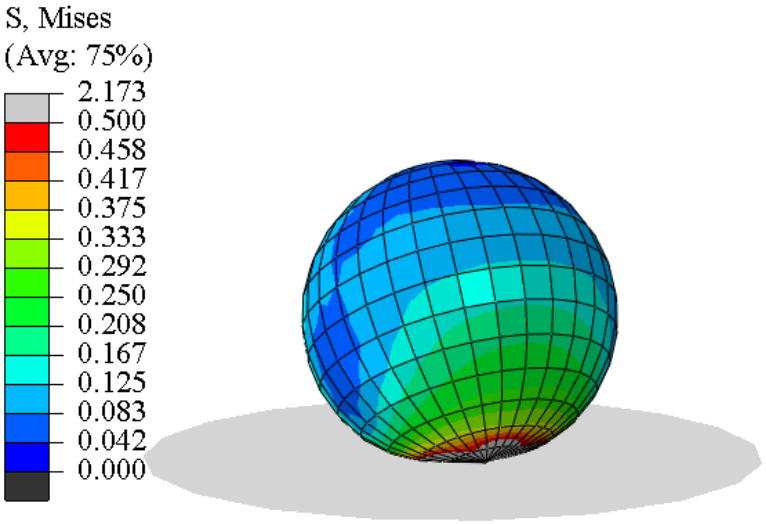
School of Engineering
Brown University

EN234: Computational methods in Structural and Solid Mechanics

Homework 1: Basic FEA with ABAQUS
Due Fri Sept 22, 2017



1. The figure shows an FEA simulation of a spinning hyperelastic sphere (available commercially as a 'superball') rebounding off a rigid surface. Before doing the analysis, you might find it helpful to watch an experimental video of such an impact in this recent publication (which also contains some relevant references). The goal of this problem is to set up a model of the impact in ABAQUS.

- The sphere is idealized as a near-incompressible neo-Hookean hyperelastic material with shear modulus μ, bulk modulus K and mass density ρ. You can find some background information about hyperelastic materials at http://solidmechanics.org/Text/Chapter3_5/Chapter3_5.php. You can also read section 19.5.1 of the ABAQUS theory manual to familiarize yourself with the ABAQUS version of the neo-Hookean material (in particular notice that their material constants C_{10} and D_1 are not the same as those in the textbook). We will focus on the limit where $K >> \mu$.
- The sphere has radius *R,* mass *m*, and is launched with initial angular velocity ω_0 and linear velocity V_0 at an angle α to the rigid surface.
- Coulomb friction, with friction coefficient *f* acts at the contact between the sphere and the rigid surface.

Your goal is to run a finite element simulation of this problem. Various quantities could be calculated in this analysis, and you should feel free to calculate anything that might be interesting. But to focus on something specific, we will calculate the angular and linear velocity of the sphere after impact, and use the results to determine the normal and tangential restitution coefficients.

Before doing any analysis, either:

1.1 Write down the equations and boundary conditions that govern the solution.

1.2 Introduce normalized measures of stress, time, and displacement as

$$\hat{\sigma}_{ij} = \sigma_{ij} / \mu \qquad \hat{u}_i = u_i / R \qquad \hat{x}_i = x_i / R \qquad \hat{t} = tV_0 / R$$

1.3 Re-write the equations in dimensionless form

Or, if you have not taken enough mechanics courses to be able to write down the equations, write down a list of the variables, and rearrange expressions for ω_1, v_1 in dimensionless form. Hence show that the solution is governed by the following dimensionless groups:

$$\frac{\rho}{\mu} V_0^2, \frac{R\omega_0}{V_0}, \alpha, f$$

2. Now set up an ABAQUS simulation of the problem. The following parameters are suggested, at least for an initial computation. You can explore variations after getting an initial simulation to work:

- The base should be a 3D analytical rigid surface. You can create it by revolving a horizontal line (the length is arbitrary, but make it at least 0.1 units long) about the axis of symmetry (the *y* axis in CAE). Place the base at position $y = -0.055$ (arbitrary units).
- You will need to add a reference point to the rigid object to be able to apply boundary conditions. In the Part module, use the Feature>Reference Point… menu to do this. It is best to put the reference point on the axis of symmetry.
- The sphere should be a 3D deformable solid with radius *R=0.05* (arbitrary units) with center at the origin. You can create the sphere by drawing a closed semicircle and revolving it through 360 degrees about the *y* axis.
- The sphere should be assigned material properties representing a hyperelastic neo-hookean material with $C_{10} = 1$ and mass density $\rho = 1$ (units are arbitrary since we are working in dimensionless form). Leave the *D* field blank (this makes ABAQUS use a default value to approximate incompressibility)
- In the assembly, place the sphere a small distance above the rigid surface. The distance is not important (if you put the rigid surface at $y = -0.055$ in the first step it should appear in the right place automatically).
- Use an implicit dynamic step, with time period of 0.4. Use a fixed time increment with a time-step of 0.005 units. You can accept defaults for all other parameters. Make sure the NLGEOM parameter is set so you run a finite strain calculation.
- In the Step module create a History output that will record the velocity of the center of mass of the sphere and its angular momentum vector. To do this (i) use the Tools…. Set menu to create an element set that contains the sphere, then (ii) in the History Output Request select 'Set' for the domain (and specify the set you just created; select Every n increments with n=1 for the frequency; and select Mass Center Kinematics and check the boxes for the displacements, velocity and angular momentum about the center of mass. Also edit the Field Output Request to save data for every increment.
- For boundary conditions, use the 'BC' option in the Load module to select the reference point on the rigid surface, then select the 'displacement/rotation' option to constrain all 6 DOF (translation and rotation) for the reference point. For initial conditions, use the 'Predefined Field' option in the Load module; select the 'Mechanical' and Velocity; and select the entire sphere. Select a Uniform distribution of velocity and the 'Translation & rotational' option, then give the sphere an initial vertical velocity (in the *y* direction) of -0.1 units (i.e. towards the rigid surface), and a horizontal velocity (in the *x* direction) of 0.07 units, together with an angular velocity (about the *z* axis) of 5 units.

- The interaction between the sphere and the surface should be hard normal contact and 'penalty' tangential contact with a friction coefficient *f=1*. The options for the contact (node region –v-surface) are not important – just use the surface to surface contact.
- Mesh the sphere with hybrid elements (check the hybrid box for all element geometries). Seed the sphere with a mesh size of about 0.01 units (this will run in a reasonable time; you can always refine the mesh further if you want more accuracy). The Hex Dominated Sweep with Advancing Front option should be able to mesh the sphere (you can try other options as well if you like).
- Use the default options for the job submission.

When you have managed to get your simulation to run (watch an animation of the contours of Mises stress, to see what is happening), plot graphs showing the (i) variation of the horizontal and vertical velocity of the center of mass of the sphere as a function of time; and (ii) the variation of the angular velocity of the sphere as a function of time. Work out how to change (i) the font size for the axis titles and labels; (ii) the background color of the window so the graph is legible.

As a solution to this homework, hand in your answers to 1.1-1.3, and your plot of the linear and angular velocity of the sphere.

3. Write a simple python script that will

(i) Read the odb file that you generated from problem (2)
(ii) Extract the (*x,y*) datapoints specifying the horizontal and vertical velocity of the center of mass of the sphere;
(iii) Save the data to a csv file
(iv) Determine the normal and tangential restitution coefficient for the impact

Check your python script by reading the csv file with Matlab and plotting the data. Hand in your plot as a solution to this problem.

4. Write a python script that will

(i) Read the model database from problem 2
(ii) Using a loop, vary the coefficient of friction from 0.1 to 1 in increments of 0.1
(iii) Determine (and save) the velocity and angular velocity of the center of mass of the sphere at the end of the impact (after the sphere has rebounded from the rigid surface)
(iv) Save a csv file that tabulates the friction coefficient values, and the velocity and angular velocities.

Read the csv file into MATLAB and plot a graph showing the variation of normal and tangential restitution coefficient at the contact as a function of the coefficient of friction. Recall that the restitution coefficient relates normal and tangential velocities at the contact point (v_{n0}, v_{t0}) (v_{n1}, v_{t1}) before and after impact through

$$e_n = -\frac{v_{n1}}{v_{n0}} \qquad e_t = -\frac{v_{t1}}{v_{t0}}$$

You can calculate the velocities at the contact point from the COM velocities and the angular velocity using rigid body kinematics equations.