EN234 Final Project

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1 Problem Description

In elastic soft homogeneous materials that admit large compressive deformations a localization of strain, wherein a mode transformation from uniform compression to highly localized compression, occurs at some critical strain ϵ^* . Using linear perturbation theory for sinusoids, Biot calculated a theoretical critical strain of a neo-Hookean half-space to be 45.6% [1]. In subsequent computational work, a sub-critical phenomenon has been discovered and extensively reported with creasing occurring at 35% compressive strain; this is currently though to be the result of strain-energy instability related to the non-linearity of the material, which thus is not captured by the linear perturbation analysis [2]. See figure 1(a) for a diagram of this type of result. Work is ongoing to explain this with rigorous theoretical treatment of the system. In experiments, Gent famously measured the critical transition to a wrinkled mode at between 28% and 42% compressive strain in an analog to a three-point bend test [3]. The experimental result centered about 35% strain suggests that this sub-critical transition is captured by the computation is physical, but convincing analyses explaining the large spread in the data are not forthcoming. Generally, the accepted reasoning is that surface roughness precipitates crease formation at asperity troughs. To investigate this hypothesis, a crease formation model was created in ABAQUS. In this idealization of the creasing process, the surface asperity is set to as a triangular surface defect in a two-dimensional plane-strain representation of the problem. The dependence of critical strain on simplified asperity geometry, base angle or depth, will be considered.

2 Methods

A diagram of the geometry used for the problem is given in figure 1. In theoretical treatments of the problem, an infinite half-space is assumed; since this is impractical in our physical experimental system we consider a defect 0.050 inches deep into a two inch cuboid specimen, which is replicated in this finite element model of the system. The model geometry is diagrammed in figure 1. The defect base angle will be parametrically varied through 45, 90, and 120 degrees and plane-strain compression is imposed. The important length scales in this problem are non-dimensionalized as $\frac{w_d}{w_b}$ and $\frac{h_d}{h_b}$ where w are widths and h are heights, and d and b stand for the defect and block respectively.

To solve the computational problem, the geometry was meshed finely about the defect, but less fine further away (in the -y-direction) from the top surface. An example mesh, from the 90° base angle case, is show in figure 2, since the mesh in the upper region is to fine to permit visualization at the scale of the entire part an inset shows the relative scale of the mesh to the defect size. Mesh elements were hybrid quadrilaterals with quadratic accuracy and reduced integration. The self contact boundary condition was enforced with the surface-to-surface contact algorithm in ABAQUS, and the contact faces were idealized as rigid frictionless surfaces. Surface separation was allowed.

The analysis was conducted on a finite deformation neo-Hookean material, with a constitutive relation given by

$$U = C_{10}(\bar{I}_1 - 3) + \frac{1}{D_1}(J_{el} - 1)^2 \tag{1}$$

where C_10 is and elastic material property, \overline{I}_1 is the first invariant of the deformation gradient, D_1 is related to the compressibility of the material, and J_{el} is the volumetric elastic strain. The material properties were set to

$$\begin{array}{c|c} C_{10} & D_1 \\ \hline 100 & 0 \end{array}$$

for all testing cases since a soft incompressible rubbery material that is used in our experiments.

The material was deformed by specifying displacement boundary conditions compressing the right and left hand sides of the such that 50% compressive strain in U_{11} was realized at the final time. The bottom surface was constrained so that motion in the U_{22} direction was not allowed. In practice, convergence of the models was extremely slow in the large base angle models due to the creasing instability in the problem. A small artificial viscosity was added to the problem to overcome complete failure to converge beyond the onset of the creasing instability. To determine the onset of creasing, the difference in displacement between the analytical solution of compression of the neo-Hookean solid and the computational result for the crease tip displacement. A marked divergence from the analytical solution indicated the presences of a the beginning of creasing. Total strain energy was also computed and output, although not marked difference was observed over the strain range utilized.



Figure 1: (a) A diagram of the stress distribution and the idealization of the crease in the material (b) The geometry of the experimental set to be simulated (c) The geometry of the imposed defect used to initiate creasing

3 Results

An example deformed configuration is pictured in figure 3 on the left. The right side of figure 3 shows the deformation process leading to creasing. In the initial portion of the deformation the defect simply closes (see (a) to (b)), continuing to deform the solid causes the closing-type motion to continue and a high-stress point develops where the top corners of the defect come into contact (se (b) to (c)), further compression yields a crease developing in the material (see (c) to (d)). Similar characteristics are observed for all base angles.

A plot of the difference in displacement from the idealized solid block, as described in the methods, is given in figure 4. In this figure, there is transition from a relatively well-matched displacement (different by a geometric factor related to the defect closing process) to a much different displacement mode. This transition point indicates the onset of creasing. For larger base angles, the transition process was less abrupt and initiated earlier, but changed mode more slowly. Total strain energies were nearly identical throughout the deformation, see figure 5, indicating (predictably) that this is not an appropriate measure to detect crease initiation for displacement controlled testing.



Figure 2: An example of the meshes used for this experiment.

4 Conclusion and Future Work

In this project blocks of hyperelastic material having surface defects were deformed in a computational environment, with the goal of determining the effect of changing the base angle of a triangular defect on the critical strain for creasing. The results indicate a small change in creasing strain with changing base angle. More strikingly, the form of the function of displacement of the crease into the block surface versus the applied strain changes dramatically with changing angle. In future, the cause of this change in characteristic will be vetted more thoroughly and effect of crease depth on this functional form and on creasing strain will be investigated to more completely map the phase space of creasing bifurcation process.

5 Works Referenced

[1] Biot, M. A. "Surface instability of rubber in compression." Applied Scientific Research, Section A 12, no. 2 (1963): 168-182.

[2] Cao, Yanping, and John W. Hutchinson. "From wrinkles to creases in elastomers: the instability and imperfectionsensitivity of wrinkling." In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, vol. 468, no. 2137, pp. 94-115. The Royal Society, 2012.

[3] Gent, A. N., and I. S. Cho. "Surface instabilities in compressed or bent rubber blocks." Rubber Chemistry and Technology 72, no. 2 (1999): 253-262.



Figure 3: Left: An example (from the 45 degree base angle case) of the stress contours in the final deformed configuration.

Right: the deformation process as the defect closes and initiates creasing.



Figure 4: Displacement difference of the crease tip from an idealized neo-Hookean solid with no defect.



Figure 5: Total strain energy as a function of strain in each test case.