EN 4
Dynamics and Vibrations

Homework 7: Damped and Forced Vibrations

1. Spinal Pendulum (revisited)

In HW #3, you analyzed via MATLAB the motion of a pendulum with a section of the spinal column at the pivot. Return to that problem, and examine the equation of motion you derived. From the EOM, predict the damped natural period and the damping coefficient analytically. Using your numerical solution as "data", extract the damping coefficient from the logarithmic decrement, and compare to the analytical result.

## 2. Engine Test Stand (revisited)

The engine test stand of HW #5 is revamped to include a shock absorber to minimize recoil and vibrations when the thrust is turned on or off.   Consider the system to be running with constant thrust T=10,000 lbs, and then to have the thrust turned off abruptly.

Design a system (i.e. choose values for k and c) so that the engine + test stand (total weight = 1500 lbs) has a deflection of less than 1 ft. and returns to equilibrium as fast as possible when the thrust is turned off.
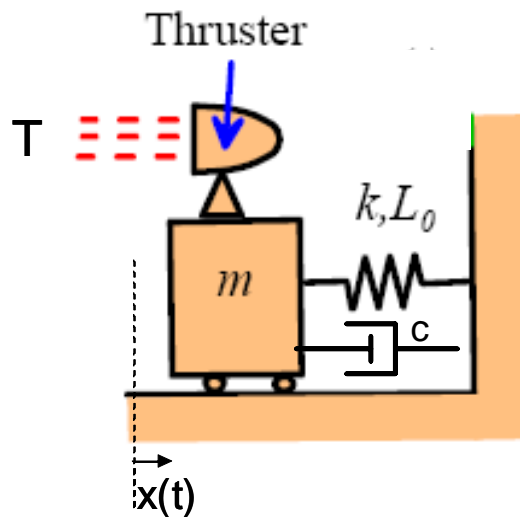
3. Design of a Seismograph

"Low Frequency Earthquake" tremors occur in the 1-5 Hz frequency range along the San Andreas fault. A simple seismograph consists of a mass/spring/dashpot system attached to a rigid frame, with the frame attached firmly to the ground. Background vibrations over a wide range of higher frequencies occur with typical amplitudes of 0.1mm, and so tremor amplitudes comparable to or smaller than this cannot be detected. Design the system (values for m, k, c) to reliably detect vibrations at a frequency of 3 Hz and having earth
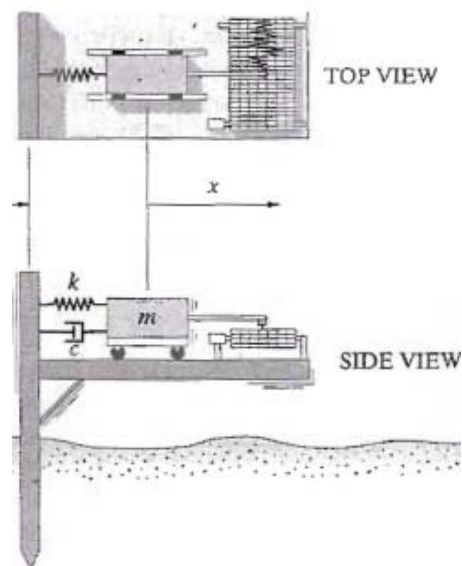
motion amplitudes of 0.01mm or larger, but also so that the maximum amplitude will not exceed 30 mm for earth motion amplitudes of 1.0 mm. **Note that the device measures motions relative to the base, not relative to a fixed point in space.**

4. Jet Engine Diagnostics (again)

The jet engine from HW #6 is mounted on an aircraft wing as shown in the figure, and is being checked for vibration levels during maintenance. To investigate the problem, the wing, engine and mounting are to be modelled as a single degree of freedom system, as shown in figure (b), with vertical motion only and assuming the wing is fixed. You have already determined the system parameters for free, damped vibrations in HW #6.
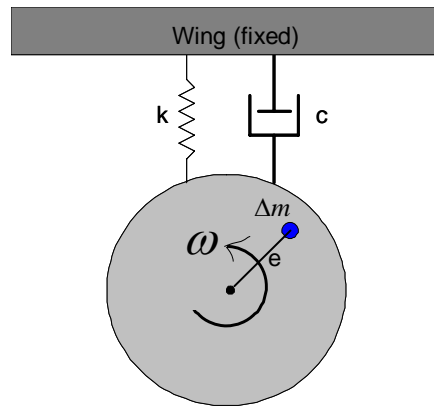
(a) Aircraft Engine   **(b) Simplified Model**

With the wing kept fixed, the engine is run at 6000 r.p.m. which is its typical operating speed at aircraft cruise conditions. At this speed, the maximum displacement amplitude of the engine's vibrational motion is found to be 20 microns (20×10^{-6} meters).

a. If the total mass of the engine is M=450 kg, use the model of figure (b) to find the effective imbalance, "Δme", in the engine (in units of kg-m). Note "Δme" is the effective mass imbalance, Δm, multiplied by its effective eccentricity, e.

b. At 6000 r.p.m., what would be the maximum force transmitted to the wing?

c. If the engine mounting could somehow be modified to double the damping constant c, without changing the spring constant k, would this increase or decrease the force transmitted to the wing when the engine is operated at this speed?

d. If the engine was run at the natural frequency of the engine/mount system, instead of 6000 r.p.m., what is the force transmitted to the wing?

5. The Two-Story Building (revisited)

Recall the equations of motion you obtained for the two-story building in Example 3 of Sec. 3.3.3. of the notes (discussed in sections). Here, we will determine the vibration frequencies analytically and then verify the resonance numerically using the already-existing MATLAB code.

a. In your EOM, discard the driving force caused by motion of the ground. This is not relevant for obtaining the natural frequencies of the building itself. Next, discard any constant terms. These correspond to shifts in the static positions of the two floors relative to whatever you called "zero" for these positions initially. They are also not relevant for obtaining the natural frequencies.

b. Put the equations in the form of a matrix equation

$$\frac{d^2}{dt^2}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} M11 & M12 \\ M21 & M22 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where y_1, y_2 are the position vectors and $M11, M12,.....$ are coefficients in the EOM.

c. Obtain the Eigenvalues of the matrix M. This corresponds to the values of λ^2 that satisfy

$$\det\begin{bmatrix} M11-\lambda^2 & M12 \\ M21 & M22-\lambda^2 \end{bmatrix} = 0$$

where det[M] is the determinant of the matrix. For a 2x2 matrix, $\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

d. Compute the natural frequencies of vibration, $\omega_n = |\lambda|$, in terms of the building parameters m and k given in the earlier HW. Use your prior MATLAB code with the earth motion to predict the positions versus time for each story of the building when the earthquake frequency equals each of the natural frequencies computed here, using the parameters from Example 3 of Sec. 3.3.3. of the notes.