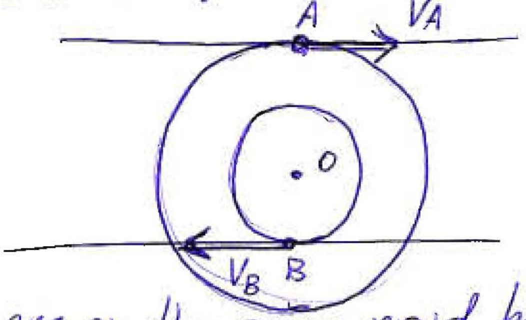
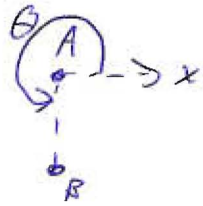
EN4 HW #9 SOLUTIONS

1. Velocities of rails correspond to velocities of wheel at the respective contact points:



Define x, y as shown

Then, since A, B are on the same rigid body we have



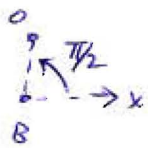
$$\dot{x}_B = \dot{x}_A - r_{AB}\,\dot{\theta}\,\sin\theta \qquad \theta = \frac{3\pi}{2}$$

so

$$-1.8\,\tfrac{m}{s} = 1.2\,\tfrac{m}{s} - (.2m)\,\dot{\theta}\,(-1)$$

and thus $\dot{\theta} = -15\,\frac{rad}{s}$ (negative means Clockwise)

Then since O is also on the rigid body we have



$$\dot{x}_O = \dot{x}_B - r_{BO}\,\dot{\theta}\,\sin\theta \qquad \theta = 90 = \frac{\pi}{2}$$

so

$$\dot{x}_O = -1.8\,\tfrac{m}{s} - (.08m)(-15\,\tfrac{rad}{s})\,1$$

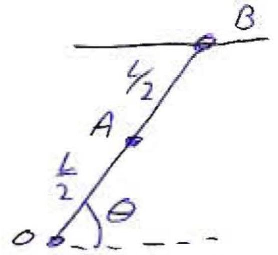
$$\Rightarrow \dot{x}_O = -0.6\,\tfrac{m}{s} \quad \text{(moving to the left)}$$

• In both a) and b) we have:

Define θ. Then

$y_B = L\sin\theta$

and $\dot{y}_B = L\dot{\theta}\cos\theta$



$\dot{y}_B$ = rate of vertical motion of platform. So, we need to find $\dot{\theta}$

Also, motion of A is $\dot{x}_A = -\frac{L}{2}\dot{\theta}\sin\theta$

$\dot{y}_A = \frac{L}{2}\dot{\theta}\cos\theta$ $\quad$ ($\dot{y}_A = \frac{1}{2}\dot{y}_B$, no surprise)

a) Using the law of cosines, we have

$$\left(\frac{3L}{4}\right)^2 = \left(\frac{L}{2}\right)^2 + (d-s)^2 - \cancel{2}\left(\frac{L}{\cancel{2}}\right)(d-s)\cos\theta$$

This relates θ to L, d, s at all times. Taking a time derivative of this equation, with L and d constant, yields

$$0 = 0 + 2(d-s)(-\dot{s}) + L(d-s)\dot{\theta}\sin\theta + L\dot{s}\cos\theta$$

Solving for $\dot{\theta}$:

$$\frac{2(d-s)\dot{s} - L\dot{s}\cos\theta}{L(d-s)\sin\theta} = \dot{\theta}$$

So $$\boxed{\dot{y}_B = L\dot{\theta}\cos\theta = \dot{s}\left[\frac{2(d-s)\cos\theta - L\cos^2\theta}{(d-s)\sin\theta}\right]}$$

We can eliminate $\cos\theta$ in favor of L, d, s using the law of cosines again:

$$\cos\theta = \frac{\left(\frac{3L}{4}\right)^2 - \left(\frac{L}{2}\right)^2 - (d-s)^2}{L(d-s)}$$

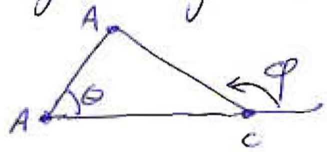
and $\sin\theta = (1-\cos^2\theta)^{1/2}$

The final result is messy but straightforward.

