



EN 4
Dynamics and Vibrations

Homework 10: Rigid Body Dynamics

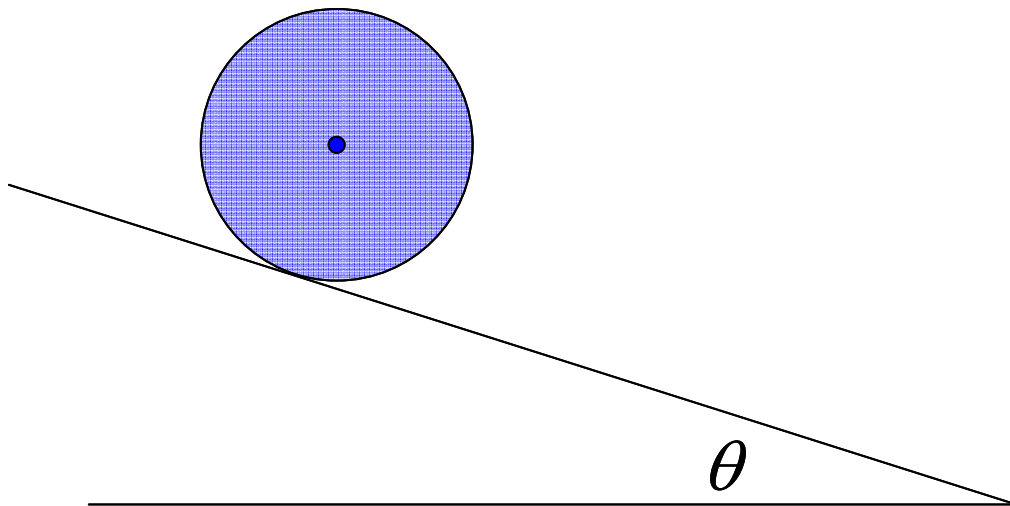
Due: **WEDNESDAY, April 29, 2009 5:00pm**

1. Galileo's Classic Problem

A cylindrical object having mass m , radius R , and moment of inertia I rolls down an incline of angle θ without slipping. What is the acceleration of the center of mass down the hill, as a function of m , I , g , and θ ?

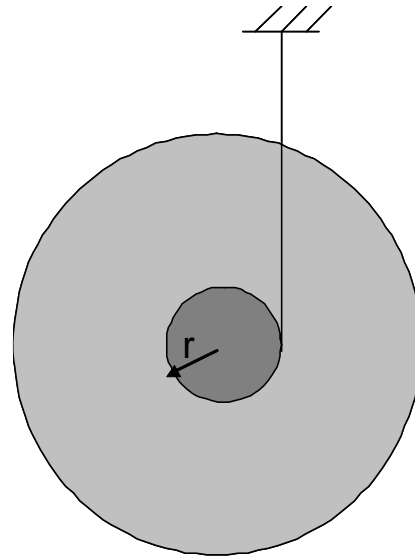
Evaluate the result in the following cases:

- (a) the mass is concentrated at the center of mass connected to a massless cylindrical frame
- (b) all the mass is concentrated at the outer rim of the cylinder
- (c) the mass is equally distributed throughout the cylinder.
- (d) How do these cases compare to the acceleration of a point particle of mass m sliding down a frictionless plane?
- (e) For case (c), if the object starts from rest, what is the velocity of the center of mass after rolling a distance d down the incline?
- (f) For case (c), now use the work-energy for relation for rigid bodies to determine the velocity after rolling a distance d down the incline. Remember that "rolling without slipping" means that the friction at the contact point is static friction (the contact point has zero velocity) and so there is no energy lost due to this static friction (energy loss only occurs when there is slipping).



2. Yo-Yo Motion

A Yo-Yo of mass m and moment of inertia $I_G = mk^2$, where k is a distance called the “radius of gyration”, moves by “rolling” along a string wound on the inner radius r , as shown.



a) What is the downward acceleration of the center of mass of the Yo-Yo and the tension in the string? (in terms of m , r , k , and g , as needed).

b) If it starts from rest, what is the rotation rate after dropping a distance L ? (in terms of m , r , k , g , and L , as needed)

c) After the Yo-Yo reaches the end of the rope, with the rope attached to a specific point on the perimeter of the inner radius, describe *briefly* and *qualitatively* what happens. Assume, for simplicity, that the unwound portion of the rope remains vertical during the entire motion.

3. Kinetic Energy Storage for Vehicles

A bus of mass $M=10,000\text{kg}$ contains a flywheel that can store kinetic energy. The flywheel is a solid disk of mass $m=1500\text{kg}$ and radius $R=0.7\text{m}$. The flywheel is initially spun up to a speed of 4000RPM (rev/minute). The bus then starts from rest and drives up a hill to a height of 20m, attaining a speed of 72 km/hr. If all of the energy comes from the flywheel, what is the rotational speed of the flywheel when the bus has reached the 20m point?