14) Failure

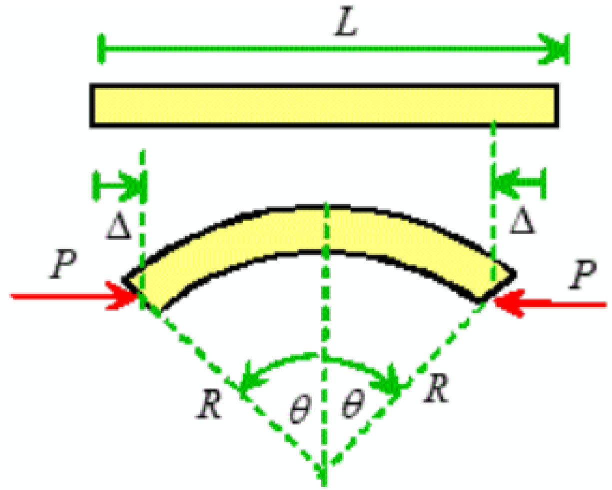
Two general forms of failure

(a) "Geometric" failure : eg elastic buckling ; necking

(b) "Material" failure : eg fracture ; fatigue ; wear

14.1 Elastic Buckling (geometric failure)

Illustrate with simple example: estimate deformed shape of axially loaded column



For simplicity assume $I_{11} = I_{22} = I$ $I_{12} = 0$

Use Rayleigh-Ritz

(1) Approximate deformed shape as circle curvature $\kappa = 1/R$

(2) Find PE ; minimize wrt κ

For beam : $\Pi = \frac{1}{2} EI \kappa^2 L - 2P\Delta$

Geometry : $\Delta = \frac{L}{2} - R\sin\theta \qquad R\theta = \frac{L}{2} \Rightarrow \theta = \frac{L}{2R}$

Hence $\Pi = \frac{1}{2}\frac{EI}{L}(\kappa L)^2 - PL\left(1 - \frac{2}{\kappa L}\sin\frac{\kappa L}{2}\right)$

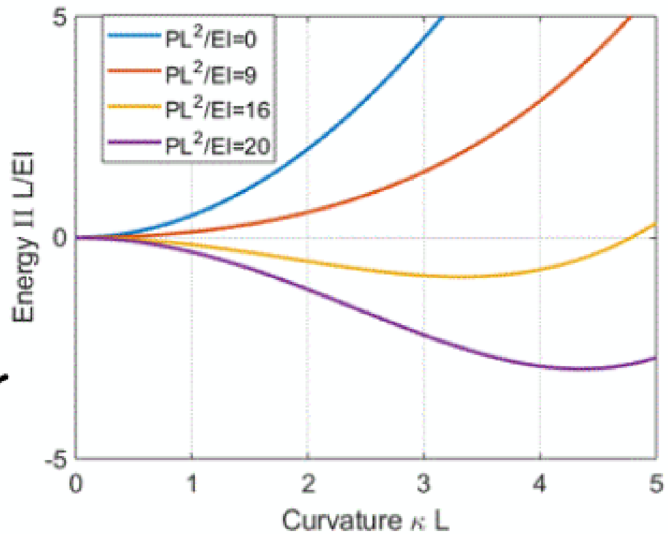
$$\Rightarrow \frac{\Pi L}{EI} = \frac{1}{2}(\kappa L)^2 - \frac{PL^2}{EI}\left(1 - \frac{2}{\kappa L}\sin\frac{\kappa L}{2}\right)$$

