Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 1: Introduction to MATLAB
Due 12:00 noon Friday January 30th

- **YOUR SOLUTION TO THIS HOMEWORK SHOULD CONSIST OF A COMMENTED MATLAB .m FILE**
- **THE ASSIGNMENT SHOULD BE SUBMITTED ELECTONICALLY BY EMAILING THE FILE AS AN ATTACHMENT TO Stephanie_gesualdi@brown.edu**
- **PLEASE PUT en4_hw1 IN THE SUBJECT OF YOUR EMAIL**
- **PLEASE NAME THE ATTACHED FILE *lastname_firstname.m***

You should make your file a function, so that you can pass variables to the ODE functions - e.g.

```
Function my_amazing_homework
       Solutions to the problems…
       Nested function for the ODE problems
end
```

1. Using a loop, create a vector called v that contains 100 equally spaced points, starting at 0 and ending at 2π

2. Using the solution to problem 1, plot a graph of $y = \cos^2(x)$

3. By using a loop and a conditional ('if…end') statement to create the array y, create a plot of
$$y = \begin{cases} \cos^2 x & \sin(x) < 0.707 \\ \sin^2 x & \sin(x) > 0.707 \end{cases}$$
as a function of *x* for $0 \le x \le 2\pi$

4. The differential equation
$$\frac{dV}{dt} = -\frac{V}{\tau}\log\left(\frac{V}{V_0}\right)$$
is called the 'Gompertz' equation. It is used to model population dynamics in biological systems – for example, it is used to model the avascular growth phase of a tumor (the period before blood vessels form to supply the tumor with nutrients). In this case *V(t)* represents the (time dependent) volume of the tumor; τ is a time constant that controls how long the tumor takes to reach its maximum size, and V_0 is the tumor volume at the end of the avascular growth phase. The same equation is used to model a number of other biological phenomena. For an example, read the paper Frenzen, C.L. and Murray, J.D., "A Cell Kinetics Justification for Gompertz' Equation**,"** SIAM J. Appl. Math. **46**, (4), pp. 614-629 (1986) (you can access the paper in Brown's library of e-journals).

   Write a sequence of matlab commands in your .m file that will integrate and plot the solution to this differential equation, with
   - Initial condition V=0.01cm^3 at time $t=0$
   - Steady state volume V_0 2cm^3
   - Time constant $\tau = 15$ days

- Plot the solution for time period $0 \le t \le 100$ days.

You should integrate the equation using the MATLAB ODE solver ode45. (It is possible to integrate the equation exactly, but the purpose of this problem is to practice solving an ODE numerically).

To do the integral, you will have to define an `equation of motion' function inside your .m file that looks something like this

```
function dVdt = growthrate(t,V)
% Function to compute the growth rate of a tumor

   enter your calculation here
end
```

and then integrate the function using a command like this:

[t,V] = ode45(@growthrate,[start_time,stop_time],Initial_volume);

5. The differential equation

$$m\frac{d^2x}{dt^2} = -\lambda \frac{dx}{dt} - kx + F_0 \sin \omega t$$

is the equation of motion for a simple vibrating system. It is used to predict the motion of a wide variety of engineering systems – the vibration of a building; an aircraft wing; a vehicle suspension; and so on. The variable *x* represents the position of the system; *m* is its mass; λ represents viscous dissipation (e.g. the effects of air resistance); *k* is the stiffness of the system and F_0, ω represents the magnitude and (angular) frequency of the external force acting on the system.

Write a sequence of commands in your MATLAB file that will integrate this equation of motion and plot the solution *x(t)* as a function of time, for the following parameters

- Mass *m* =1500 kg
- Stiffness *k* = 25 kN/m (BE CAREFUL WITH UNITS – NOTE THE kN)
- Viscous dissipation $\lambda = 300$ Ns/m
- Force amplitude 1kN
- Load frequency $\omega = 10$ radians per second
- Initial conditions $x = 0$ $\frac{dx}{dt} = 0$ at time *t=0*
- Time interval $0 \le t \le 30$ sec

To solve this problem, you will first have to convert the equation into a form that MATLAB can integrate – this means that you must introduce a new variable

$$v = \frac{dx}{dt}$$

and write the equation of motion in the form

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} v \\ \text{function of } v \text{ and } x \end{bmatrix}$$

where you must calculate the function of *v* and *x* from the equation of motion. You must then create a function that will calculate

```
function dxdt = eom(t,x)
% The array x contains variables [x,v]; dxdt is d([v,x])/dt
    enter your calculation of dxdt here
end
```

and use the ODE solver ode45 to integrate the equation of motion.