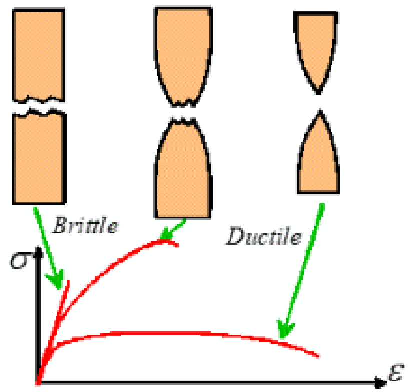
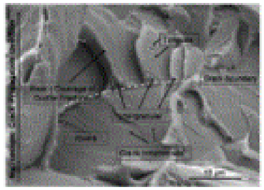
Review

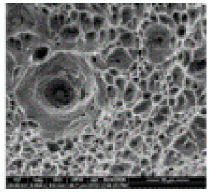
Failure under monotonic loading

1. Brittle failure – little permanent deformation prior to failure; faceted failure surface
2. Ductile failure – extensive permanent deformation prior to failure; dimpled fracture surface





Cleavage fracture (brittle)



Dimpled rupture (ductile)

Brittle failure criteria

$\{\sigma_1, \sigma_2, \sigma_3\}$ Principal stresses $\sigma_1 > \sigma_2 > \sigma_3$

$\tau = (\sigma_1 - \sigma_3)/2 \qquad \sigma_m = (\sigma_1 + \sigma_3)/2$

Isotropic failure criteria

$\sigma_1 > 0$ Failure by fracture when $\sigma_1 = \sigma_f$

$\sigma_1 < 0$ Failure by crushing (eg Mohr Coulomb criterion)

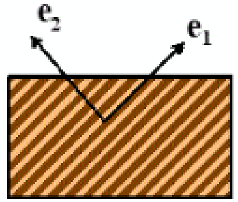
Anisotropic failure criteria (eg Tsai-Hill for composite laminates)

Basis parallel/perpendicular to fibers $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$

Plane stress state $\{\sigma_{11}, \sigma_{22}, \sigma_{12}\}$

Failure criterion

$$\left(\frac{\sigma_{11}}{\sigma_{TS1}}\right)^2 + \left(\frac{\sigma_{22}}{\sigma_{TS2}}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{\sigma_{TS1}} + \frac{\sigma_{12}}{\sigma_{SS}} = 1$$

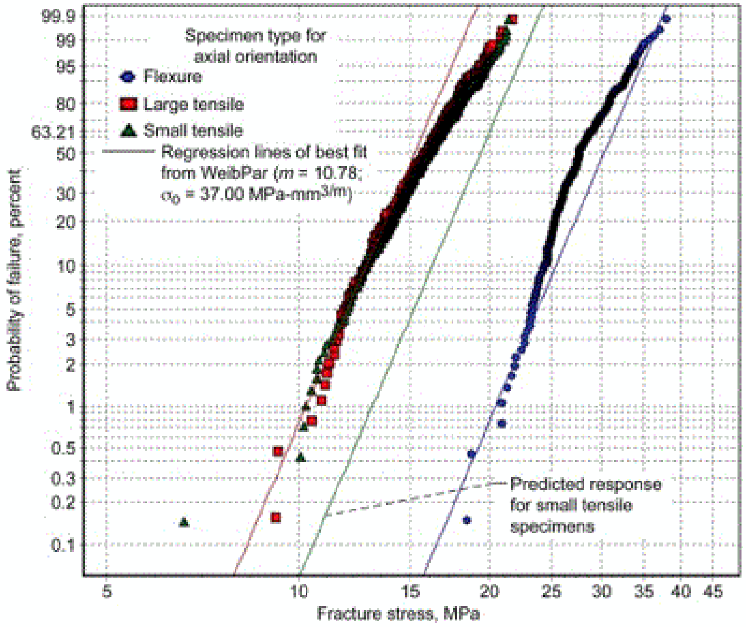


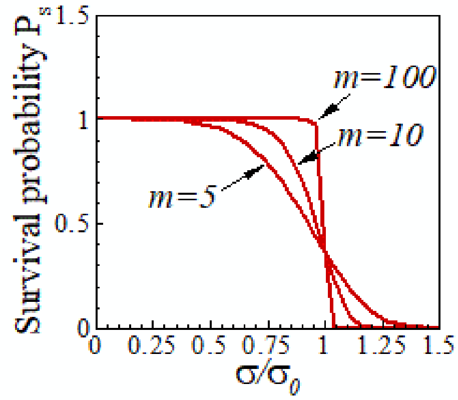
Probabilistic Brittle Failure criterion (Weibull statistics)

