School of Engineering
Brown University

EN2210: Continuum Mechanics

Midterm Examination
Wed Nov 7 2012

NAME: ______________________________________

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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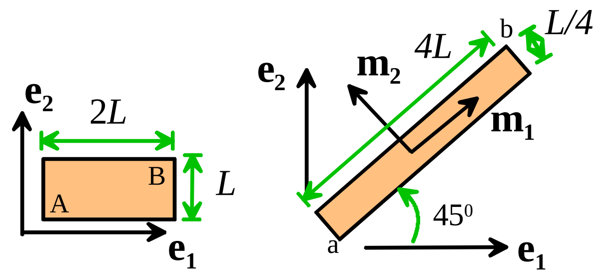
1 (14 points) _____________

2. (5 points) _____________

3. (11 points) _____________

TOTAL (30 points)___________

1. The figure shows the reference and deformed configurations for a solid. The out-of-plane dimensions are unchanged. Points a and b are the positions of points A and B after deformation. Determine



1.1 The right stretch tensor **U**, expressed as components in $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. (A 2x2 matrix is sufficient). There is no need for lengthy calculations – you may write down the result by inspection.

[2 POINTS]

1.2 The rotation tensor **R** in the polar decomposition of the deformation gradient **F=RU=VR**

[**2 POINTS]**

1.3 The deformation gradient, expressed as components in $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$. Try to do this without using the basis-change formulas.

[3 POINTS]

1.4 If the material has a constitutive relation such that the material stress is related to Lagrange strain by:

$$\Sigma_{ij} = \mu E_{ij}$$

where μ is a material constant, show that the Cauchy stress is related to the left Cauchy-Green stretch tensor **V** by

$$\boldsymbol{\sigma} = \frac{\mu}{2J}(\mathbf{V}^4 - \mathbf{V}^2)$$

where **I** is the identity tensor.

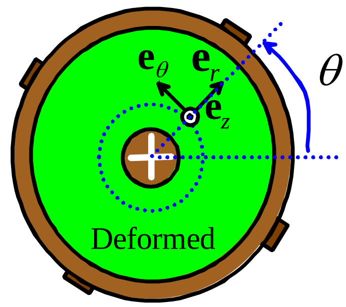
[5 POINTS]

1.5 Hence, determine an expression for the Cauchy stress components in $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$ and write down the values of the principal Cauchy stresses.

[2 POINTS]

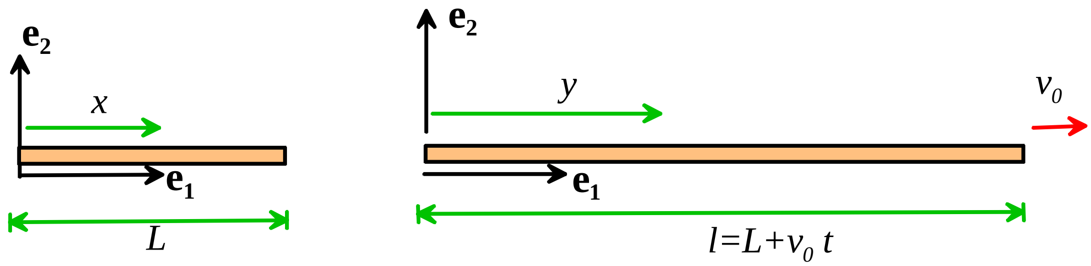
2. The figure shows the shear viscometer considered in a recent homework problem. The stress distribution in the fluid is known to have the form $\boldsymbol{\sigma} = \sigma_{r\theta}(\mathbf{e}_r \otimes \mathbf{e}_\theta + \mathbf{e}_\theta \otimes \mathbf{e}_r)$, where $\sigma_{r\theta}$ is a function of *r*. Assume that the stress state is in static equilibrium and neglect body forces. By considering a virtual velocity field of the form $\mathbf{v} = \delta v_\theta(r)\mathbf{e}_\theta$, use the principle of virtual work to show that the equilibrium equation for the stress field reduces to

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2\sigma_{r\theta}}{r} = 0$$



[5 POINTS]

3. A rubber band has initial length L. One end of the band is held fixed. For time $t>0$ the other end is pulled at constant speed v_0. Following the usual convention, let x denote position in the reference configuration, and let y denote position in the deformed configuration. Assume one dimensional, homogeneous deformation.



3.1 Write down the (horizontal component) of position y of a material particle as a function of its initial position x and time t.

[1 POINT]

3.2 Hence, determine the (horizontal) velocity distribution as both a function of x and a function of y.

[2 POINTs]

3.3 Find the deformation gradient (you only need to state the one nonzero component)

[1 POINT]

3.4 Find the velocity gradient (again, only one component is needed)

[1 POINT]

3.5 Suppose that a fly walks along the rubber band with speed w relative to the band. Calculate the acceleration of the fly as a function of time and other relevant variables.

[2 POINTS]

3.6 Suppose that the fly is at $x=y=0$ at time $t=0$. Find how long it takes for the fly to walk to the other end of the rubber band, in terms of L, v_0 and w. It is easiest to do this by calculating dx/dt for the fly.

[4 POINTS]