Final Project: Indentation Simulation

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Aim

The project requires a simulation of rigid spherical indenter indenting into a flat block of viscoelastic material. The results from the simulation will be compared with experimental data to see if the proposed constitutive model holds true.

Model

The indentation simulation is done using abaqus. An axisymmetric model is generated for the simulation. The basic geometry and the boundary conditions for the abaqus model are shown in Figure 1.

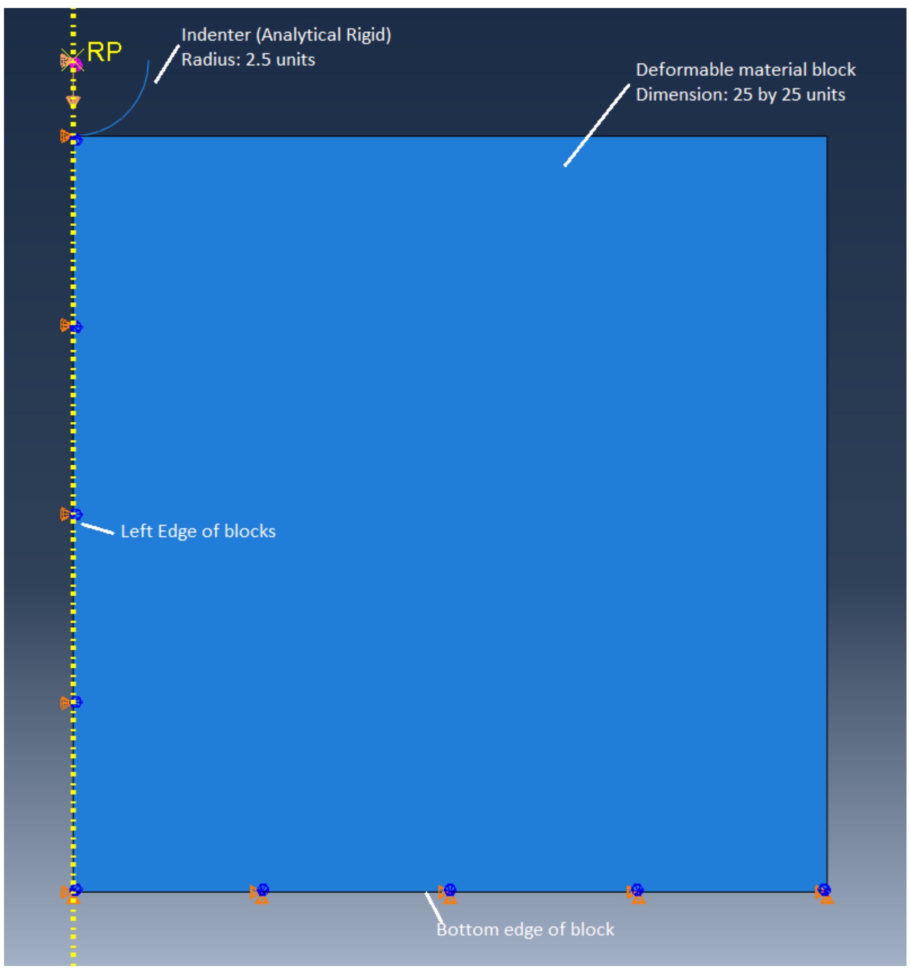

Figure 1 The basic geometry and assembly of model in abaqus.

Since, the indentation was at small scale, the model was scaled down to micrometer scale. Hence, in the model, *1 unit = 1μm*. The indenter is modeled as an analytical rigid sphere with a radius of *2.5 units*. The material block is modeled as a square of dimension *25 × 25 units*. The meshes on the material block are created such that it is finer near the region of contact and is slightly coarser as you go further away from it. The element type used for the material block is CAX4R. The step used is static general with a time duration of 30 seconds. For the interaction property between the indenter and the material block is frictionless tangential behavior. Then a surface-to-surface-contact interaction is created between them with the indenter being the master and material block being slave.