Experimental Observations

(1) Large scatter in failure stress

(2) Failure stress depends on specimen volume & geometry





Fracture statistics in graphite https://doi.org/10.1016/j.carbon.2013.02.054

Approach to predicting failure: fit survival probability with "Weibull distribution"

$$P_s = \exp\left\{ -\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m \right\}$$

(σ_0, m) material props

V_0 - specimen volume

Measuring σ_0, m

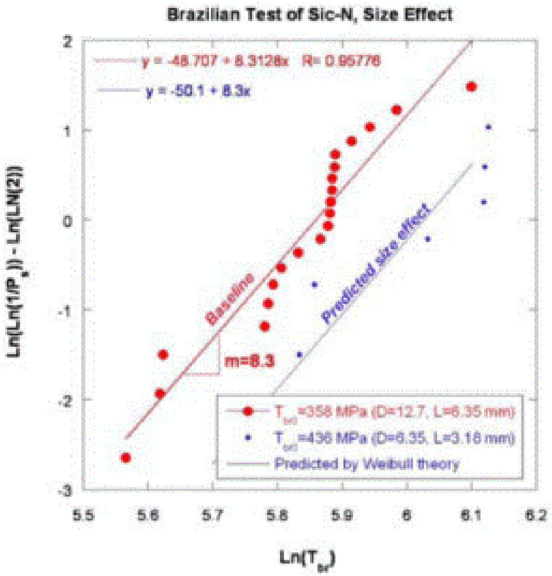
Do large number of tests; measure P_s as function of σ for specimens with volume $V = V_0$

To find σ_0, m note $\log P_s = -\underbrace{\cancel{\frac{V}{V_0}}}_{=1}\left(\frac{\sigma}{\sigma_0}\right)^m = -\left(\frac{\sigma}{\sigma_0}\right)^m$

$$\Rightarrow \log\left(\log\left(\frac{1}{P_s}\right)\right) = m\log\sigma - m\log\sigma_0$$

Slope gives m

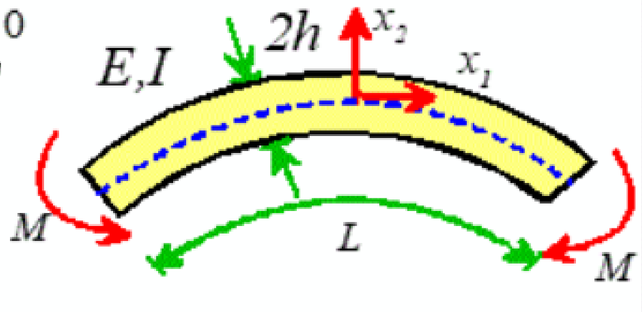
Intercept gives $m\log\sigma_0$



Example: A brittle beam with Youngs modulus *E*, inertia $I_{22} = I_{11} = I,\ I_{12} = 0$ and length *L* and rectangular cross-section 2hxb is bent by a moment *M*

The beam has Weibull parameters V_0, σ_0, m

Find a formula for the probability for the beam to survive



Use formula: Extend to multiaxial loading with nonuniform stress

Survival probability

$$\log(P_s) = -\frac{1}{V_0 \sigma_0^m} \int_V \left(\langle \sigma_1 \rangle^m + \langle \sigma_2 \rangle^m + \langle \sigma_3 \rangle^m \right) dV$$

where $(\sigma_1, \sigma_2, \sigma_3)$ are principal stresses

$$\langle x \rangle = \begin{cases} x & x \geq 0 \\ 0 & x \leq 0 \end{cases}$$

Apply to beam $\sigma_{11} = \frac{M x_2}{I}$ all other $\sigma_{ij} = 0$

Hence $\log(P_s) = -\frac{1}{V_0 \sigma_0^m} \int_0^b \int_0^L \int_{-h}^{h} \left\langle \frac{M x_2}{I} \right\rangle^m dx_2 \, dx_1 \, dx_3$

$$= - \frac{\overbrace{bLh}^{V/2}}{V_0 \sigma_0^m} \frac{1}{(m+1)} \left(\frac{Mh}{I} \right)^m$$