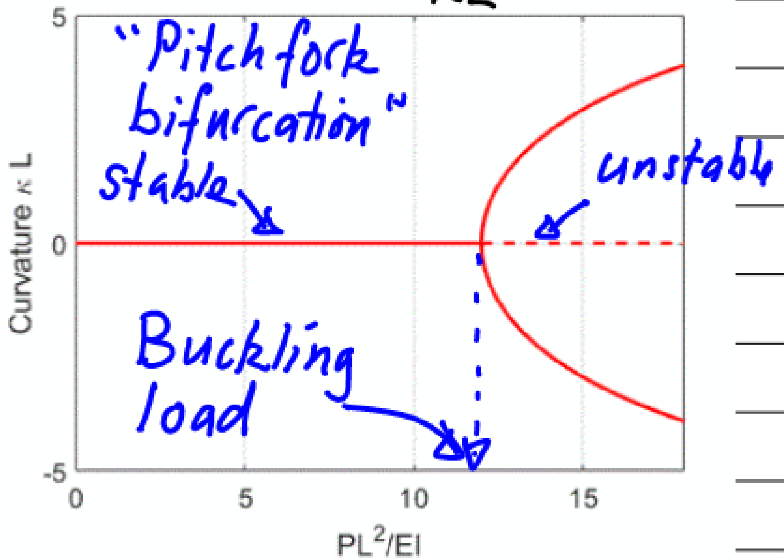
Note for $\frac{PL^2}{EI} < 11$

we find Π min

occurs for $\kappa L = 0$

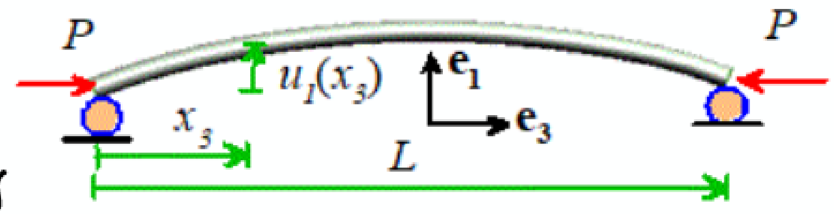
$\frac{PL^2}{EI} > 11 \quad \Pi_{min}$ for $\kappa L \neq 0$





Calculating exact buckling loads for beams

Solve pinned-pinned beam as an example



General procedure: look for solutions to equilibrium eq for beam with $u_1 \neq 0$: solutions occur for special values of P (buckling loads)

Beam eq: $$EI \frac{d^4 u_1}{dx_3^4} - T_3 \frac{d^2 u_1}{dx_3^2} = 0 \quad (*) \qquad \frac{dT_3}{dx_3} = 0 \Rightarrow T_3 = -P$$

General sol: $$u_1 = A \sin kx_3 + B \cos kx_3 + Cx_3 + D$$

Subst sol into $*$: $$(k^4 EI - Pk^2)(A \sin kx_3 + B \cos kx_3) = 0$$

$$\Rightarrow \quad P = k^2 EI \quad \text{for nonzero sol}$$

Find k from boundary conditions

(a) $u_1 = 0$ at $x_3 = 0$ $x_3 = L$

(b) $M_2 = EI \dfrac{d^2 u_1}{dx_3^2} = 0$ at $x_3 = 0$, $x_3 = L$ (free rotation)

Re-write these in matrix form

$$\begin{array}{l} u_1 = 0 \quad x_3 = 0 \rightarrow \\ u_1 = 0 \quad x_3 = L \\ u_1'' = 0 \quad x_3 = 0 \\ u_1'' = 0 \quad x_3 = L \end{array} \underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 \\ \sin kL & \cos kL & L & 1 \\ 0 & -k^2 & 0 & 0 \\ -k^2 \sin kL & -k^2 \cos kL & 0 & 0 \end{bmatrix}}_{[H]} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\det [H] = 0$ for nonzero solution

$\Rightarrow k^4 \sin kL = 0$ (Matlab?)

Hence $kL = n\pi$ for $n = 1, 2, 3$ etc

Hence $P = \dfrac{\pi^2 EI}{L^2} n^2$ ← P for many "buckling modes"

Buckling mode with lowest P is $P_{crit} = \dfrac{\pi^2 EI}{L^2}$

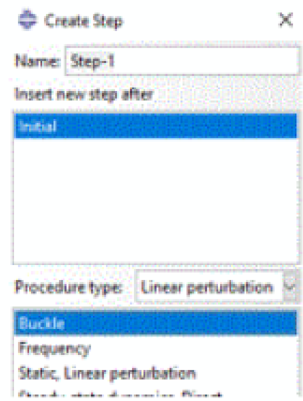
To find buckling mode can subst for k into H and look for null vector

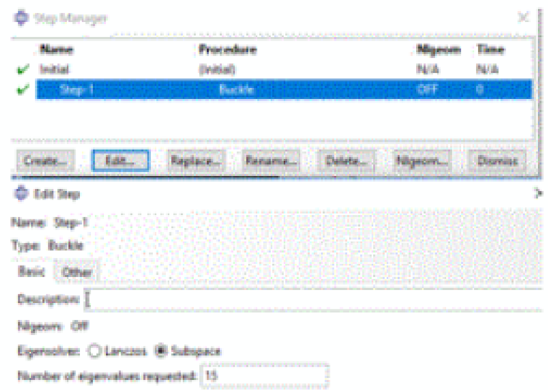
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & L & 1 \\ 0 & -k^2 & 0 & 0 \\ 0 & -k^2 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

