EN 4
Dynamics and Vibrations

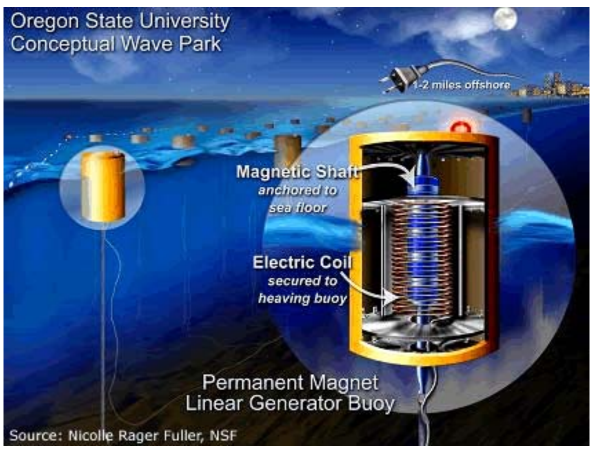
Homework 6: Damped and Forced Vibrations

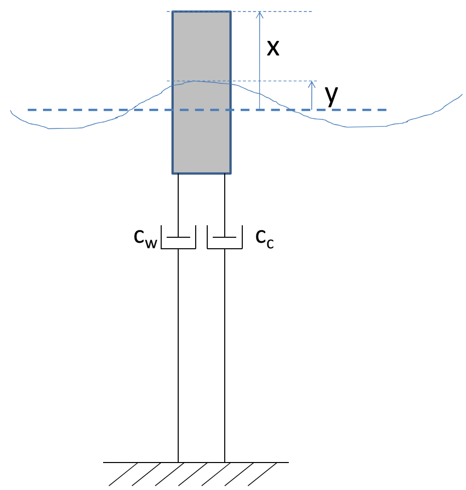
1. More MATLAB

Generalize the MATLAB code created last week to include an additional sinusoidal forcing function. Using the system parameters from last week, generate and plot numerical solutions for the Magnification Factor as a function of the ratio of (forcing frequency)/(natural frequency) over a range covering the resonance condition.

2. Design of an Ocean Wave Energy-Harvesting Device

The idea of extracting energy from naturally occurring fluctuations in the earth's dynamics is increasing, with wind energy and wave energy the most popular. Here, you will design a basic wave energy harvester to extract energy from ocean waves. A pretty picture of such a system is:





The picture shows a magnetic shaft surrounded by an electric coil. The shaft is attached to a cable fixed to the ocean bottom, while the coil is attached to the buoy. As the buoy bobs up and down due to the ocean waves, the coil passes back and forth across the magnetic shaft and generates an electric current (you will learn about this in EN 52). That current is captured by various electronics and sent back to shore. Thus, mechanical energy in the wave is converted into electrical energy.

We would like to design the buoy (dimensions, density, damping of the electrical system) to extract the maximum amount of electrical energy possible, given knowledge of the most typical wave behavior in the chosen location. We model the system above as a mass suspended in the water and connected to two viscous dampers, one representing the damping due to the ocean water, c_w, and the second one representing the damping due to the electrical system, c_c. It is the energy dissipated in c_c that we are interested in, since that is the electrical energy created. A sketch of the engineering model is shown above.

The cross-sectional area of the buoy is A, the length is L, the density of the buoy material is ρ_b, and the density of water is ρ_w.

The forces on the buoy are gravity, the forces due to the viscous damping, and the buoyancy force. The buoyancy force is an upward force equal to the weight of the volume of water displaced by the buoy.

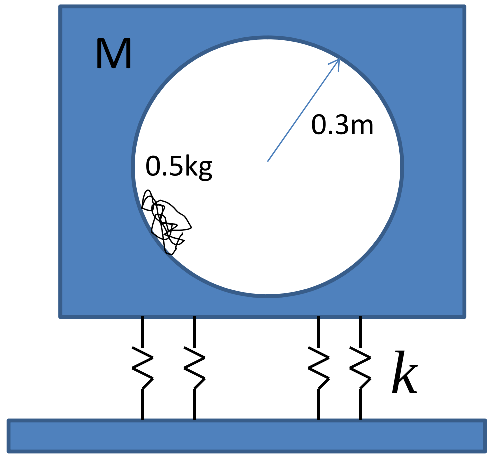
Define the position of the top of the buoy as x, and the height of the water relative to flat water, as y, as shown in the figure above.

a. Draw the free body diagram for the forces acting on the buoy. Express the buoyancy force in terms of x, y, and buoy and water properties. Express the force of gravity in terms of the buoy properties.
b. Determine the general equation of motion for the buoy position x when the water height is y.
c. Determine the static position of the buoy x_o in terms of the buoy dimensions and the buoy and water densities.
d. Define a new buoy height variable as $z=x-x_o$ and obtain the equation of motion for z.
e. Write down the function y(t) for a wave having frequency ω and wave amplitude y_{max}.
f. Combining d. and e., put the equation of motion in the "standard form" for a forced vibrational system, and express the natural frequency, the viscous damping coefficient, and the magnitude of the driving force F_o, in terms of the material, system, and wave parameters of this problem.
g. The steady-state solution can be written as $z(t) = Z\sin(\omega t + \phi)$. Write down the equation for the steady-state amplitude Z of the buoy motion in terms of the parameters.
h. Assuming the viscous damping factor ς is small, determine the natural frequency for the buoy that will maximize the steady-state amplitude of the buoy motion, and then determine the combination of buoy dimensions and density that can achieve this natural frequency.
i. Express the maximum amplitude $Z_{\max}$ in terms of ς.
j. Express the power dissipated by the electrical system (i.e. by c_c) in terms of the $Z_{\max}$ and other system parameters. Then compute the *average power dissipated per period of the vibration* in terms of the parameters.
k. Using the result of i. in j., and considering c_w to be fixed, find the value of c_c that will maximize the power dissipated, and thus maximize the electrical power generated, in terms of any other parameters of the buoy and water.

