1 Background

The circulatory system is an interesting mechanical engineering workhouse. The pump is in charge of sending a non-Newtonian fluid, blood, to the body so that oxygen can be squeezed out of little carriers found in the blood (red blood cells) and so that waste can be removed. Here I examine one of the mechanisms by which the vasculature system helps the operation of this paradigm.

Veins are the particular focus of this paper because their buckling is not well understood. Veins are low pressure, low fluid velocity tubes that often require one way valves to return blood to the heart. An additional mechanism by which veins bias blood flow is through buckling. Many have described the buckling trend by qualitatively estimating the change in cross-sectional geometry during the buckling process, but few have numerically confirmed this. Finite Element Analysis could be a critical tool in elucidating this phenomenon because of its flexibility to boundary valued problems such as this that do not have well developed analytical solutions.

2 Formulation

I desire to find the critical parameters for the failure and buckling of this system. Before defining this yield criteria, I know that the solution will be governed by a few geometric non-dimensional variables. The non-dimensional critical thickness will be defined as the ratio between the thickness and the radius of the major access of the cylinder:

$$R_c = \frac{t}{R_a}, \tag{1}$$

The geometry of the cross-section will be elliptical to bias the buckling, so there will also be some critical eccentricity that will dictate when the vessel will buckle in a way to bifurcate rather than collapse.

For buckling analysis, we can start with the von Mises yield criteria, shown below:

$$\sigma_y = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2) + (\sigma_2 - \sigma_3) + (\sigma_3 - \sigma_1)} \tag{2}$$

But let us switch into cylindrical coordinates and start by imagining the 2D symmetric problem of analyzing a quarter or half of the cross-section during the test. Here we will set the out of plane stress to zero. We will also, as many others often do for thin walled structures, set the radial stress to zero.

$$\sigma_z = \sigma_r = 0 \tag{3}$$

$$\sigma_y = \sigma_\theta^2 \tag{4}$$

From geometry, we can find this hoop stress for a circular thin tube of thickness t and radius r:

$$\sigma_\theta = \frac{Pr}{t} \tag{5}$$

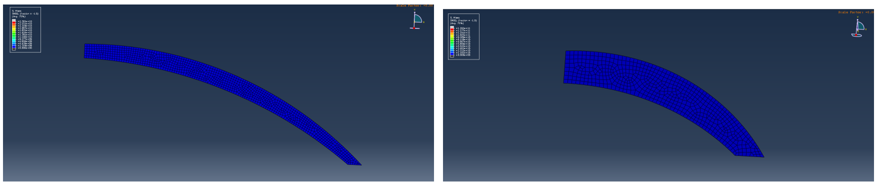
Which gives the yield stress in terms of the internal pressure and the non-dimensional critical thickness of the wall:

$$\sigma_y = (\frac{P}{R_c})^2 \tag{6}$$

To create this buckling, a constant pressure was applied within the vessel. This means that the yield stress is inversely proportional to the thickness squared. This is a very strong dependence, and we will observe this in the solution later. In the following analyis, I used Abaqus Implicit Dynamics to analyze the buckling of various geometries of vein walls. I chose to use 2D shell elements, and applied shell edge tractions on the inside of the lumen of the vein. This pressure was constant to analyze the bifurcation up to the point of self-contact. Self-contact could not be added to shell elements, but we can assume the return of the vessel to non-buckled geometry is symmetric to its deformation.

3 Results and Discussion

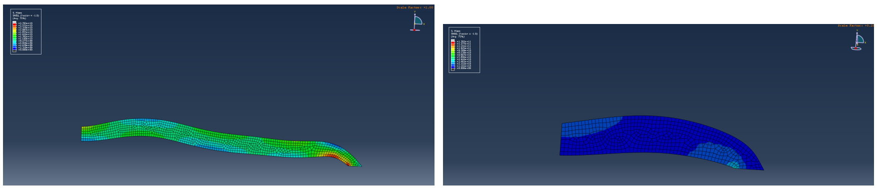
I started this project by asking if the vein will buckle or collapse, and it turns out that the critical parameter here is the relative wall thickness. To see this, I analyzed one quarter region of the cross-sectional area to use symmetry for well-defined boundary conditions. Shown below are two modelled vein walls with 5% and 14% wall thicknesses.