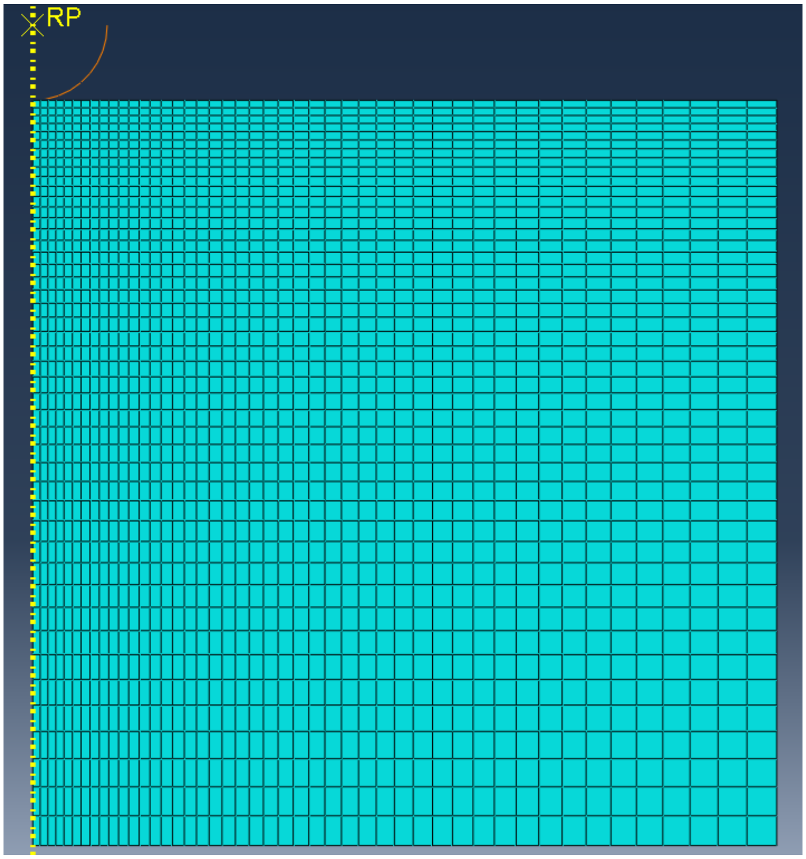

Figure 2 Initial configuration for the model with mesh.

The boundary conditions:

1. Bottom edge of the material block created from the initial step: U1 = U2 = UR3 = 0
2. Left edge of the material block created from the initial step: U1 = UR3 = 0
3. Reference point on the indenter created from the initial step: U1 = UR3 = 0
4. Reference point on the indenter from the contact step: V2 = -0.01342 unit/seconds. This boundary condition pushes the indenter into the material block.

Analysis of elastic material block

The Abaqus simulation is first done with the indenter indenting into an isotropic linear elastic material. The reactant force acting on the indenter is plotted against the indentation depth. The results from the

Abaqus simulation are compared with the analytical solution obtained from the Hertzian contact mechanics. The Hertzian solution for indentation problem is:

$$F = \frac{4}{3} \times \frac{E}{1-\nu^2} R^{\frac{1}{2}} d^{\frac{3}{2}}$$

Here, *F* is reactant force acting on the indenter, *E* is Young's elastic modulus, *v* is poisons ratio, *R* is the radius of indenter and *d* is indentation depth.

The material parameters used for the analytical solution and abaqus simulation are:

Young's elastic modulus, E = 100 GPa
Poisons ration, *v* = 0.3
Radius of indenter, *R* = 2.5 µm

The von mises stress produced in the material block as a results of indentation are showed in Figure 3 and Figure 4. Also, the result of reactant force acting on the indenter versus the indentation depth are compared for FEA simulation and analytical solution. In the Figure 5, it is seen that FEA results match very well with the analytical solution. Hence, the model setup seems to be right.

