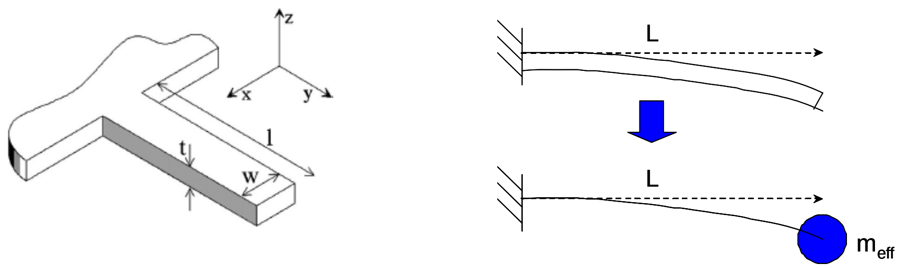
EN 4
Dynamics and Vibrations

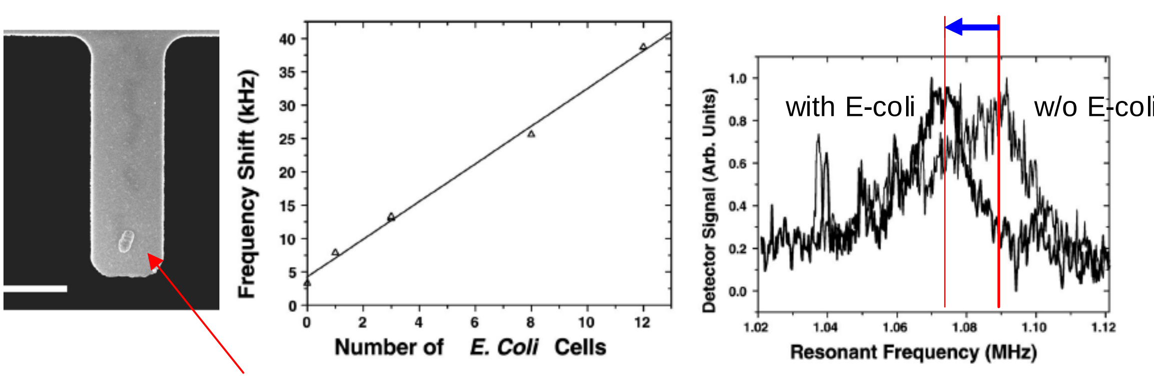
Homework 6: Free and Damped Vibrations

1. *Detection of E-coli by change in the natural frequency of a nanobeam resonator.* Nanofabrication techniques have been used to create cantilever beams of Silicon-Nitride.



Left: Schematic w=10 micrometers; t=320 nanometers; l = 25 micrometers
Right: Approximation of beam as a mass at the end of a massless beam spring.

a) Because the beam itself has mass, it has a natural frequency given by $\omega_n = 1.03\sqrt{\dfrac{Ewt^3}{l^3 m}}$ where m is the mass of the beam, with E=110x10^9 N/m^2 and mass density $\rho = 3.4 g/cm^3$ for this type of Silicon Nitride. The force-displacement law for downward deflection of the beam is $F = \dfrac{3EI}{L^3}x$, where I is the area moment of inertia (see lecture notes). To make things simple, treat the cantilever beam as if it is a spring with an effective mass m_{eff} concentrated at the end of the beam (right figure). Determine the appropriate effective mass m_{eff}, in terms of the true mass m, so that the natural frequency is the same as the true value for the beam.



Left: Top view (one E-coli near the end of beam) Middle: Frequency shift in kiloHz vs number of E. Coli cells on the beam. Shift at 0 is due to an antibody coating used to make the E Coli stick to the beam; neglect this. Right: Vibration amplitude versus frequency; the natural frequency is near the peak of the response, and the shift in frequency is approximately indicated.

[see B. Ilic, et al., J. Vac. Sci. Tech. B, 19, p. 2825 (2001)]

b) A single E-coli bacterium becomes stuck near the end of the beam. As a result of the change in mass, the natural frequency of vibration of the system is changed. The frequency response of the system is shown in the figure. From this data and the information and results in part a), determine the mass of the E-coli.

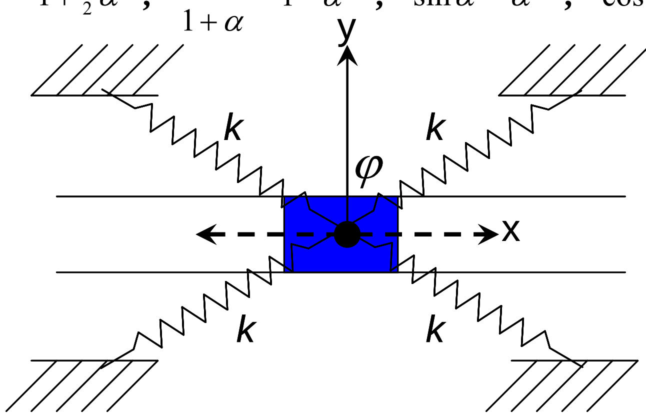
2. A mass m is confined to move in the channel shown. The mass is connected to four springs of spring constant k oriented at angles $\varphi = 60^o$ as shown when the system is in static equilibrium. In the equilibrium state, the spring lengths are L. Ignoring friction, determine the natural frequency of vibration of the system in terms of k, m, and φ.

The unstretched spring length is not needed. If you find it useful to introduce an initial spring length, you may do so, but it should not appear in your solution.