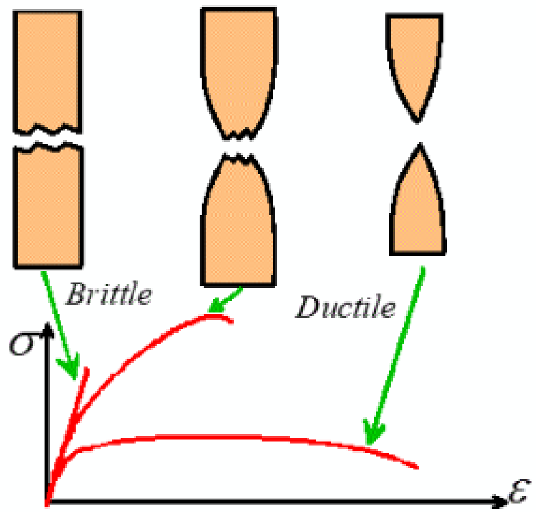
note $\frac{Mh}{I} = \sigma_{max}$

$$P_s = \exp\left\{ - \frac{V}{2V_0 (m+1)} \left(\frac{\sigma_{max}}{\sigma_0} \right)^m \right\}$$

Survival probability in bending > survival probability in tension

Failure criteria for ductile materials

Failure occurs by nucleation & growth of voids ; driven by plastic strain
Voids grow faster in regions of high hydrostatic tension



(1) Simple strain based failure criterion

Let ε_e = Von Mises plastic strain

$\varepsilon_e < \varepsilon_f$ safe

$\varepsilon_e = \varepsilon_f$ fail

ε_f material prop

Use torsion test to measure ε_f

[Not very accurate]

(2) Johnson - Cook failure model

- accounts for temperature, strain rate, hydrostatic stress

Let σ_e - Von-Mises stress

$\sigma_h = \sigma_{kk}/3$: hydrostatic stress

$\dot{\varepsilon}_e$ - Von Mises plastic strain rate

Define "Damage parameter" D

$$D = \int_0^t \frac{\dot{\varepsilon}_e \, dt}{\left[d_1 + d_2 \exp\left(\frac{d_3 \sigma_h}{\sigma_e}\right)\right]\left[1 + d_4 \frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_0}\right]}$$

$(\dot{\varepsilon}_0, d_1, d_2, d_3, d_4)$ material properties

Lots of experiments needed to determine these

Failure criterion

$D < 1$ everywhere safe

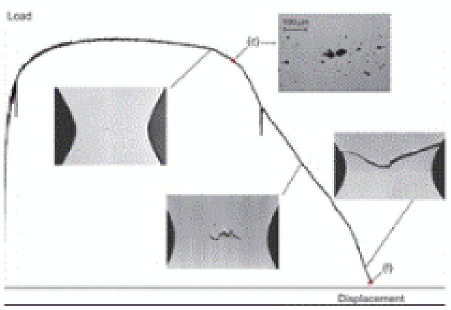
$D = 1$ fail

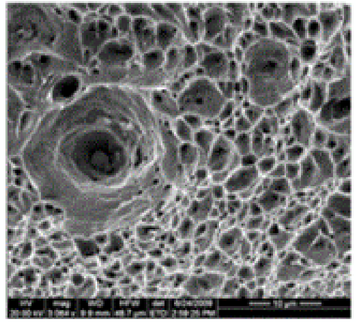
ABAQUS can delete elements when $D = 1$ to simulate fracture