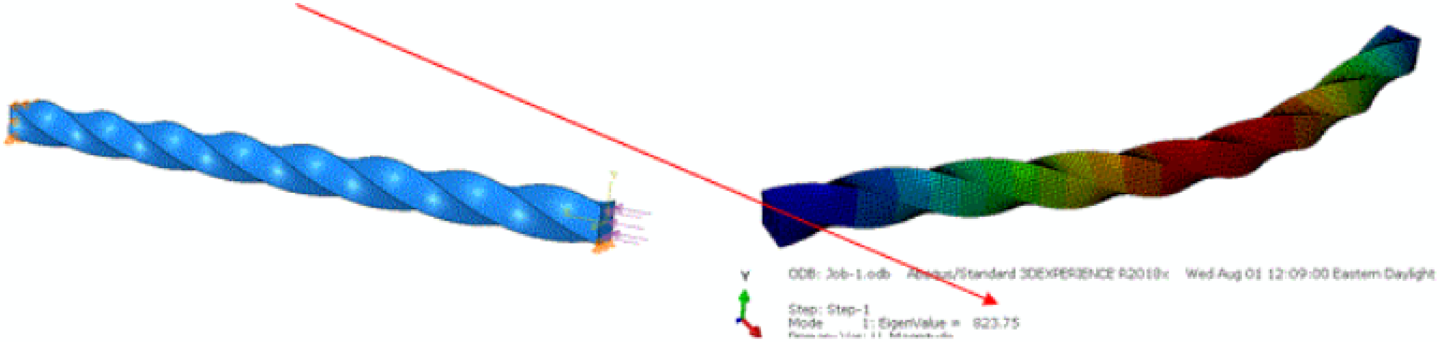
$A \neq 0 \quad B = C = D = 0 \Rightarrow u_1 = A \sin\left(\dfrac{\pi n x_3}{L}\right)$

Calculating buckling loads with ABAQUS

1. Set up geometry, properties, section, etc in usual way.
2. Create part instance in assembly in usual way
3. Create a new step after optional static step, then in Step menu select ‘Linear Perturbation’ procedure, and select ‘Buckle’. Can select # buckling modes
4. Apply boundary conditions in usual way. Be sure to include a load that will cause buckling. The load can have an arbitrary magnitude – ABAQUS will compute how much the load needs to be multiplied by to cause buckling.
5. Mesh solid – be careful with element choice (usually best to avoid reduced integration/incompatible modes as they have artificial deformation modes; also if elements will lock that will cause serious problems). Large # buckling modes will require fine mesh. Experiment with different element types.
6. Run job in usual way
7. Bucklin mode shapes are displayed in Visualization Module. The ‘Eigenvalue’ is the scale factor applied to the loads





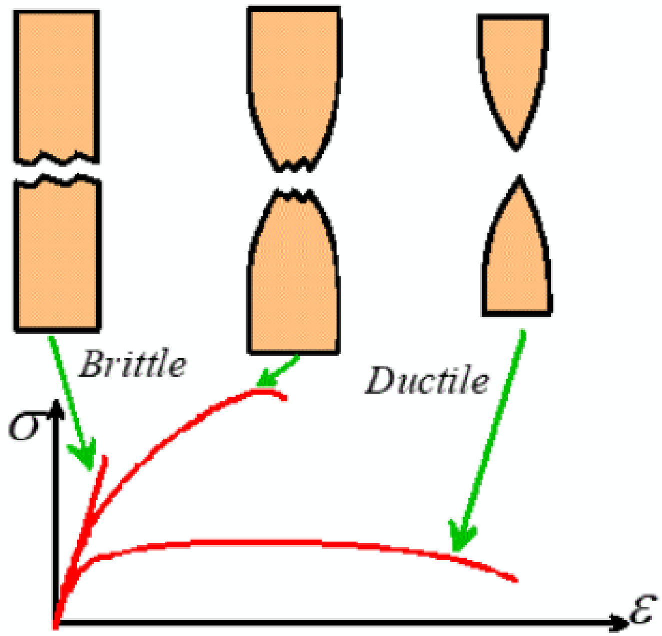


14.2 Material failure under monotonic loading

Two types of failure

(a) "Brittle": little permanent deformation
faceted fracture surface

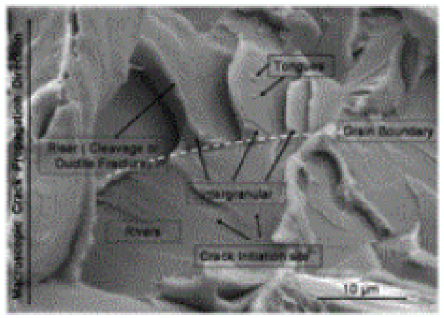
(b) "Ductile": lots of permanent deformation
necking might precede
material failure
dimpled fracture
surface

