



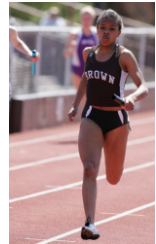
Division of Engineering
Brown University

EN40: Dynamics and Vibrations

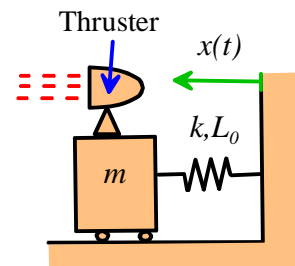
Homework 5: Linear and Angular Momentum

This homework will not be graded.

1. In a representative stride, a runner has a foot in contact with the ground for 0.1sec and is airborne for 0.2 sec. (see, e.g. Weyand *et al* 'Ambulatory estimates of maximal aerobic power from foot-ground contact times and heart rates in running humans,' *J. Appl. Physiol*, **91**, 451-458 2001). Use the impulse-momentum equation to estimate the average vertical contact force that acts on your feet while you run.



2. As background to this problem, read Koizumi, Komurasaki and Arakawa, (2004) 'Development of thrust stand for low impulse measurement from micro-thrusters,' *review of scientific instruments* **75** (10), 3185-3190. The goal of this problem is to illustrate the principle of their instrument using a simplified idealization, shown in the figure. Assume that

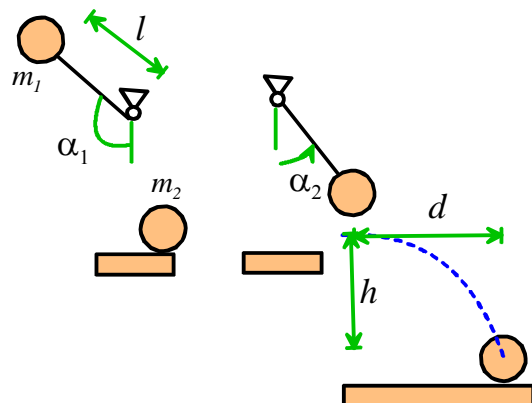


1. The system starts at rest, with $x = L_0$, where L_0 is the unstretched length of the spring.
2. At $t=0$ the thruster is fired. The thruster exerts a force on the mass for only a very short time, so the mass can be assumed to be stationary during firing.
3. The thruster starts the mass moving to the right. The length of the spring $x(t)$ is recorded, and in particular the length d at the instant of maximum spring compression is measured.

- a. Use energy conservation to write down a relationship between the d and the speed of the mass just after the thruster fired.
- b. Hence, calculate a formula for the impulse of the thruster, in terms of d , L_0 , k and m .

3. The figure shows an experimental apparatus for measuring the restitution coefficient of, e.g. a golf-ball or a bowling ball. It uses the following procedure

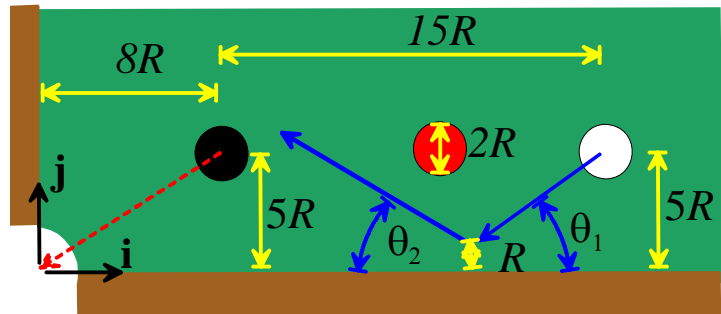
- A pendulum (a golf-club head, e.g.) is swung from a known initial angle α_1 and dropped from rest so as to strike the ball
- The angle of follow-through α_2 of the pendulum is recorded
- The distance traveled by the ball d is measured



Your goal is to derive a formula that can be used to determine the restitution coefficient e from the measured data.

