Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Dynamics of Particles
Due Friday Feb 6, 2009

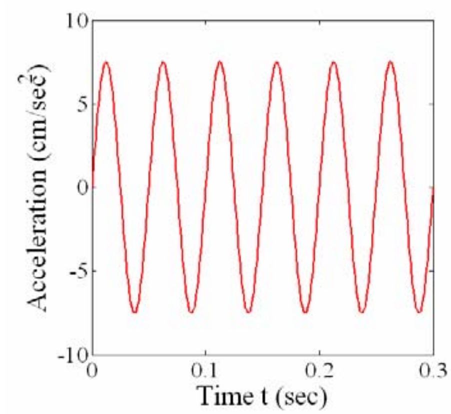
1. The design specifications for an elevator specify (i) the maximum acceleration (or deceleration) of the elevator $a_{\max}$; and (ii) the maximum speed of the elevator $V_{\max}$.

a. Derive a formula that predicts the shortest possible time for the elevator to travel between two floors that are a distance d apart, in terms of d, $a_{\max}$ and $V_{\max}$.

b. Using the data in the article http://www.elevator-world.com/magazine/archive01/0309-003.shtml, (specifically the section labeled Atmospheric Pressure Control, which gives the height, max speed, and the time to reach the top) estimate the acceleration of the elevators in the Taipei tower

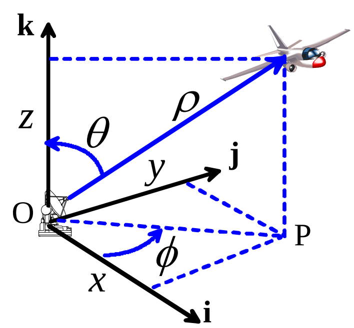
2. The figure shows a vibration measurement from an accelerometer. The vibration can be assumed to be harmonic. Use the graph to estimate:

1. The amplitude of the acceleration
2. The period of oscillation
3. The angular frequency of oscillation (in radians per second)
4. The amplitude of the velocity of the accelerometer
5. The amplitude of the displacement of the accelerometer.



3. A ground-based radar system is used to track the position of an aircraft. The radar records the distance $\rho(t)$ of the aircraft from the tracking station, and the two angles $\theta(t)$ and $\phi(t)$, as functions of time.

a. Using elementary trigonometry, write down the position vector of the aircraft as components in the basis shown, in terms of $\rho(t)$ and the two angles $\theta(t)$ and $\phi(t)$. To find the x and y components of position, it is helpful to first calculate the length of OP.



b. Using MAPLE, calculate a formula for the velocity vector of the aircraft, in terms of $\rho(t)$ and the two angles $\theta(t)$ and $\phi(t)$, and their time derivatives

$$\frac{d\rho}{dt}, \frac{d\theta}{dt}, \frac{d\phi}{dt}$$

There is no need to submit your MAPLE calculations – just give the solution.

c. Use MAPLE to find a formula for the speed of the aircraft in terms of these quantities (if you load the VectorCalculus package, the function DotProduct(v,v) can compute the dot product of velocity with itself – see the online notes for an example). Again, no need to submit the MAPLE – just report your solution.

4. Repeat the calculation done in class to estimate the minimum thrust that must be produced by the engines of an aircraft in order to take off from the deck of an aircraft carrier – but extend your calculations to account for the motion of the aircraft carrier.

a. Derive formulas for the required acceleration of the aircraft, and the time required to reach takeoff speed, in terms of the runway length d, the take-off speed v_t and the speed of the aircraft carrier V.

b. Draw a free body diagram and write down the equation of motion for the aircraft, expressing your solution in vector form.

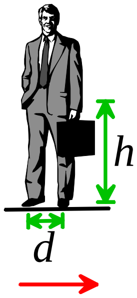
c. Hence, calculate a formula for the required engine thrust.

d. Using data given in class, and taking V=35 knots, estimate a value for the required thrust. What is the percentage reduction in thrust due to the motion of the aircraft carrier?

5. Estimate the maximum allowable acceleration of a `people mover' to ensure that a person can remain standing as the vehicle starts and stops. (the figure is from www.leitner-lifts.com)

a. Draw a free body diagram showing the forces acting on a person in the vehicle. Assume that the person stands sideways, and (stupidly) has both hands free.

b. Write down the equations of motion for the person (you will need both **F**=m**a** and **M**=**0. Be sure to take moments about the center of mass!!**). Express your answers in vector form, in terms of the height h of the person's center of mass above the ground, and the distance d between his or her feet.

