EN234: Computational methods in Structural and Solid Mechanics
Homework 1: Basic FEA with ABAQUS
Due Wed Sept 23, 2015

School of Engineering
Brown University


1. The figure shows an FEA simulation of a rigid sphere rebounding of a soft rubber thin film.

- The sphere is idealized as a near-incompressible neo-Hookean hyperelastic material with shear modulus $\mu$, bulk modulus $K$ and mass density $\rho$. You can find some background information about hyperelastic materials at http://solidmechanics.org/Text/Chapter3_5/Chapter3_5.php. You can also read section 19.5.1 of the ABAQUS theory manual to familiarize yourself with the ABAQUS version of the neo-Hookean material (in particular notice that their material constants $C_{10}$ and $D_{1}$ are not the same as those in the textbook). We will focus on the limit where $K \gg \mu$.
- The sphere has radius $R$, mass $m$, and is launched with initial velocity $V_{0}$ towards the rigid surface.

Your goal is to run a finite element simulation of this problem. Various quantities could be calculated in this analysis, and you should feel free to calculate anything that might be interesting. But to focus on something specific, we will calculate (i) The time that the sphere is in contact with the surface $t_{0}$; and (ii) the force of the impact $F(t)$.

Before doing any analysis, either:
1.1 Write down the equations and boundary conditions that govern the solution. You will need equations for the film and for the rigid body motion of the sphere.
1.2 Introduce normalized measures of stress, time, and displacement as

$$
\hat{\sigma}_{i j}=\sigma_{i j} / \mu \quad \hat{u}_{i}=u_{i} / R \quad \hat{x}_{i}=x_{i} / R \quad \hat{t}=t V_{0} / R
$$

Or, if you have not taken enough mechanics courses to be able to write down the equations, write down a list of the variables, and rearrange expressions for $t_{0}$ and $F(t)$ in dimensionless form

Hence show that the solution is governed by two dimensionless groups:

$$
\frac{\mu R^{3}}{m V_{0}^{2}}, \frac{\rho}{\mu} V_{0}^{2}
$$

in addition to dimensionless geometric variables $R / H, L / H$
In the quasi-static limit $\rho V_{0}^{2} / \mu \rightarrow 0$, so only the first parameter is important.

Now set up an ABAQUS simulation of the problem. The following parameters are suggested, at least for an initial computation. You can explore variations after getting an initial simulation to work:

- The sphere should be a 2D axisymmetric discrete rigid surface (analytical rigid surface is OK too, but some people seem to be unable to get ABAQUS to accept a 180 degree semi-circle with this option). Use a radius $R=0.05$ (arbitrary units).
- You will need to add a reference point to the rigid object to be able to apply boundary conditions. In the Part module, use the Feature>Reference Point... menu to do this. It is best to put the reference point on the axis of symmetry. Assign the sphere a mass of 0.2 units. (To do this use Special>Inertia in the property editor, and assign the mass to the Reference Point)
- The film should be rectangular, with a thickness 0.05 units. The length is arbitrary - select a sufficiently large value to ensure that the solution will not be influenced by the choice. The film should be assigned material properties representing a hyperelastic neo-hookean material with $C_{10}=1$ and mass density $\rho=1$ ( units are arbitrary since we are working in dimensionless form). Leave the $D$ field blank (this makes ABAQUS use a default value to approximate incompressibility)
- In the assembly, place the sphere a small distance above the thin film surface. The distance is not important.
- The interaction between the sphere and the surface should be hard normal contact and frictionless tangential contact. The options for the contact (node region -v-surface ) are not important - just use the surface to surface contact.
- For boundary conditions, constrain the radial displacement on the axis of symmetry of both solids, and assign zero vertical displacement to the base of the film. Assign the sphere a downwards vertical velocity of 0.05 units (in the Load module use Predefined Field; select the Initial step in the window at the top, and check the Mechanical radio button - this will then allow you to select Velocity). Constrain the rotation of the reference point on the sphere to prevent rigid body rotation.
- Use an explicit dynamic step, with time period somewhere between 1 and 5, depending on how far above the film you start the sphere (the analysis should be long enough to simulate the whole contact event). You can accept defaults for all other parameters. Make sure the NLGEOM parameter is set so you run a finite strain calculation. In the Step module create a History output that will record the contact force during the collision - use Output > History Output Requests > Create, and select Interaction for the Domain; then you can select the surface interaction created while defining the contact. You can accept the default variables that will be saved, but you won't need most of them....
- Mesh the solid with reduced integration axisymmetric stress linear quadrilateral elements (the default for quadrilateral elements in an explicit dynamic analysis. Choose a mesh size that you think is sensible. If you used a discrete rigid surface instead of an analytical rigid surface you will need to mesh the rigid sphere as well as the block. To do this, seed the sphere in the usual way, then simply mesh the part (or instance if you made the mesh independent).
- Use the default options for the job submission.

When you have managed to get your simulation to run (watch an animation of the contours of Mises stress, to see what is happening), plot a graph showing the variation of the vertical reaction force acting on the rigid surface with time. Work out how to change (i) the font size for the axis titles and labels; (ii) the background color of the window so the graph is legible. In addition, choose appropriate dimensionless measures of force and time for the plot and modify the data to plot the dimensionless force and time (to do this you have to (i) Create an X-Y data set using Tools>XY Data>Create and select the History output; then modify the data using Tools $>$ XY Data $>$ Operate on XY data.

Also, calculate the total impulse exerted on the sphere during the collision.
As a solution to this homework, hand in your answers to 1.1-1.2, your plot of dimensionless force-v-time; and the number you compute for a suitable dimensionless measure of the impulse.

## Optional problems:

As further exploration, repeat the calculation for a few values of $\frac{\mu R^{3}}{m V_{0}^{2}}, \frac{\rho}{\mu} V_{0}^{2}$

More advanced ABAQUS users could try writing a python script to automatically run a series of computations varying $m$ and $\rho$, and hence plot a dimensionless graph of the contact time and maximum impact force as a function of $\frac{\mu R^{3}}{m V_{0}^{2}}$ for several values of $\frac{\rho}{\mu} V_{0}^{2}$.