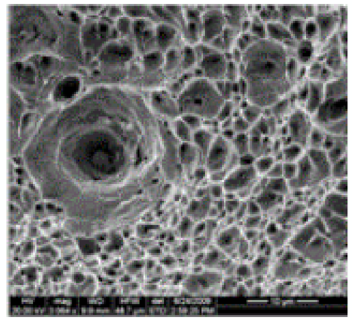


Different failure criteria are used

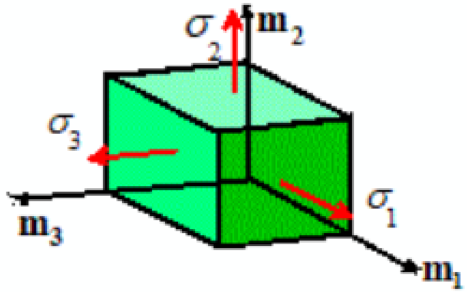
(a) Stress-based criteria

(b) Strain-based "


Cleavage fracture (brittle)


Dimpled rupture (ductile)

(a) Isotropic brittle fracture criteria
(use for glass, ceramics, concrete etc)



(1) Find principal stresses $\{\sigma_1, \sigma_2, \sigma_3\}$

(2) Assume $\sigma_1 > \sigma_2 > \sigma_3$

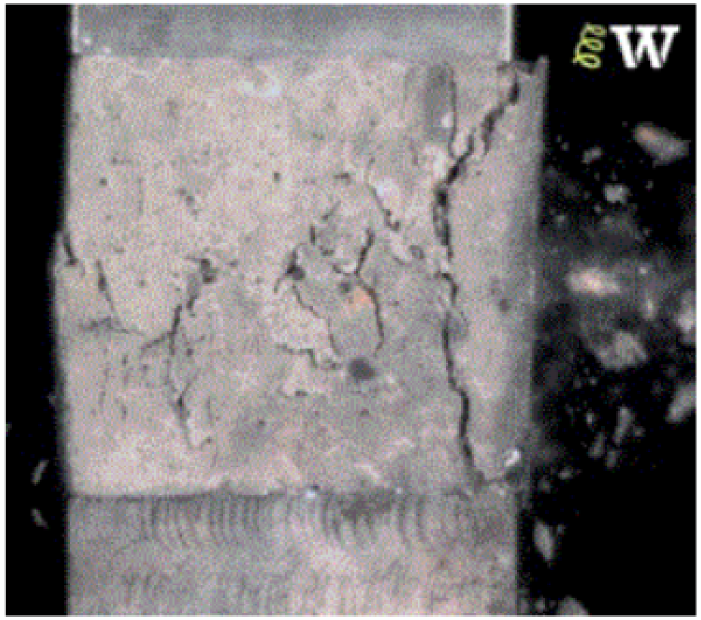
If $\sigma_1 > 0$ failure by fracture

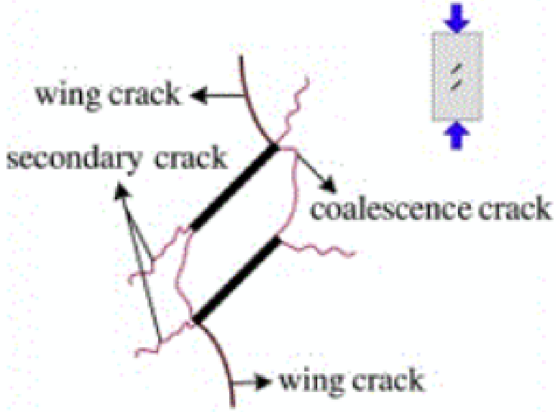
$\sigma < \sigma_{fract}$ safe
$\sigma = \sigma_{fract}$ fail