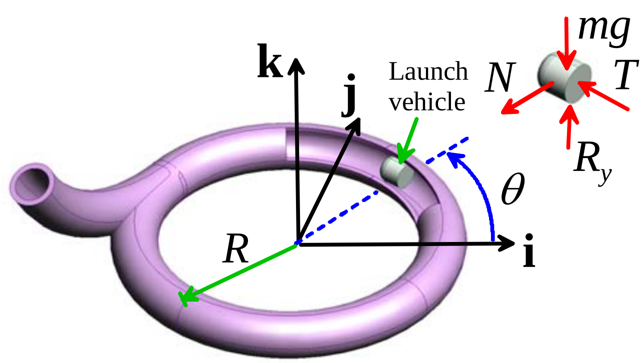


c. Hence, find a formula for the maximum acceleration in terms of h and d.

d. Estimate values for h and d and so calculate a value for the acceleration.

6. There is currently great interest in the aerospace industry in developing alternatives to chemically powered rocket motors for launching satellites into space. One such proposal is to build a large circular electromagnetic rail gun. The gun would consist of a large vacuum tube (the vacuum is needed to eliminate air resistance), arranged in a circular ring. A magnetic levitation system would be used to keep the launch vehicle from touching the sides of the ring, and an electromagnetic linear motor would accelerate the satellite to launch speed. On reaching launch speed the vehicle is permitted to enter the exit tube – thereafter it flies as a ballistic missile. A very brief description of this facility, due to be tested in 2009, can be found at http://www.launchpnt.com/portfolio/space-launch.html

The goal of this problem is to work through some preliminary design calculations for the device: in particular, we will estimate the time required to accelerate the satellite to its launch speed, and the forces acting on the satellite as it travels around the ring.



Note that

- The position of the projectile can be conveniently described in terms of the angle $\theta(t)$ shown in the figure.
- The linear motor applies a tangential force T to the projectile, and a magnetic levitation system applies a normal force N that will force the projectile to travel in a circle, as well as a vertical; reaction force R_y. The tangential force T is constant, but N must vary with time.

a. Write down the position vector of the projectile in terms of the ring radius R and the angle $\theta(t)$.

b. Find a formula for the velocity vector of the projectile in terms of R and the time derivatives of $\theta(t)$.

c. Find a formula for the magnitude of the velocity vector, in terms of R and the time derivatives of $\theta(t)$.

d. Find a formula for the acceleration vector of the projectile, in terms of R and the time derivatives of $\theta(t)$.

e. Write down the force vector acting on the projectile in terms of N, T, and other relevant variables expressing your answer as components in the basis shown

f. Write down **F** =m **a** for the projectile, and combine the components of the equation of motion to show that

$$\frac{d^2\theta}{dt^2}=\frac{T}{mR} \qquad\qquad N=mR\left(\frac{d\theta}{dt}\right)^2$$

g. Assume that the vehicle starts at rest with $\theta=0$. Find expressions for $\theta(t)$ and $d\theta/dt$ in terms of time and the variables T, m and R. (You should be able to solve the equation of motion by hand, but if you are stuck, use the MAPLE 'dsolve' function). Hence deduce a formula for the variation of the normal force N with time.

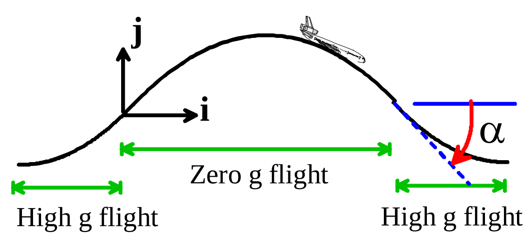
h. Using your solutions to (c) and (g), find a formula for the speed of the launch vehicle as a function of time.

i. Using the following data, calculate (i) The total time required to accelerate the launch vehicle to launch speed; (ii) the total number of times the vehicle must travel around the ring; (iii) The maximum value of the reaction force *N*.