(a) 5%   (b) 14%

Figure 1: Different wall dimensions

It turns out that the buckling mechanism for these two geometries was different. The thin walled geometry exhibited a deformed shape that appears to slightly bifurcate, while the thicker walled geometry clearly collapses. Note that these plots are for the same pressures.



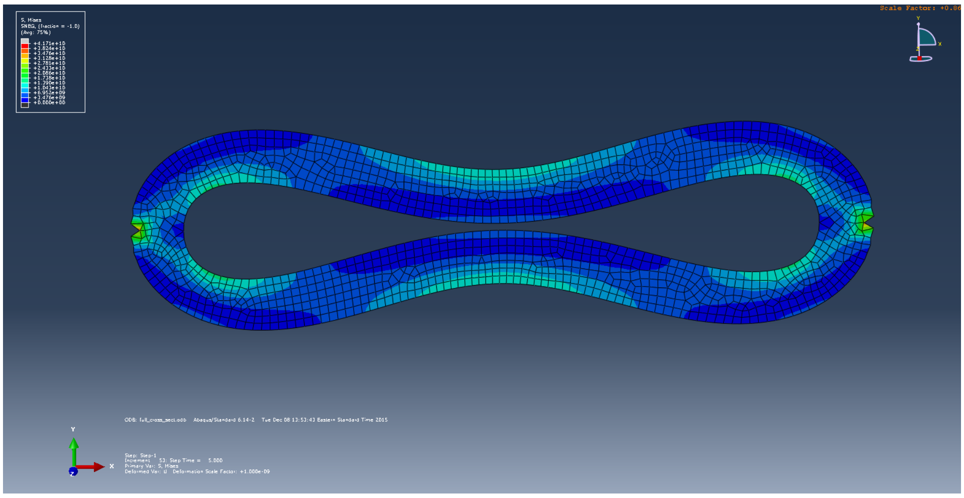
(a) 5%   (b) 14%

Figure 2: Different wall dimensions, deformed

It is interesting to note the elastic waves propagate through the geometry (more notable in the thin walled geometry). The mechanism for bifurcation must be that these elastic waves coalesce at the middle of the curve. Further, the choice of an elastic material can be validated for thin walled vessels. All of the pressure is handled with the hoop stress, and the elements do not deform greatly in the radial diirection. For the thicker vessel (14% of the radius) the wall does seem to deform in the radial direction, hinting towards a collapse mechanism rather than the bifurcation.

From here, I put together a full plane of the vein cross-section to examine the development of the cross-sectional area over the course of the deformation. Shown below in Figure 3 is the deformed vein wall cross-section after buckling. This calculation was performed with the wider vein geometry because my lab intended to use these dimensions for an experimental aspect of this system, and for this reason, I had to use a much higher pressure. For a 3-fold increase in thickness, a 10-fold increase in pressure was required to cause the structure to buckle. This reflects the aforementioned relationship between yield stress and the thickness squared. On the experimental side of this project, the next step is to optimize for the ideal thickness by considering what pressure ranges they can apply to the system and how thin they can fabricate the geometry from the latex mold injection.

Figure 3: Bifurcation of the vein



I now wish to briefly describe what steps need to be taken to implement this analysis in FEA-EN234. This problem is clearly an exercise in finite strain elasticity (hyperelastic material). Deformation must be considered relative to the deformed configuration, and one would need to select the constitutive equation wisely. This stress-strain relationship will then be included into the equilibrium equation to derive a weak form where one can then go through the usual machinary to use interpolation functions that lead to the solution.