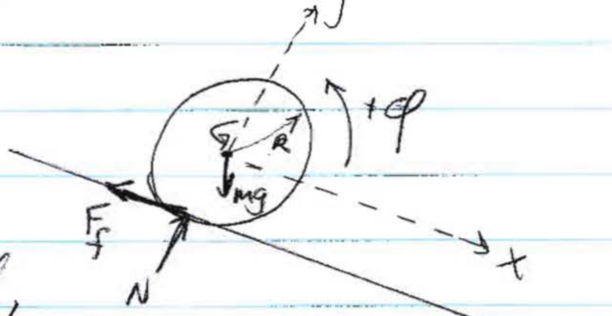
EN 4 HW #10 SOLUTIONS



1. Proceed by usual set of steps
   1. Choose coordinates: x–y shown
   2. Define motion of G: x_G, y_G
   3. Draw F.B.D.: gravity, normal, and friction, which is <u>static</u> and opposes the potential slippage.
   4. Set $\vec{F} = m\vec{a}_G$, $\sum \vec{M}_G = I_G \vec{\alpha}$:

      (a) $\sum F_x = -F_f + mg\sin\theta = m\ddot{x}_G$

      (b) $\sum F_y = N - mg\cos\theta = m\ddot{y}_G$

      (c) $\sum \vec{M}_G = -F_f R\hat{k} = I_G \ddot{\varphi}\hat{k}$

   5. Identify kinematic constraints: $\ddot{y}_G = 0$
      $\ddot{x}_G = -R\ddot{\varphi}$ (Rolling)
      x_G downhill is $\dot{\theta}$ CW, not CCW.
   6. Sub in and solve:

      (c): $F_f = -\dfrac{I_G\ddot{\varphi}}{R} = +I_G\ddot{x}_G/R^2$

      (c) into (a): $-I_G\ddot{x}_G/R^2 + mg\sin\theta = m\ddot{x}_G$

      $$\Rightarrow \boxed{\ddot{x}_G = \frac{mg\sin\theta}{m + I_G/R^2}}$$

(a) Here, $I_G = 0 \Rightarrow \ddot{x}_G = g\sin\theta$

(b) Here, $I_G = mR^2 \Rightarrow \ddot{x}_G = \frac{1}{2}g\sin\theta$

(c) Here, $I_G = \frac{1}{2}mR^2 \Rightarrow \ddot{x}_G = \frac{2}{3}g\sin\theta$

(d) A point particle would be the same as (a) ($I_G = 0$). Cases b, c have <u>smaller</u> acceleration because linear motion is accompanied by rotation.

(e) Basic Kinematics, since $\ddot{x}_G$ is constant

Either use directly $V_{Gx} = \sqrt{2 d a_{Gx}}$

or consider motion in time:

$$V_{Gx} = \cancel{V_{Gx0}}^{\,0} + a_{Gx} t$$

$$x_G = \cancel{x_{G0}}^{\,0} + \cancel{V_{Gx0}}^{\,0} t + \frac{1}{2} a_{Gx} t^2$$

so at

$x_G = d$, we have $t = \sqrt{\dfrac{2d}{a_{Gx}}}$ and thus $V_{Gx} = \sqrt{2 d a_{Gx}}$

So, $V_{Gx} = \sqrt{2d \cdot \frac{2}{3} g \sin\theta} = \sqrt{\frac{4}{3} g d \sin\theta}$

(f) Static friction f_f and N do no work, so energy is conserved.

$$T_f + V_{gf} = T_i + V_{gi}$$

Take $y_f = 0$, $y_i = d \sin\theta$, $T_i = 0$ (at rest) so

$$\frac{1}{2} m V_{Gx}^2 + \frac{1}{2} I_G \omega^2 + 0 = 0 + mgd\sin\theta$$

For a rolling wheel, $V_{Gx} = R\omega$ so

$$\frac{1}{2} m V_{Gx}^2 + \frac{1}{2} I_G \left(\frac{V_G^2}{R^2}\right) = mgd\sin\theta$$

$$\frac{1}{2}\left(m + I_G/R^2\right) V_{Gx}^2 = mgd\sin\theta$$

Case (c): $I_G = \frac{1}{2} mR^2$ so $\frac{1}{2}\left(\frac{3}{2} \cancel{m}\right) V_{Gx}^2 = \cancel{m} g d\sin\theta$

$$\Rightarrow V_{Gx} = \sqrt{\frac{4}{3} g d \sin\theta}$$

same as above, of course

2. The Yo-Yo is like the cable-reel problem turned 90°, and is similar to problem 1 in many respects.

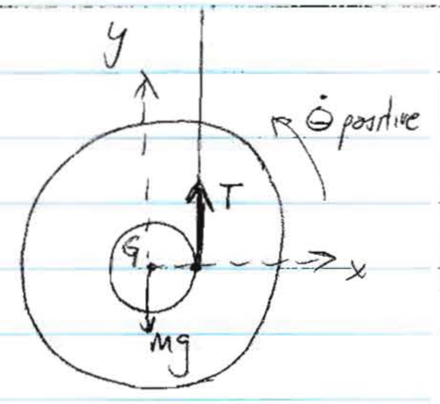
Follow steps:

a) 1. Choose x-y coords as shown

2. Define motion of G: x_G, y_G

3. Draw F.B.D.: gravity, tension

4. Set $\vec{F} = m\vec{a}_G$, $\Sigma \vec{M}_G = \underline{I}_G \vec{\alpha}$



(a) $\Sigma F_x = 0 = m\ddot{x}_G$

(b) $\Sigma F_y = T - mg = m\ddot{y}_G$

(c) $\Sigma \vec{M}_G = RT\hat{k} = I_G \ddot{\theta}\hat{k}$

5. Kinematics: like a rolling wheel: $\ddot{y}_G = -R\ddot{\theta}$ (y up ⇔ CW rotation)

6. Sub in and solve

(c). $T = \frac{I_G \ddot{\theta}}{R} = \frac{-I_G \ddot{y}_G}{R^2}$

(c) ⇒ (b): $-\frac{I_G}{R^2}\ddot{y}_G - mg = m\ddot{y}_G$

$$\Rightarrow \ddot{y}_G = \frac{-mg}{m + I_G/R^2}$$ "−" ⇒ falling down ✓

$I_G \equiv mk^2$ so $\boxed{\ddot{y}_G = \frac{-g}{1 + k^2/R^2}}$

Then $T = -\frac{mk^2}{R^2}\frac{-g}{1 + k^2/R^2} = \underline{\underline{mg\left(\frac{k^2}{R^2 + k^2}\right)}}$

b) Use energy, again. T does no work (T not acting over any distance even though it seems like it.)

So, with $y_i = 0$, we have

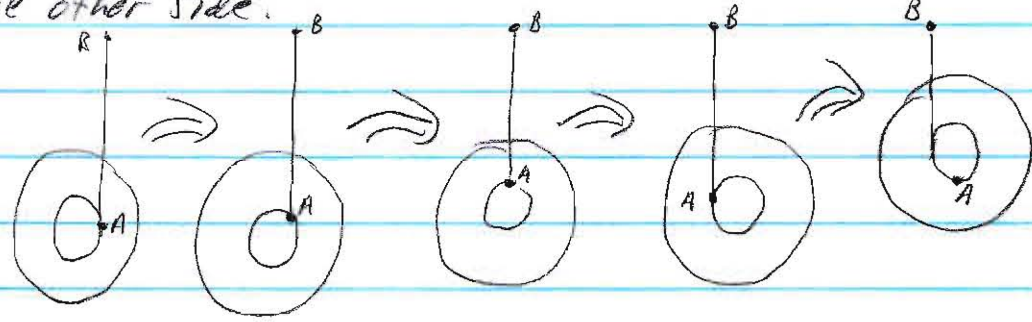
$$\underbrace{\tfrac{1}{2}mV_{Gi}^2 + \tfrac{1}{2}I_{G}\omega_i^2}_{0,\text{ at rest}} + 0 = \tfrac{1}{2}mV_{Gf}^2 + \tfrac{1}{2}I_G\omega_f^2 - mgL$$

Again, we have $V_G = R\omega$ so

$$\frac{1}{2}m(R\omega)^2 + \frac{1}{2}mk^2\omega^2 = mgL$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{2mgL}{R^2+k^2}}}$$

(c) At end of rope, string is attached to a fixed point on the Yoyo. The Yoyo then pivots around this point and starts rolling up the other side.



[A full analysis is needed to examine X motion of the attachment point A, however. It is really point B that is fixed.]

3. Initially, we have kinetic energy in the flywheel. Some of this energy accelerates the bus and drives it uphill. Treating the bus and flywheel as a system, we have

$$T_i + V_{gi} = T_f + V_{gf}$$

$$\underbrace{\frac{1}{2}I_{Gfw}\omega_{fwi}^2}_{\text{initial spinning}} + 0 = \underbrace{\frac{1}{2}I_{Gfw}\omega_{fwf}^2}_{\text{final spinning}} + \underbrace{\frac{1}{2}M_b V_b^2}_{\text{Kinetic energy of bus}} + \underbrace{M_b g y_b}_{\text{potential energy of bus.}}$$

[Mass of bus includes the flywheel] Plugging in, we have

$$\frac{1}{2}\left(\frac{1}{2}1500\text{kg}\ 0.7^2\text{m}^2\right)\left(\frac{4000\cdot 2\pi}{60\text{s}}\right)^2 = \frac{1}{2}\left(\frac{1}{2}1500\text{kg}\ .7^2\text{m}^2\right)\omega_{fwf}^2 + \frac{1}{2}10^4\text{kg}\left(\frac{72\times 10^3}{3600}\frac{\text{m}}{\text{s}}\right)^2$$

$$+ 10^4\text{kg}\left(9.81\frac{\text{m}}{\text{s}^2}\right)(20\text{m})$$

$$32.24\times10^6\ \frac{kg\text{-}m^2}{s^2} = 183.75\ kg\text{-}m^2\ W_{fwf}^2 + 2\times10^6\ kg\frac{m^2}{s^2}$$

$$+\ 1.962\times10^6\ kg\text{-}m^2/s^2$$

$$\Rightarrow \quad W_{fwf} = 392\ \frac{rad}{s} = 62\ \frac{rev}{s} = \underline{\underline{3720\ \frac{rev}{min}}}$$

Plenty of energy in the flywheel!!