- Ring radius *R* – 1.5 miles
- Vehicle mass 220 lb
- Linear motor thrust 300N
- Launch speed 2km/sec

7. The `Vomit Comet' is a specially equipped aircraft used by NASA and others to simulate weightlessness for short periods of time (the aircraft is actually no longer a Comet – NASA retired their Comet in 2004. They now sub-contract flights to a commercial company http://www.gozerog.com/ who fly a Boeing 727). The goal of this problem is to estimate the maximum length of time that the weightless period can last; and to estimate the air-space required for the maneuver. To provide some perspective on this problem, you could download and read the paper: F. Karmali and M. Shelhamer, (2008) "The dynamics of parabolic flight: Flight characteristics and passenger percepts," Acta Aeronautica, **63**, pp.594-602 (you can access this paper in Brown's database of electronic journals).

The figure shows the flight profile: it consists of a 'high-g' phase, followed by a 'zero-g' phase. During the 'zero g' phase the aircraft has constant horizontal speed, and has acceleration vector $\mathbf{a} = -g\mathbf{j}$.



a. Explain *briefly* (no more than 4 sentences, possibly with the aid of a free body diagram!) why the passengers in the aircraft feel weightless during the `zero-g' phase.

b. Consider the flight of the aircraft during the zero-g phase. Assume that the aircraft enters the 'zero g' trajectory at time *t=0* and at this instant has position vector $\mathbf{r} = \mathbf{0}$ and velocity vector $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$. Write down formulas for the velocity and position vectors as functions of time.

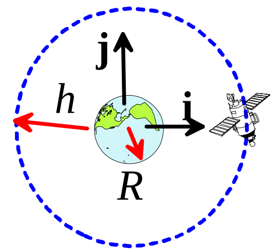
c. Using the following information, estimate (i) the total time of the zero-g portion of flight, (ii) the altitude change during zero-g flight; and (iii) the horizontal distance traveled by the aircraft during zero g flight. (if you are curious, you could compare the results of (i) with the actual data in Karmali and Shelhamer (2008)).
   - The maximum allowable true airspeed speed during zero g flight is 360 knots
   - The minimum allowable true airspeed is 245 knots.

d. Calculate the pitch angle α of the aircraft at the end of the zero g trajectory

e. Is an engine thrust required to maintain "weightless" flight? If so, why?

8. The goal of this problem is to estimate (very crudely) the velocity required for the ballistic launch vehicle described in Problem 7 to reach Low Earth Orbit (LEO). Assume that

- The vehicle is launched in a direction perpendicular to the earth's surface (parallel to the **i** direction)
- Air resistance can be neglected
- The earth's rotation can be neglected.



a. Draw a free body diagram showing the forces acting on the launch vehicle.

b. Write down the force vector acting on the launch vehicle, expressing your answer in terms of the gravitational *G*, the mass of the earth *M*, the mass of the launch vehicle *m*, and the distance *x* from the center of the earth.

c. Write down Newton's law of motion for the launch vehicle, and hence show that the distance from the earth satisfies the differential equation

$$\frac{d^2x}{dt^2} + \frac{GM}{x^2} = 0$$

d. Show that the equation of motion can be re-written as

$$v_x \frac{dv_x}{dx} + \frac{GM}{x^2} = 0 \quad \text{where} \quad v_x = \frac{dx}{dt}$$

e. Solve this equation of motion for v_x, with initial conditions $v_x = V_0$ at $x=R$ to obtain a formula for the speed of the launch vehicle v_x in terms of *x* and other parameters. You can do this by hand, or you can use the MAPLE 'dsolve' function.

f. Finally, estimate a value for the launch speed, assuming that

- The earth's radius is 6378.145km
- The Gravitational parameter $\mu = GM = 3.986012 \times 10^5 \ \text{km}^3\text{s}^{-1}$
- The LEO altitude is 250km above the earth's surface.
- The vehicle has zero velocity when it reaches LEO altitude (a rocket must be fired at this instant to start the satellite in its orbit)

In practice a spacecraft would not be launched this way – instead it would be put on a path *tangent* to the earth's surface, placing it in an elliptical transfer orbit. You would still have to fire a rocket when the spacecraft reaches its maximum altitude (otherwise it will just return to the earth, which would scare the pants off the people in the launch facility). But a much smaller and lighter rocket motor is required with this method of launch.