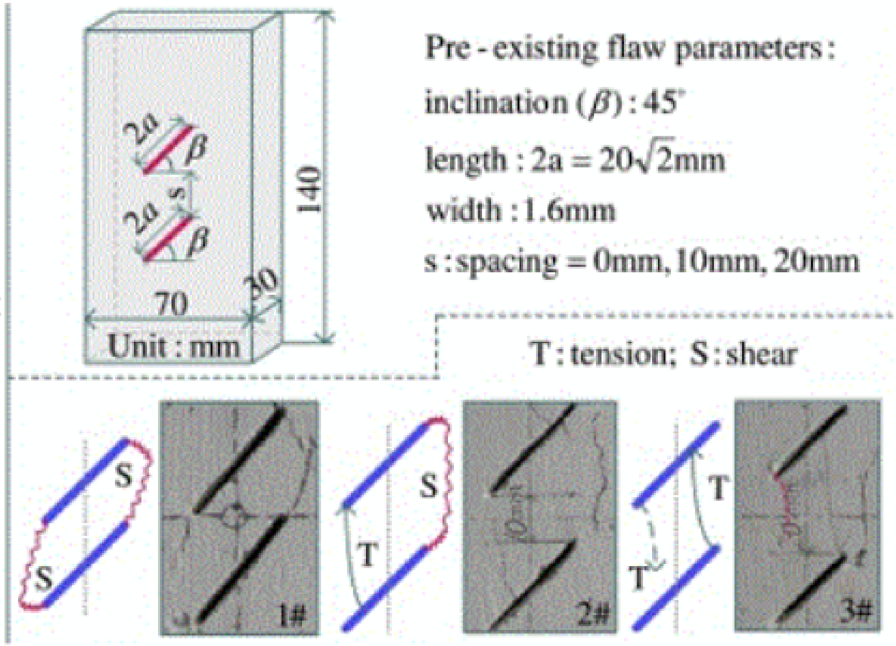
σ_{fract} is a material prop

[Advanced methods:
(1) Statistical methods "Weibull statistics"
(2) Fracture mechanics: measure crack size calculate critical stress required for crack to grow]

(3) For $\sigma_1 < 0$ failure will occur by crushing







Many criteria exist; eg "Mohr - Coulomb" criterion

Define $\tau = (\sigma_1 - \sigma_3)/2$ max shear stress

$\sigma_m = (\sigma_1 + \sigma_3)/2$ Normal stress on plane with max shear

Failure criterion

$$\tau + \sigma_m \sin\phi - \sigma_c \cos\phi \;\begin{cases} < 0 & \text{safe} \\ = 0 & \text{failure} \end{cases}$$

$\{\phi, \sigma_c\}$ material properties

$\tan\phi$ = friction coefficient between crack faces
σ_c = measure of shear strength

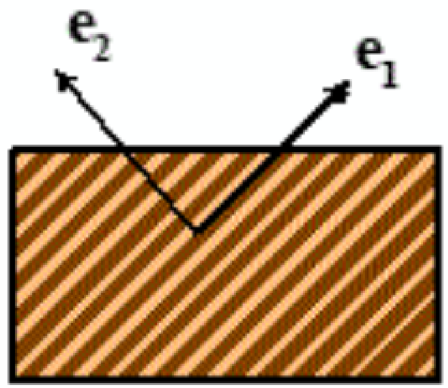
Measure $\{\phi, \sigma_c\}$ experimentally

Need 2 tests eg torsion $(\sigma_m = 0)$
Uniaxial compression $\sigma_m = \sigma/2$
$\tau = \sigma/2$

(b) Anisotropic brittle fracture criteria (used for composites)

Example: "Tsai-Hill" criterion for laminated fiber-reinforced composites

Find stress state; express as components in basis parallel & perpendicular to fibers $\{\underset{\sim}{e}_1, \underset{\sim}{e}_2\}$



$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

Fracture criterion

$$\left(\frac{\sigma_{11}}{\sigma_{TS1}}\right)^2 + \left(\frac{\sigma_{22}}{\sigma_{TS2}}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{(\sigma_{TS1})^2} + \left(\frac{\sigma_{12}}{\sigma_{SS}}\right)^2 \begin{array}{l} < 1 \text{ safe} \\ \geq 1 \text{ fail} \end{array}$$

Here $\{\sigma_{TS1}, \sigma_{TS2}, \sigma_{SS}\}$ are material props

Measure with 3 expts

(1) Load along fiber $\Rightarrow$ get σ_{TS1}

(2) " transverse to " $\Rightarrow$ σ_{TS2}

(3) Load at 45° to fiber $\Rightarrow$ $\sigma_{11} = \sigma_{22} = \sigma_{12} = \sigma$

Deduce σ_{SS}