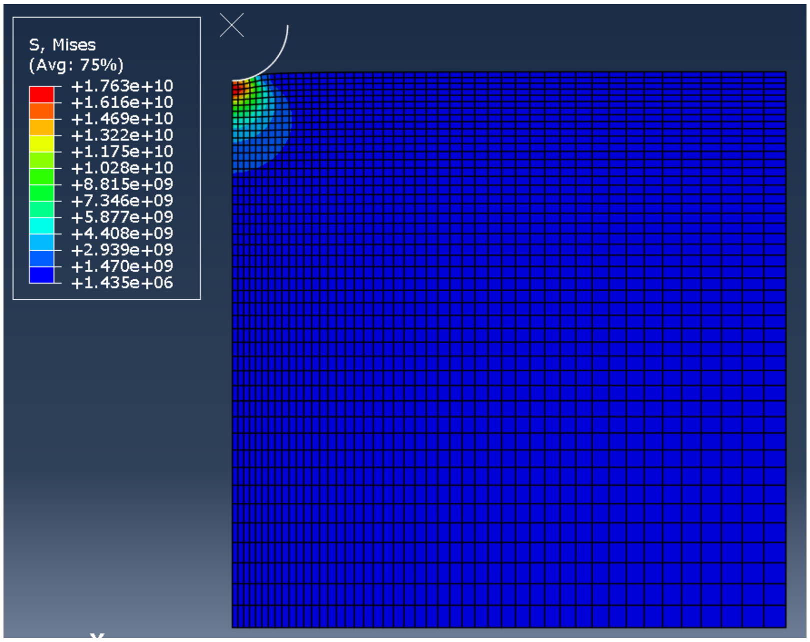

Figure 3 Von-mises stress produced in the material block modeled as linear-isotropic material. (Note that the units of von-mises stress are in kg/µm.s²)

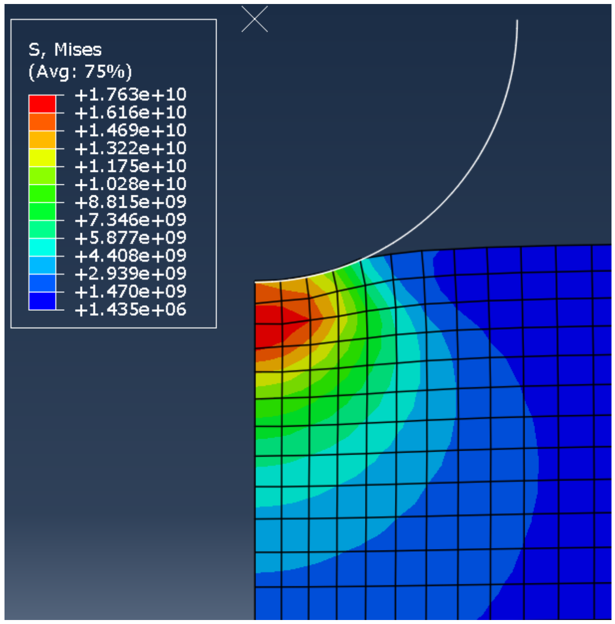

Figure 4 Close-up of von-mises stress produced in the material block modeled as linear-isotropic material. (Note that the units of von-mises stress are in kg/µm.s²)

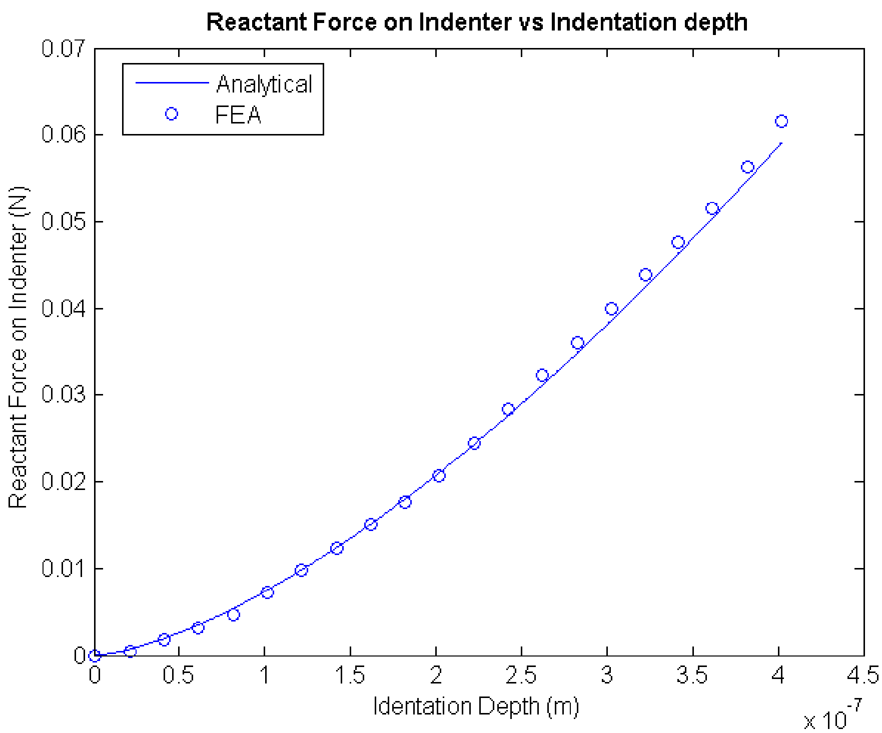

Figure 5 Reactant force acting on indenter versus the indentation depth for a linear-isotropic material

Theory of constitutive model

8-Chain model

The stress of 8-chain model (also called as Arruda-Boyce model) is given by [1,2]:

$$\sigma_A = \frac{\mu}{J\,\overline{\lambda^*}} \frac{\mathcal{L}^{-1}\left(\overline{\lambda^*}/_{\lambda_L}\right)}{\mathcal{L}^{-1}\left(1/_{\lambda_L}\right)} dev[\boldsymbol{b}^*] + \kappa(J-1)\boldsymbol{I}$$

Here, $\boldsymbol{b}^*$ is the distortional left Cauchy-Green tensor, and the applied chain stretch is given by:

$$\overline{\lambda^*} = \sqrt{\frac{tr(b^*)}{3}}$$

Table 1 Material parameters used by 8-chain model

Sr. No.	Symbol	Description
1	μ	Shear modulus
2	λ_L	Locking Stretch
3	K	Bulk Modulus

Bergstrom-Boyce model

In Bergstrom-Boyce model [3,4,5,6], the deformation gradient acts on two parallel networks, as seen in the Figure 6. The network A follows 8-chain model and the network B is a combination of elastic and viscoelastic components.

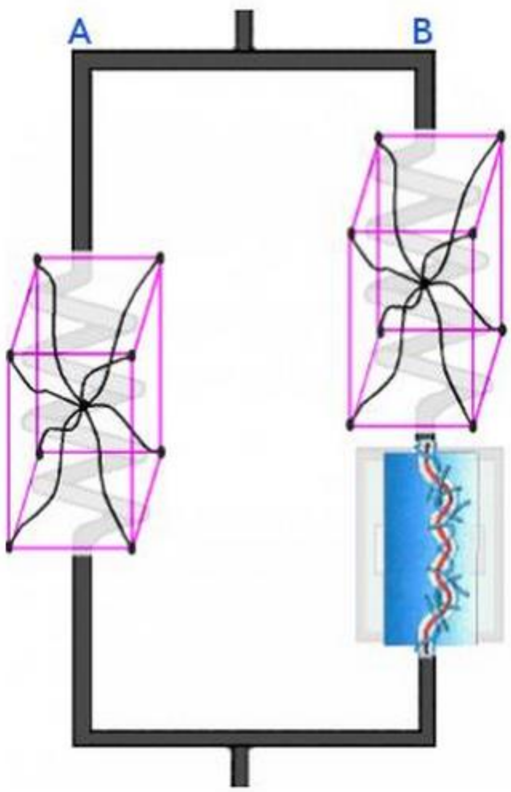

Figure 6 Rheological representation of Bergstrom-Boyce model (Source: http://polymerfem.com/content.php?77-bergstrom-boyce-model)