Porous plasticity 'Gurson' model in ABAQUS

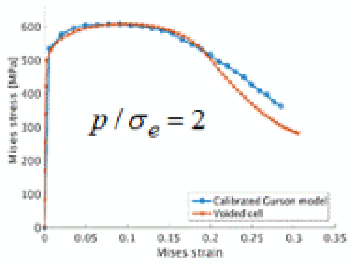
Failure mechanism in ductile metals

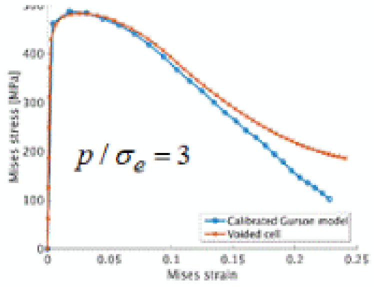
1. Specimen necks
2. Hydrostatic (tensile) stress increases in neck
3. Voids nucleate at 2nd phase particles
4. Voids grow
5. Voids coalesce to form crack

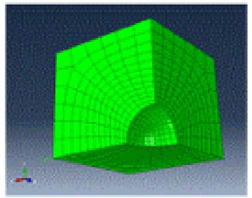


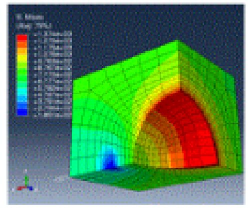


Simulations of void growth









Gurson (porous plasticity model)

Mises stress   Void volume fraction   Hydrostatic stress

Plastic strain rate magnitude $\dot{\varepsilon}_e = g(\sigma_e, p, \sigma_0, f^*) = \dot{\varepsilon}_0 \left[\left(\frac{\sigma_e}{Y(\varepsilon_e)} \right)^2 + 2 q_1 f^* \cosh\left(q_2 \frac{3p}{2Y(\varepsilon_e)} \right) - (q_1 f^*)^2 - 1 \right]^{m/2}$

Yield stress

Void growth and nucleation $\dot{f} = (1-f)\dot{\varepsilon}^p_{kk} + N_v \dot{\varepsilon}_e$

Void nucleation rate $N_v = \begin{cases} \frac{f_N}{s_N\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left[\frac{\varepsilon^p_m - \varepsilon_N}{s_N} \right] \right) \dot{\varepsilon}_e & p > 0 \\ 0 & p < 0 \end{cases}$

Void coalescence $f^* = \begin{cases} f & f < f_c \\ f_c + (1/q_1 - f_c)(f - f_c)/(f_F - f_c) & f \geq f_c \end{cases}$