- By considering its trajectory, derive an expression for the velocity of the ball just after it is struck, in terms of the height h , the measured distance d and the gravitational acceleration.
- Derive an expression for the speed of the mass on the end of the pendulum just after it strikes the ball, in terms of α_2 , l and the gravitational acceleration.
- Derive an expression for the speed of the mass on the end of the pendulum just before it strikes the ball, in terms of α_1 , l and the gravitational acceleration.
- Hence, deduce a formula for the coefficient of restitution.

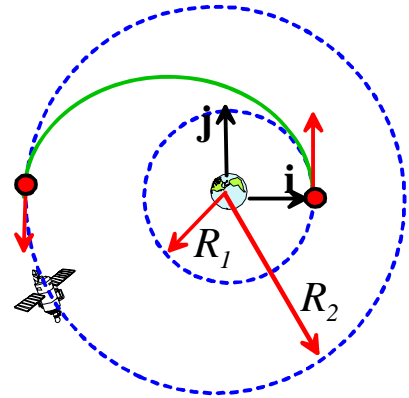
4. **Virtual Snooker.** The figure shows three balls on a snooker table. All balls have radius R . The impact of the cue ball (white) with the cushion has a restitution coefficient of 0.8; the impact of two balls has a restitution coefficient of 0.9. Your goal is to calculate an initial velocity for the (white) cue ball that will pot the black ball. Note that you are not allowed to hit the red ball, and so can't make a direct shot – you will have to play the shot off the cushion.



- Write down the coordinates of the cue ball at the instant that it strikes the black ball, assuming that it is on a trajectory to knock the black ball directly into the pocket.
- Consider the rebound of the cue ball off the cushion, as shown in the figure. Show that the angle of incidence is related to the angle of rebound by $\tan \theta_2 = e \tan \theta_1$ (Note that the trajectory of the center of the balls is offset from the cushion by the ball radius R).
- Hence, calculate the initial velocity of the cue ball that will pot the black ball (The magnitude of the velocity is arbitrary).
- To check your calculation [click here](#) to download a MATLAB simulation of the shot. Save the code in a file called *snooker.m* and then run it with
`>>snooker(Vx,Vy)`
 where V_x , V_y are the initial velocities of the cue ball (the magnitude is arbitrary – in fact the code will scale your velocity to have unit magnitude for convenience in doing the animation).
- Set up a trick shot of your own design, and edit the MATLAB code to animate it. The code is a bit flakey – MATLAB will occasionally miss a collision, for example – so don't be too ambitious. If you email your MATLAB code to Stephanie_Gesualdi@brown.edu we will web post your shot...

5. A 'Hohmann Transfer' is a maneuver for changing the radius of a satellite orbit. It uses the following procedure:

- The satellite typically starts in a low circular orbit, with radius R_1
- At some convenient time, a rocket is fired to increase the speed of the satellite, without changing its direction of motion. This places the satellite on an elliptical orbit, with perigee R_1 and apogee R_2
- The satellite is allowed to complete one half of the elliptical orbit. When it reaches its apogee, a rocket is fired again, to increase its speed a second time, without changing its direction. This places the satellite in a circular orbit, with radius R_2



- Suppose that the satellite is in a circular orbit, and moves with speed v . By (i) calculating the acceleration vector of the satellite; (ii) drawing a free body diagram for the satellite, and using Newton's law of motion, calculate a formula relating the speed of a satellite v to the radius of its orbit, in terms of the gravitational parameter $\mu = GM$.
- Consider the satellite in its elliptical transfer orbit. Use energy conservation and angular momentum conservation to calculate a formula for the speed of the satellite v_p at the perigee of the satellite orbit, and the speed v_a at the apogee of the satellite orbit, in terms of $\mu = GM$, and R_1, R_2
- Hence, calculate formulae for the changes in speed of the satellite during the two rocket burns
- A satellite in 'Geo-synchronous' orbit around the earth completes one orbit in 24 hours. Calculate the radius of this orbit
- Hence, calculate the changes in speed required to raise a satellite from low earth orbit (250 km above the earth's surface) to geosynchronous orbit