The stress (σ_A) for network A is given by 8-chain model. Network B also follows 8-chain model but it has different shear modulus. The stress for network B is given by:

$$\sigma_B = \frac{s\,\mu}{J_B^e\,\overline{\lambda_B^{c*}}} \frac{\mathcal{L}^{-1}\left(\overline{\lambda_B^{e*}}/\lambda_L\right)}{\mathcal{L}^{-1}\left(1/\lambda_L\right)}\, dev[\boldsymbol{b}_B^{e*}] + \kappa(J_B^e - 1)\boldsymbol{I}$$

Hence, the total Cauchy stress is given by sum of stress from network A and network B.

$$\sigma = \sigma_A + \sigma_B$$

The velocity gradient in the network B is given by.

$$L_B = L_B^e + (\overline{D}_B^v + \overline{W}_B^v)$$

The rate of viscous deformation of network B is described by:

$$\overline{D}_B^v = \dot{\gamma}_B\,(\sigma_B, b_B^{e*}) N_B^V$$

The viscous flow rate equation is given by:

$$\dot{\gamma}_B^v = \dot{\gamma}\,(\overline{\lambda_B^v} - 1 + \xi)^C \left[R\left(\frac{\tau}{\tau_{base}} - \dot{\tau}_{cut}\right)\right]^m$$

The effective stress driving the viscous flow is given by:

$$\tau = \|dev[\sigma_B]\| = \sqrt{tr[\sigma_B' \sigma_B']}$$

Table 2 Material parameters for Bergstrom-Boyce model

Sr. No.	Symbol	Value
1	μ	Shear modulus of network A
2	λ_L	Locking stretch
3	K	Bulk Modulus
4	s	Relative stiffness of network B
5	ξ	Strain adjustment factor
6	C	Strain exponential
7	τ_{base}	Flow resistance
8	M	Stress exponential
9	τ_{cut}	Normalized cut-off stress for flow

Analysis for viscoelastic material

Bergstrom-Boyce model:

The indentation experiments were performed on an Agarose material which follows Bergstrom-Boyce (BB) model at macro-scale. The material parameters for BB-model were found from uniaxial compression test at the macro-scale level. It of interest to see if the model follows the same constitutive model at micro-scale.

Table 3 Material parameter for Bergstrom-model for material at macro-scale.

Sr. No.	Symbol	Value
1	μ	3532.21 Pa
2	λ_L	1.01327
3	K	26228.5 Pa
4	s	5.02899
5	ξ	0.499664
6	C	-5.99838 e-10
7	τ_{base}	102023
8	M	1.39367
9	τ_{cut}	0.01

To test the validity, indentation experiments were performed at micro-scale. From the experiments, the data of reactant force acting on the indenter versus the indentation depth was observed. Now, an FEA model is to be setup with the material properties followed at macro-scale and simulate an indentation model at micro-scale level, to see if the results match with the experimental results.

For abaqus simulation, Bergstrom-Boyce model is available as an inbuilt feature in abaqus under the material parameter as 'hysteresis.' An abaqus simulation was tried for BB-model but the abaqus solution couldn't converge. Various options were explored to get the simulation to work, but the solution couldn't converge. On further investigation, it was found that other people also had problem with indentation simulation for BB-model using hysteresis feature in Abaqus. But Dr. Bergstrom was able to run the same simulation with his UMAT for BB-model. So it might be possible that there is some error in abaqus's implementation of BB-model using 'hysteresis.'

8-Chain model:

Now, the material block is modeled using 8-chain model. The simulation is performed using the 8-chain model using the same material parameter that were obtained for BB-model at macro-scale. As discussed earlier, the reactant force acting on the rigid sphere indenter was plotted against the indentation depth.