Element deleted when $f = f_F$

Material Properties

Yield and hardening $Y(\varepsilon_e)$

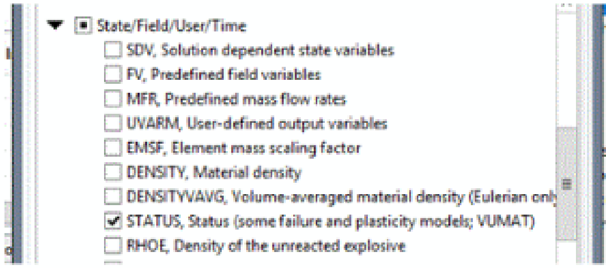
Void growth rate q_1, q_2

Void coalescence f_c, f_F

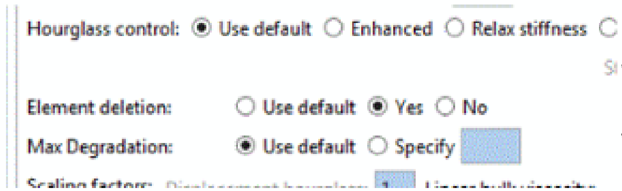
Void nucleation rate f_N, s_N, ε_N

Running ABAQUS/Explicit with Gurson Model

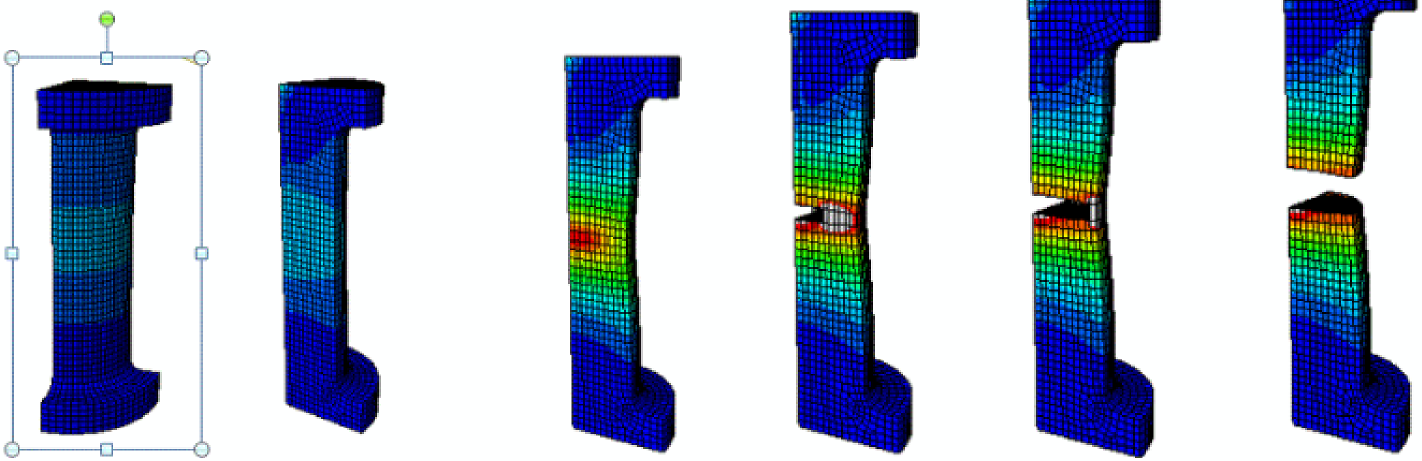
1. Set up geometry, section, etc in usual way.
2. In the Material module select Porous Metal Plasticity
3. Enter values for q1,q2,q3 (can use 1 for each)
4. Use the Suboptions button to define $f_N, s_N, \varepsilon_N \quad f_c, f_F$
5. Define the initial value of r=1-f in the 'Relative Density'
6. Create part instance in assembly in usual way
7. Create an explicit dynamic step. To enable element deletion, use Results->Field Output and in the dialog make sure the 'Status' option is checked



8. Apply boundary conditions in usual way
9. Mesh solid – in the Element Type menu check the box for 'Element Deletion'



10. Run job in usual way
11. WARNING: Simulations with models like this are always mesh sensitive once material starts to soften – softening rate gets faster as mesh is refined.

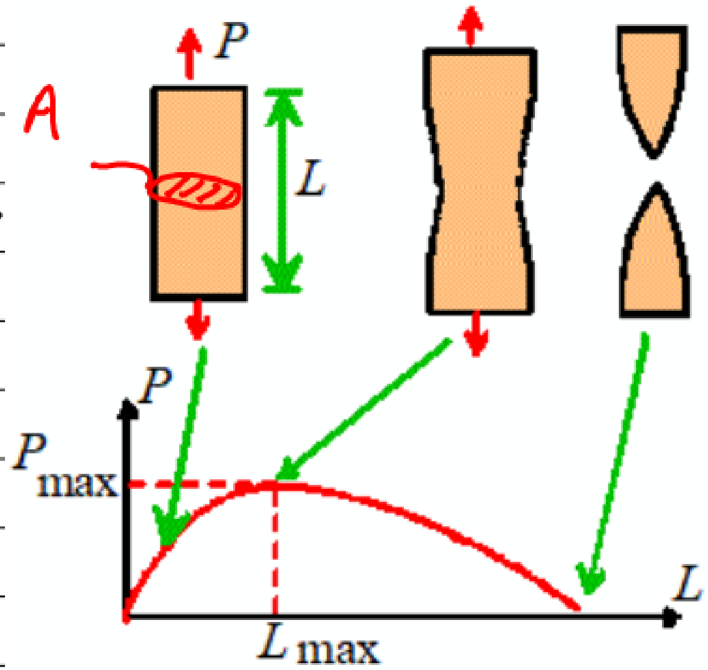


Ductile failure by necking & strain localization

In ductile materials failure can occur by material losing strength ; or geometric changes (or both)