To find the EOM and put it into the "standard form" for a vibrations problem, you will need to finding the EOM for *small* displacements x, i.e. x/L<<1, away from the static equilibrium point. In doing so, you will have to expand various exact expressions in terms of the small quantity (x/L) and neglect terms that are proportional to $(x/L)^2$,…

Possibly useful expressions for a small dimensionless quantity α:

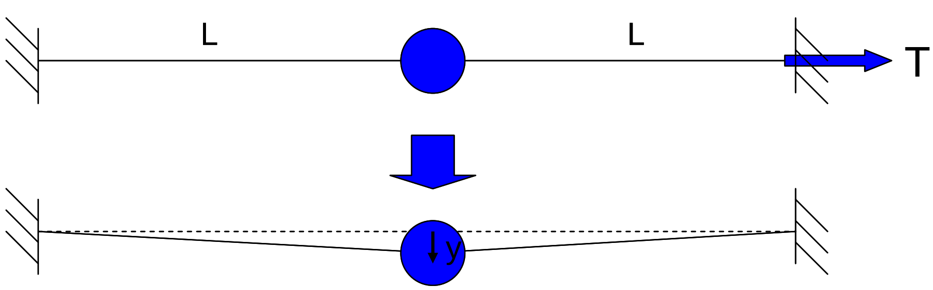
$$(1+\alpha)^{1/2} \sim 1+\tfrac{1}{2}\alpha \quad ; \quad \frac{1}{1+\alpha} \sim 1-\alpha \quad ; \quad \sin\alpha \sim \alpha \quad ; \quad \cos\alpha \sim 1-\tfrac{1}{2}\alpha^2$$



3. **Homemade bass guitar.**

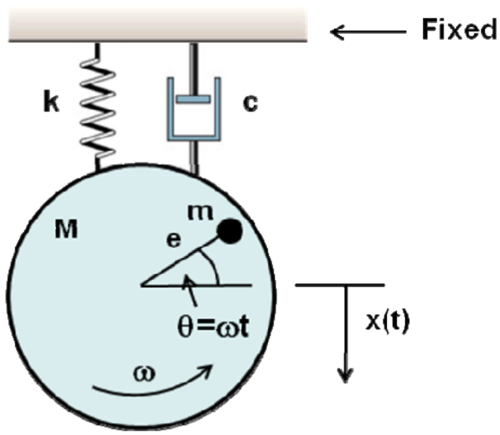
Two wires of negligible mass are attached to a mass m at the middle, as shown. The wires are then stretched so that there is a tension T in each wire, as shown in the figure. What is the frequency of vibration of the mass if it is displaced laterally by a small distance y and released? Express your answer in terms of m, T, and L only.

As in Prob. 1, you should develop an EOM that is valid for small y only (y/L<<1). To begin design of a bass guitar (low frequencies), but using only very light-weight piano wire usually used for high notes and a tension of 750N typical for pianos, determine the mass m needed to obtain the A one octave below middle C (i.e. the A at frequency 110 Hz) using a length of 1m for the "guitar".

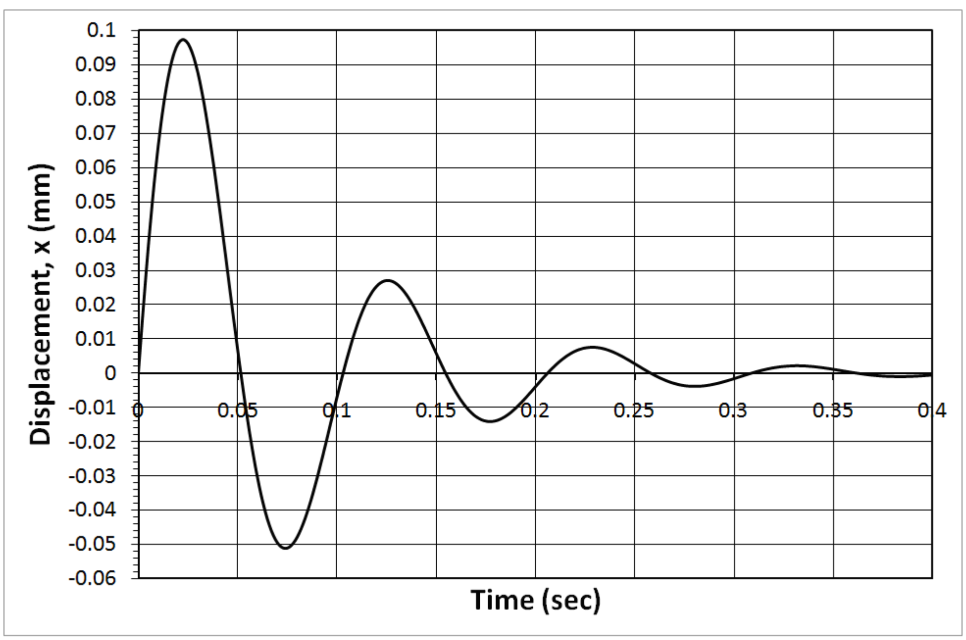


4. Jet Engine Diagnostics. An aircraft engine, mounted on an aircraft wing as shown in figure (a), is being checked for vibration levels during maintenance. To investigate the problem, the wing, engine and mounting are to be modelled as a single degree of freedom system, as shown in figure (b), with vertical motion only and assuming the wing is fixed. To investigate the effective properties of the mount, a device is built to hold the wing fixed, and an impulse hammer is used to supply an initial velocity to the engine. The resulting free vibration of the engine is shown in figure (c).

(a) Aircraft Engine



(b) Simplified Model



(c) Free (unforced) response to an initial impulse

From this data, find approximate values for the damped natural period τ_d, the viscous damping coefficient ζ, and the natural frequency ω_n.