Table 4 Material parameter for 8-chain model for material block using data obtained from BB-model at macro-scale

Sr. No.	Symbol	Value
1	μ	3532.21 Pa
2	λ_L	1.01327
3	K	26228.5 Pa

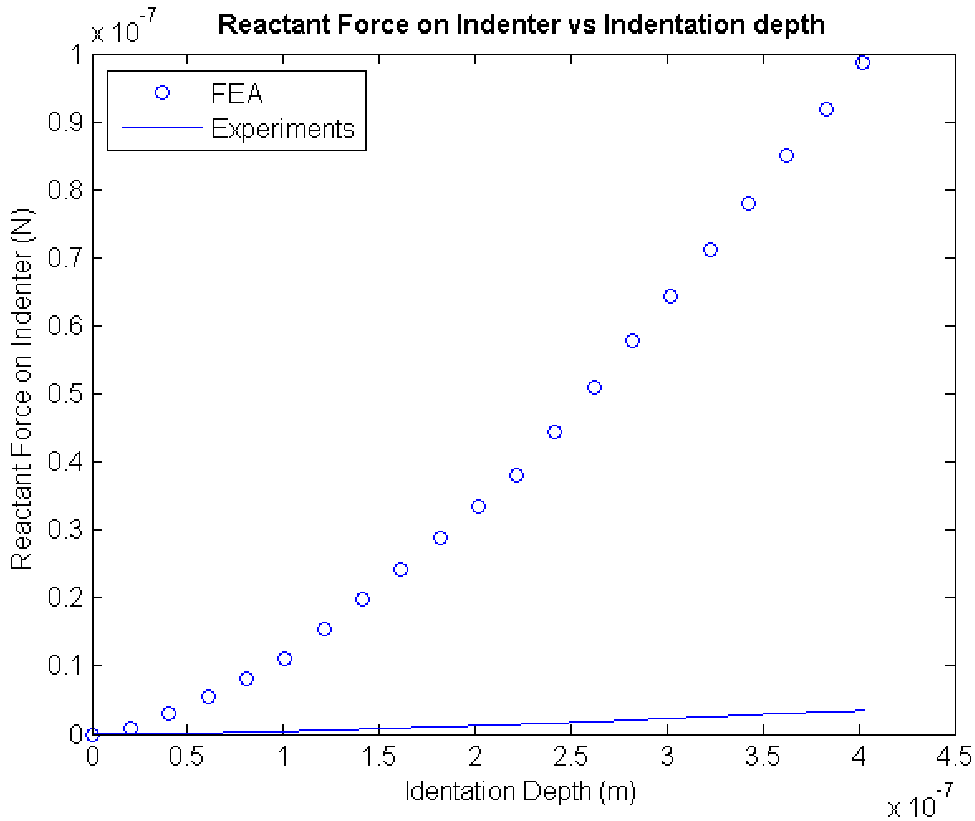

Figure 7 Reactant force on indenter vs the indentation depth for material following 8-chain model with parameters borrowed from BB-model.

As expected, the plots didn't match. This is because we know that μ, κ and λ_L values that we borrowed from the BB-model just describe the material model for network A and not the combination of the whole model. We need to model the whole material as a single network following 8-chain model and see if we the plots match with the experimental material. To find the new material parameter, it was assumed that the value of λ_L and v remain same. Accordingly, the value of μ and κ were found. The abaqus simulation result versus the experimental data for this new parameter for 8-chain model are show in Figure 8.