Predicting necking in tension

Predict P_{max} in uniaxial tension test



Assume material has true stress-v-true strain curve

$$\sigma = Y(\varepsilon) \qquad \varepsilon = \log(L/L_0)$$

Note $\frac{dP}{dL} = 0$ when $P = P_{max}$

Note also $P = A\sigma$

$$\frac{dP}{dL} = \frac{dA}{dL}\sigma + A\frac{d\sigma}{d\varepsilon}\frac{d\varepsilon}{dL} \qquad \frac{d\varepsilon}{dL} = \frac{1}{L}$$

To find $\frac{dA}{dL}$ assume vol is constant $\Rightarrow AL = \text{const}$

$$\Rightarrow \frac{dA}{dL}L + A = 0 \qquad \Rightarrow \frac{dA}{dL} = -\frac{A}{L}$$

$$\Rightarrow -\frac{A}{L}\sigma + \frac{A}{L}\frac{d\sigma}{d\varepsilon} = 0$$

At P_{max} $\boxed{\sigma = \frac{d\sigma}{d\varepsilon}}$ "Considere criterion"

Examples:

Linear hardening $\quad Y = Y_0 + h\varepsilon$

$$\Rightarrow \quad Y_0 + h\varepsilon_f = h \Rightarrow \quad \varepsilon_f = 1 - \frac{Y_0}{h}$$

Power-law $\quad Y = Y_0\left(\frac{\varepsilon}{\varepsilon_0}\right)^m \quad m < 1$

$$\frac{d\sigma}{d\varepsilon} = \sigma \Rightarrow \quad \frac{m\,Y_0}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon_0}\right)^m = Y_0\left(\frac{\varepsilon}{\varepsilon_0}\right)^m$$

$\Rightarrow \quad \varepsilon_f = m$ at point of failure

High hardening, low flow stress makes material resist necking

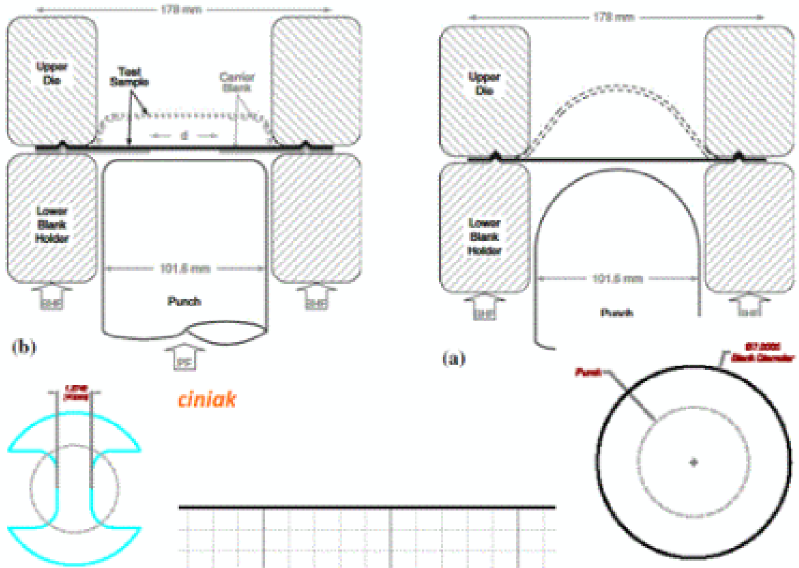
In general parts can fail by local thinning

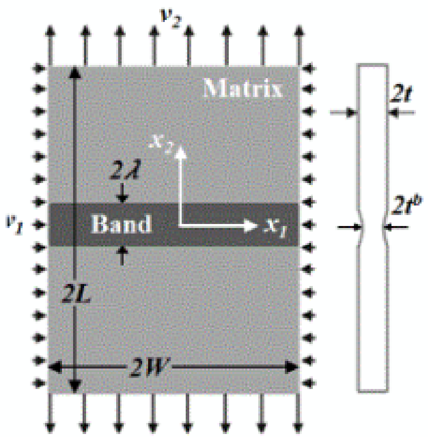
Necking in sheet metals – Forming Limit Diagram