Another solution follows from considering rigid body $\overline{AC}$, so

$\dot{x}_A = \dot{x}_c - \frac{L}{2}\dot{\varphi}\sin\varphi$

$\dot{y}_A = \dot{y}_c + \frac{L}{2}\dot{\varphi}\cos\varphi$



with $\dot{y}_c = 0$, $\dot{x}_c = -\dot{s}$. Equating the expressions for $\dot{x}_A$, $\dot{y}_A$ we have

$$\dot{x}_A: \quad -\frac{L}{2}\dot{\theta}\sin\theta = -\dot{s} - \frac{L}{2}\dot{\varphi}\sin\varphi$$

$$\dot{y}_A: \quad \frac{L}{2}\dot{\theta}\cos\theta = \frac{L}{2}\dot{\varphi}\cos\varphi$$

Eliminating $\dot{\varphi}$, we have

$$\frac{\dot{s} - \frac{L}{2}\dot{\theta}\sin\theta}{\frac{L}{2}\dot{\theta}\cos\theta} = \tan\varphi$$

Solving for $\dot{\theta}$, we get

$$\dot{\theta} = \frac{2\dot{s}}{L(\sin\theta + \cos\theta\tan\varphi)} \Rightarrow \boxed{\dot{y}_B = \frac{2\dot{s}}{\tan\theta + \tan\varphi}}$$

We can use the law of cosines to find $\cos\theta$, as before, and $\cos\varphi$ and then get $\sin\theta$ and $\sin\varphi$, in terms of L, d, and s.

b) In this case, we use the first approach used in part a.

$$s^2 = \left(\frac{L}{2}\right)^2 + d^2 - 2\left(\frac{L}{2}\right)d\cos\theta$$

$$\Rightarrow 2s\dot{s} = 0 + 0 + Ld\dot{\theta}\sin\theta$$

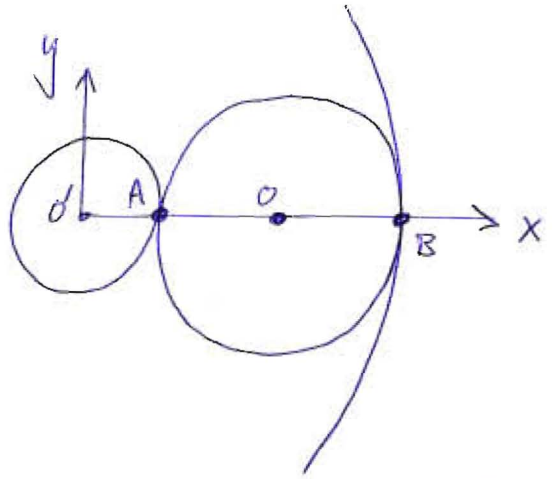
$$\Rightarrow \dot{\theta} = \frac{+2s\dot{s}}{Ld\sin\theta}$$

$$\Rightarrow \boxed{\dot{y}_B = \frac{+2s\dot{s}}{d\tan\theta}}$$

Eliminate $\tan\theta$ using the law of cosines above.

SEE PLOT ON ADDITIONAL FILE SHOWING DIFFERENCES.

3.

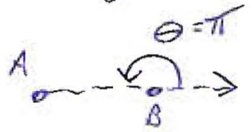


Identify points A, B at contacts of sun, planet gears and planet, ring gears, respectively. Let O be at center of planet gear.

a) ω_s is given, $\omega_r = 0$. So, velocity at A is $\dot{y}_A = r_s \omega_s$

velocity at B is $\dot{y}_B = 0 \quad (\omega_r = 0)$

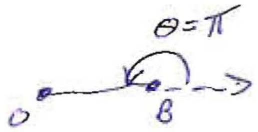
Now relate $\dot{y}_A, \dot{y}_B$: $\dot{y}_A = \dot{y}_B + r_{AB}\dot{\theta}\cos\theta \qquad \theta = \pi$



$$\Rightarrow r_s \omega_s = 0 + 2r_p \dot{\theta}(-1)$$

$$\Rightarrow \dot{\theta} = -\frac{r_s}{2r_p}\omega_s$$

Now determine $\dot{y}_o$: $\dot{y}_o = \dot{y}_B + r_{BO}\dot{\theta}\cos\theta$



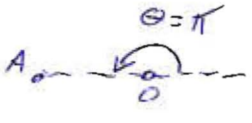
$$\dot{y}_o = 0 + r_p\left(\frac{-r_s}{2r_p}\omega_s\right)(-1) = \frac{r_s}{2}\omega_s \quad \text{(upward ✓)}$$

O rotates around O':

$$\dot{y}_o = (r_s + r_p)\omega_{pp} \Rightarrow \boxed{\omega_{pp} = \frac{r_s \omega_s}{2(r_s + r_p)}} \quad \text{(CCW)}$$

b). Now planet plate is fixed, so $\dot{y}_o = 0$.

So $\dot{y}_A = \dot{y}_o + r_{AO}\dot{\theta}\cos\theta$



$$\Rightarrow r_s \omega_s = 0 + r_p \dot{\theta}(-1)$$

$$\Rightarrow \dot{\theta} = -r_s \omega_s / r_p \quad \text{(CW)} ✓$$

Then, $\dot{y}_B = \dot{y}_o + r_{oB}\dot{\theta}\cos\theta = 0 + r_p\left(-\frac{r_s \omega_s}{r_p}\right)(1) = -r_s \omega_s$ (down; same speed as A, as expected)

Ring gear rotates around O' so $\dot{y}_B = (r_s + 2r_p)\omega_r \Rightarrow \boxed{\omega_r = \frac{-r_s \omega_s}{r_s + 2r_p}}$ CW