Table 5 New material parameter found for 8-chain model for the material block

Sr. No.	Symbol	Value
1	μ	632.21 Pa
2	λ_L	1.01327
3	K	4694.16 Pa

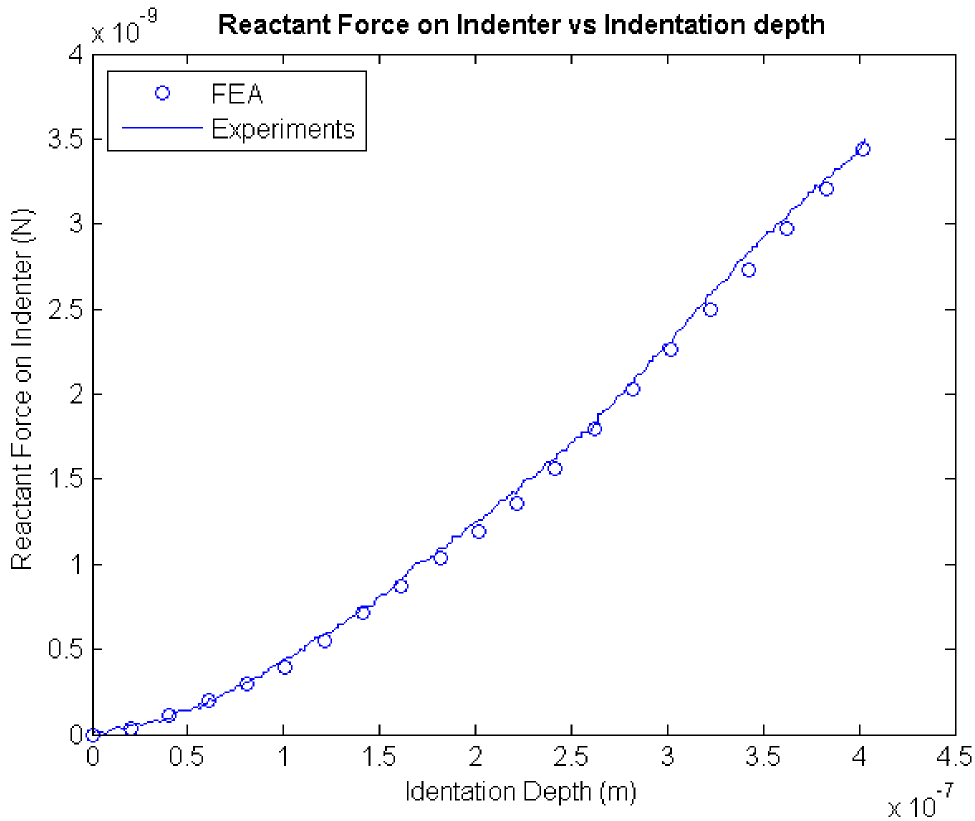

Figure 8 Reactant force acting on the indenter versus the indentation depth for new material parameters found for 8-chain model.

It is seen that the 8-chain model can be used to describe the material behavior of the material block. The requirement for new material parameter is also associated with the fact that the Agarose material is porous. Hence, at macro-scale the material property depend on both the property of material and the pressure term due to the space occupied by the fluid in the pores. But when we shift our attention to micro-scale the effect of pressure term may disappear as the size of indenter is smaller than the pore size.

Reference

[1] E. M. Arruda and M. C. Boyce. A three-dimensional constitutive model for the large stretch behavior of rubber elastic materials. J. Mech. Phys. Solids., 41(2):389–412, 1993.

[2] J. S. Bergström. Large Strain Time-Dependent Behavior of Elastomeric Materials. PhD thesis, MIT, 1999.

[3] J. S. Bergström and M. C. Boyce. Constitutive modeling of the large strain time-dependent behavior of elastomers. J. Mech. Phys. Solids, 46:931–954, 1998.

[4] J. S. Bergström and M. C. Boyce. Mechanical behavior of particle filled elastomers. Rubber Chem. Technol., 72:633–656, 1999.

[5] J. S. Bergström and M. C. Boyce. Large strain time-dependent behavior of filled elastomers. Mechanics of Materials, 32:620–644, 2000.

[6] J. S. Bergström and M. C. Boyce. Constitutive modeling of the time-dependent and cyclic loading of elastomers and application to soft biological tissues. Mechanics of Materials, 33:523–530, 2001.