/H$

$$\underline{u} = \underline{u}^a (1-\eta)(1-\xi) + \underline{u}^b \xi(1-\eta) + \underline{u}^c \xi\eta + \underline{u}^d \eta(1-\xi)$$

Linear : ABAQUS also has quadratic elements

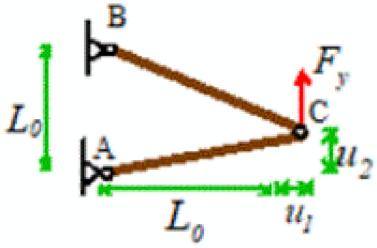
(3) Integration Points ; special points inside element

Stresses & strains calculated @ int pts

"Reduced" integration - uses fewer points
- needed for near incompressible materials

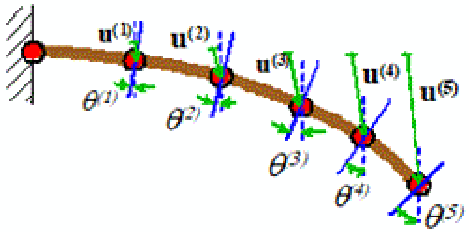
Special Element Types

For some solids with special shapes
we can simplify stress / strain fields

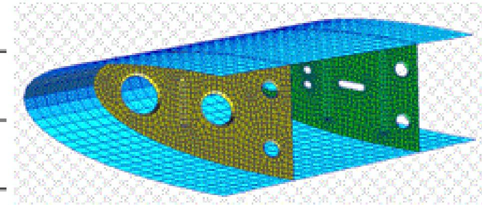


Deformed

Truss element:
can stretch or rotate
Specify X-sect area



Beam element
Elements can bend
Stretch and twist
Nodes have rotations and displacements
Specify x-sect geometry

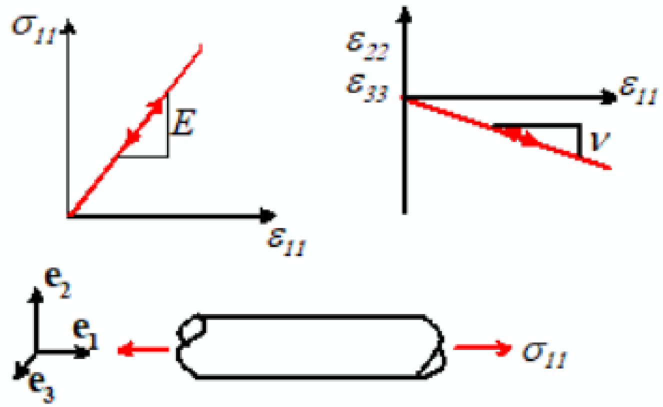


Shell / Plate elements

- same idea as beam in 2D

Specify thickness

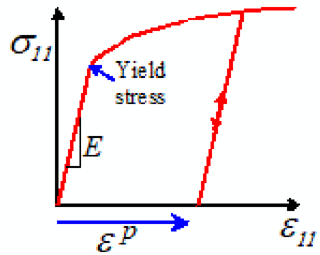
2.2 Material Models



Linear Elasticity (small strains)

$$\varepsilon_{11} = \frac{\delta l}{l} = \frac{1}{E}\sigma_{11} \qquad \varepsilon_{22} = \varepsilon_{33} = -\frac{\nu}{E}\sigma_{11}$$

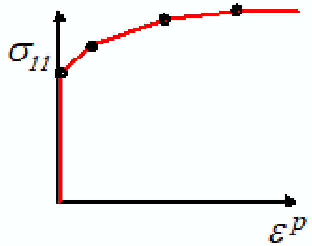
(Also anisotropic materials)



Elastic - plastic (strains can be large)

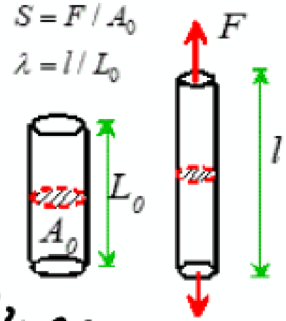
Material is nonlinear & irreversible

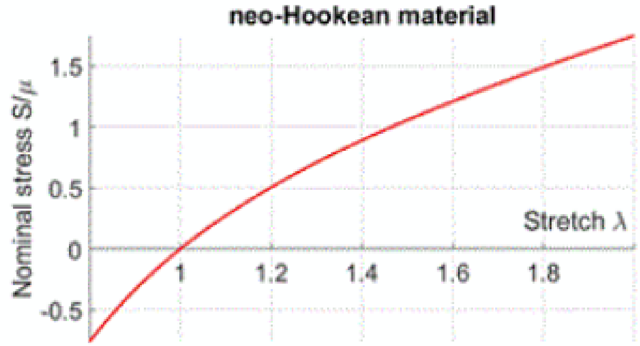
Specify yield stress -v- plastic strain in ABAQUS with table.



Hyperelasticity (large strain elasticity)

Rubber material; large deformations; nonlinear σ-ε curve; reversible



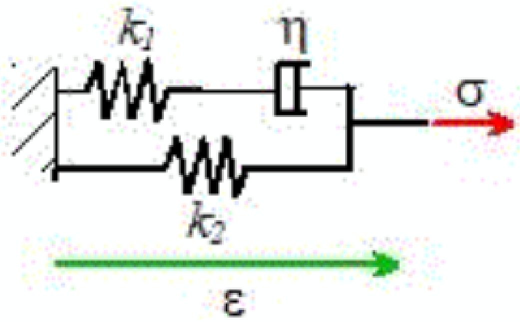


Need to use stress & strain measures for large deformations

Usually approximated as near-incompressible

Viscoelasticity (rate dependent, can be linear or not)

Stress-strain behavior specified by ODE - for example shown



$$\frac{\eta}{k_2}\frac{d\sigma}{dt} + \sigma = k_2\varepsilon + \left(1 + \frac{k_1}{k_2}\right)\eta\frac{d\varepsilon}{dt}$$

Or can specify variation of elastic moduli with time