1. Background:

Biological tissues are primarily viscoelastic in nature, meaning that their mechanical behavior, i.e. responses to an applied stress or strain, are time dependent. I created a model of a segment of cartilage tissue in ABAQUS using FEA in order to illustrate its viscoelastic properties. I assumed geometrically linear behavior of the material, meaning that the constitutive equations satisfy the conditions for superposition and proportionality. I then modeled creep behavior using ABAQUS’s built-in time-dependent linear viscoelasticity material model.

2. Theory:

For small displacements and relatively small times \( t \), the response of a viscoelastic material is dominantly elastic for constant strain rates and that for larger values of \( t \), the response is dominantly or entirely viscous.

The Kelvin-Voight model can accurately describe creep behavior in tissue. This model represents a viscoelastic material as a spring and dashpot in parallel and is especially accurate in predicting creep behavior and modeling small stretches.

The governing constitutive equation for the elastic spring and viscous dashpot is:

\[
F = c \varepsilon + c \eta \dot{\varepsilon}
\]

with retardation time

\[
\tau = \frac{c \eta}{c}
\]

this becomes

\[
\frac{F}{c \eta} = 1 \left( \varepsilon + \dot{\varepsilon} \right)
\]
And the general solution to this differential equation is given by

\[ \varepsilon(t) = \frac{1}{\eta} \int_0^t e^{-\frac{t-\xi}{\tau}} F(\xi) \, d\xi \]

For a constant force \( F \), the strain response simplifies to

\[ \varepsilon(t) = \frac{F}{c} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

(Source: Oomens, Brekelmans, and Baaijens).

Prony series parameters \( g, k, \) and \( \tau \) were needed to characterize the creep displacement behavior in ABAQUS. Shear and bulk moduli were then calculated using these parameters (Keenan) in order to compute the analytical solution:

\[ G_{eq}(t) = G_0 \left[ 1 - g \left( 1 - e^{-\frac{t}{\tau}} \right) \right] \]

\[ K_{eq}(t) = K_0 \left[ 1 - k \left( 1 - e^{-\frac{t}{\tau}} \right) \right] \]

with

\[ G_0 = \frac{E}{2(1 + \nu)} \]

\[ K_0 = \frac{E}{3(1 - 2\nu)} \]

3. ABAQUS Implementation:

Prony series parameters were obtained from a study by Keenan et. al. which modeled creep indentation in human patella cartilage. The parameters \( g = 0.744, k = 0.978, \) and \( \tau = 13.3 \) seconds were input into the viscoelastic material model. An initial Young’s modulus of \( E_0 = 3.78 \) MPa and Poisson’s ratio of \( \nu = 0.47 \) were used. All parameters were averaged values from a dataset of donors.

Boundary conditions involved applying a constant force of -0.35N on the bottom of the tissue model in U2. The top face was constrained in U2, the left side was constrained in U3, and the back was constrained in U1. The displacement remained small compared to the total length of the tissue sample model.
4. Results

For small strain, the FEM and Analytic results are in exact agreement, indicating that the above material model is valid for small deformations and can be effectively implemented in a single-element model in ABAQUS.
5. Conclusion

FEA analysis is in agreement with analytical results for small strain. This model works well for small strains, but would likely be ineffective in modeling large displacements and strains.

Works Cited:
