School of Engineering
Brown University

EN2210: Continuum Mechanics

Homework 3: Kinematics
Due Wednesday Oct 5, 2016

1. The infinitesimal strain field in a long cylinder containing a hole at its center is given by

$$\varepsilon_{31} = -bx_2 / r^2 \qquad \varepsilon_{32} = bx_1 / r^2 \qquad r = \sqrt{x_1^2 + x_2^2}$$

(a) Show that the strain field satisfies the equations of compatibility.

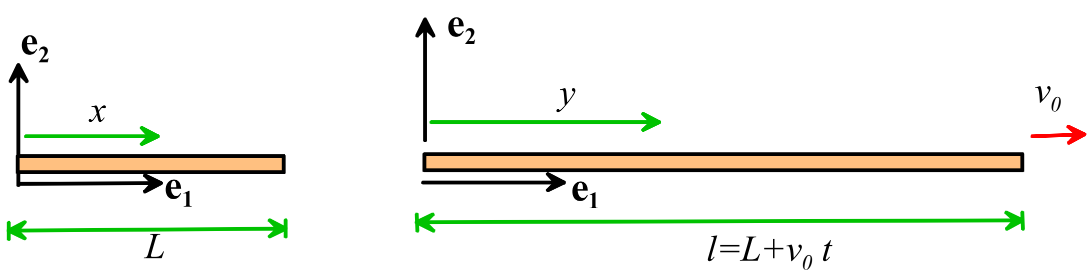
(b) Show that the strain field is consistent with a displacement field of the form $u_3 = \theta$, where $\theta = 2b\tan^{-1} x_2 / x_1$. Note that although the strain field is compatible, the displacement field is *multiple valued* – i.e. the displacements are not equal at $\theta = 2\pi$ and $\theta = 0$, which supposedly represent the same point in the solid. Of course, displacement fields like this do exist in solids – they are caused by dislocations in a crystal.

2. Calculate the displacement field that generates the following 3D infinitesimal strain field

$$\varepsilon_{ij} = (1+\nu)(x_k x_k \delta_{ij} + 2x_i x_j) - (3-\nu)\delta_{ij}$$

(it is easier to do this using the method of integrating strain components than the formal path integral)

3. A rubber band has initial length *L*. One end of the band is held fixed. For time *t>0* the other end is pulled at constant speed v_0. Following the usual convention, let *x* denote position in the reference configuration, and let *y* denote position in the deformed configuration. Assume one dimensional deformation.



3.1 Write down the position *y* of a material particle as a function of its initial position *x* and time *t*.

3.2 Hence, determine the velocity distribution as both a function of *x* and a function of *y*.

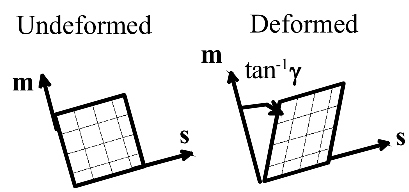
3.3 Find the deformation gradient (you only need to state the one nonzero component)

3.4 Find the velocity gradient

3.5 Suppose that a fly walks along the rubber band with speed w relative to the band. Calculate the acceleration of the fly as a function of time and other relevant variables.

3.6 Suppose that the fly is at $x=y=0$ at time $t=0$. Find how long it takes for the fly to walk to the other end of the rubber band, in terms of L, v_0 and w. It is easiest to do this by calculating dx/dt for the fly.

4. A single crystal deforms by shearing on a single active slip system as illustrated in the figure. The crystal is loaded so that the slip direction $\mathbf{s}$ and normal to the slip plane $\mathbf{m}$ maintain a constant direction during the deformation



(a) Show that the deformation gradient can be expressed in terms of the components of the slip direction $\mathbf{s}$ and the normal to the slip plane $\mathbf{m}$ as $F_{ij} = \delta_{ij} + \gamma s_i m_j$ where γ denotes the shear, as illustrated in the figure.

(b) Suppose shearing proceeds at some rate $\dot{\gamma}$. At the instant when $\gamma = 0$, calculate (i) the velocity gradient tensor; (ii) the stretch rate tensor and (iii) the spin tensor associated with the deformation.

(c) Find an expression for the stretch rate and angular velocity of a material fiber parallel to a unit vector $\mathbf{n}$ in the deformed solid, in terms of $\dot{\gamma}, \mathbf{s}, \mathbf{m}$.

5. Derive the identities relating accelerations to velocity gradient, stretch rate and vorticity

$$a_i = \left.\frac{\partial v_i}{dt}\right|_{\mathbf{y}} + \frac{1}{2}\frac{d}{dy_i}(v_k v_k) + 2W_{ij}v_j$$

$$\in_{ijk}\frac{\partial a_k}{\partial y_j} = \left.\frac{\partial \omega_i}{\partial t}\right|_{\mathbf{x}=const} - D_{ij}\omega_j + \frac{\partial v_k}{\partial y_k}\omega_i$$

6. Show that $\mathbf{D} = \mathbf{F}^{-T}\dfrac{d\mathbf{E}}{dt}\mathbf{F}^{-1}$

7. Let $\mathbf{n}$ be a unit vector parallel to infinitesimal material fiber in a deforming solid. Show that

$$\frac{d\mathbf{n}}{dt} = \mathbf{Dn} + \mathbf{Wn} - (\mathbf{n}\cdot\mathbf{Dn})\mathbf{n}$$

8. Derive the transport formula

$$\frac{d}{dt}\int_S \phi n_i dA = \int_S \left(\delta_{ij}\left.\frac{\partial \phi}{\partial t}\right|_{\mathbf{x}=const} + \delta_{ij}\phi\frac{\partial v_k}{\partial y_k} - \phi\frac{\partial v_j}{\partial y_i} \right) n_j dA$$