EN234: Computational methods in Structural and Solid Mechanics

Homework 3: Interpolation and integration
Due Wed Oct 2, 2013

School of Engineering
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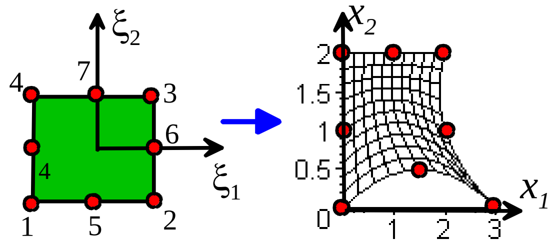
1. Consider the mapped, 8 noded isoparametric element illustrated in the figure (shape functions are given in solidmechanics.org or you can just cut and paste them out of the MATLAB 2D or 3D linear elastic code). Write a simple program to plot contours of the determinant $\det(dx_i / d\xi_j)$ as a function of ξ_1, ξ_2



- $x_1^{(1)} = 0,\ \ x_2^{(1)} = 0,\ \ x_1^{(2)} = 2.0,\ \ x_2^{(2)} = 2.0,$
  $x_1^{(3)} = 2.0,\ \ x_2^{(3)} = 6.0,\ \ x_1^{(4)} = 0.0,\ \ x_2^{(4)} = 4.0,$
  $x_1^{(5)} = 1.0,\ \ x_2^{(5)} = 1.0,\ \ x_1^{(6)} = 2.0,\ \ x_2^{(6)} = 4.0,$
  $x_1^{(7)} = 1.0,\ \ x_2^{(7)} = 5.0,\ \ x_1^{(8)} = 0.0,\ \ x_2^{(8)} = 2.0$
- $x_1^{(1)} = 0,\ \ x_2^{(1)} = 0,\ \ x_1^{(2)} = 2.0,\ \ x_2^{(2)} = 0.,\ \ x_1^{(3)} = 3.0,\ \ x_2^{(3)} = 0.0,\ \ x_1^{(4)} = 0.0,\ \ x_2^{(4)} = 2.0$
  $x_1^{(5)} = 1.7,\ \ x_2^{(5)} = 0.50,\ \ x_1^{(6)} = 2.0,\ \ x_2^{(6)} = 1.0,\ \ x_1^{(7)} = 1.0,\ \ x_2^{(7)} = 2.0,\ \ x_1^{(8)} = 0.0,\ \ x_2^{(8)} = 1.0$

Note that for the latter case, there is a region in the element where $\det\left(\partial x_i / \partial \xi_j\right) < 0$. This is unphysical. Consequently, if elements with curved sides are used in a mesh, they must be designed carefully to avoid this behavior. In addition, quadratic elements can perform poorly in large displacement analyses.

Hand in your contour plots as a solution to this problem

2. Write a simple code that will read an input file consisting of a 2D or 3D FEA mesh of quadrilateral or hexahedral elements (i.e. nodal points and element connectivity), and an origin, and will then compute (i) the components of inertia tensor (in the global coordinate system) for the solid about the fixed point and; (ii) the directions of the principal axes of inertia and the magnitudes of the principal axes of inertia. Recall that the inertia tensor has components

$$I_{ij} = (\delta_{ij} Q_{kk} - Q_{ij}) \qquad Q_{ij} = \int_V x_i x_j dV$$

Optional: Modify your code to find the coordinates of the center of mass, and the axes of inertia about the center of mass.

You should use the FEA interpolation and integration scheme to do the calculation:

1. Loop over the elements
2. For the *k*th element, compute the contribution to the volume or area integral using Gaussian quadrature over the standard square or cube, mapped to the actual spatial configuration. What order of integration is needed to evaluate the integrals?
3. Add the element contribution to the total inertia tensor

Hand in solutions to some example problems to demonstrate that your code is working as a solution to this problem.