'Forming Limit Diagram' measured using both Marciniak and Nakajima experiments

Predicted with 'M-K' analysis – calculate strain that causes necking in a thin sheet with a small defect

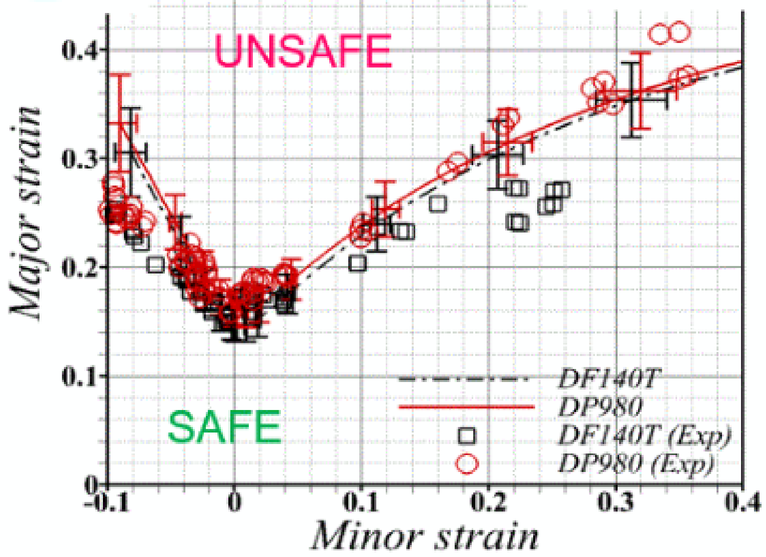
FLD can be defined in ABAQUS as a failure criterion





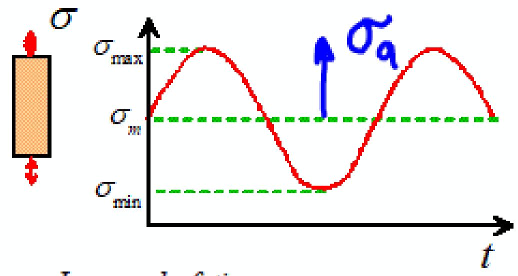
Major Strain ε_{22}

Minor Strain ε_{11}



¶4.3 Simple criteria for failure under cyclic loading

Typical behavior in cyclic tension:

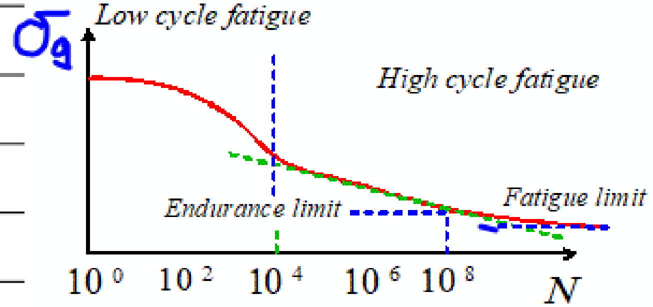


(1) "Low Cycle" fatigue

$N < 10^4$

Yield stress exceeded

Plastic strain controlled



(2) "High Cycle" fatigue

$N > 10^4$

Below yield

Stress controlled

cycles to failure

"Endurance Limit" : critical stress amplitude to survive 10^8 cycles

"Fatigue Limit" : critical stress " for ∞ life

Failure criterion for high cycle fatigue
"Basquin's law"

$$N^b \sigma_a = C$$

(b, C) material props

where $\sigma_a = (\sigma_{max} - \sigma_{min})/2$

C depends on mean stress $\sigma_m = (\sigma_{max} + \sigma_{min})/2$

Goodman's rule $\quad C = C_0 \left(1 - \dfrac{\sigma_m}{\sigma_{UTS}} \right)$

Where C_0 is value of C for fully reversed loading
$\sigma_m = 0$

σ_{UTS} = Tensile strength

Low Cycle Fatigue criterion

Low cycle fatigue controlled by plastic strain amplitude

$$N^d \Delta\varepsilon^p = D$$

where (d, D) are material properties

"Coffin - Manson" law.