You now have the design parameters for an optimal wave energy conversion device, within the limits of the simple design and approximations considered in this problem.

3. That Unbalanced Washing Machine over in Perkins

A 35kg washing machine, including wet clothing, sits on four supporting springs, which have a static displacement of 0.01m. What is the amplitude of vibrations if a wet 0.5kg clump of clothes is stuck to the inside of the washer shown, when the washer is in the "spin" cycle rotating at 275 RPM (revolutions per minute)?

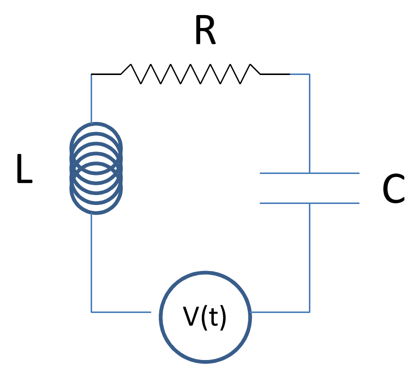


4. Electrical Circuit Theory (getting a jump on EN 52)

In electrical circuits, the electrical charge movers through various components according to various laws of physics.

- For a capacitor of capacitance C (units of Farads) under a voltage V, the charge on the plates of the capacitor is q=VC.
- For a resistor of resistance R (units of Ohms), the voltage and current are related by Ohm's Law V=IR, where I=dq/dt.
- For an inductor (a coil) of inductance L (units of Henrys), the voltage across it is V=L dI/dt.

A basic "RLC Circuit" consists of an inductor, capacitor, and resistor all in series with an AC voltage source $V(t) = V_o \sin(\omega t)$. The sum of the voltages around the circuit must be equal to that of the voltage source, so that the equation governing the system is



$$L\frac{dI}{dt} + RI + \frac{1}{C}q = V_o \sin(\omega t)$$

a. Write down the "equation of motion" for the charge *q* as a function of time, *q(t)*.

b. What is the natural frequency ω_n of charge oscillations in terms of R, L, C, V_o, and/or ω?

c. What is the approximate steady-state amplitude of the steady-state current *I(t)* when $\omega = \omega_n$, in terms of R, L, C, and/or V_o ?

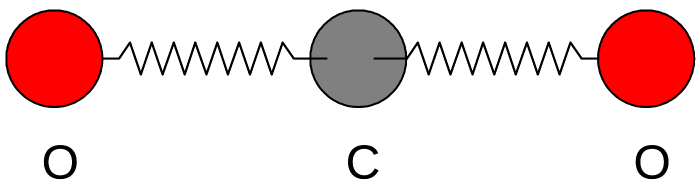
5. Greenhouse Gases

Molecules are groups of atoms connected through (conservative) interatomic potential functions. Since the atoms have mass, the molecules have natural frequencies and associated modes of vibration. For molecules having charged atoms (+ and -), an applied electromagnetic field can act has a driving force on the molecule, and energy can be absorbed into the molecule. The absorption is particularly high at resonance – when the frequency of the electromagnetic field (which, depending on the frequency, we might call a microwave, a radio wave, an infrared wave, visible light wave, or ultraviolet wave). Greenhouse gases are molecules that absorb *infrared radiation* generated by the heat of the earth. Carbon dioxide (CO_2) is a Greenhouse Gas because it has vibrational frequencies that are in the range of the radiation frequencies emitted by the earth.

For an introduction, and lots more, on Greenhouse Gases and CO_2 see:

http://books.google.com/books?id=ZfoUIfhX3YIC

Carbon dioxide (CO_2) is a "linear" molecular



For models of vibrations in CO_2, see http://www.chemtube3d.com/vibrationsCO2.htm.
Of particular interest are the vibrations at 1373 cm^{-1}, 2438 cm^{-1}. These values are the wavenumber $= 2\pi / \lambda$, where λ is the wavelength. The frequency is $\omega = 2\pi c / \lambda$ where c is the speed of light.

In HW 6, you showed that a model for atomic bonding between C and O yields an effective spring constant for vibrations and, using the average of the C and O masses, an estimated natural frequency of vibration. This week, we will examine the vibrations of CO_2 in more detail and elucidate its role as a Greenhouse Gas.

- **a.** Consider linear motion, along the axis of the O-C-O bonds only, of the atoms. Use the equilibrium position of each atom as an origin for that atom. Sketch the atoms in some non-equilbrium state with positive positions x_1, x_2, x_3 from their origins.
- **b.** Treating the molecule as three equal masses connected by two springs of stiffness k (see last week), determine the equations of motion for each atom.
- **c.** The forces in the EOMs in b. should only involved differences between the atom positions (the absolute position of the molecule in space has no relation to its vibrations). So, define two new variables y and z corresponding to the distances between the two pairs of atoms (O-C and C-O) and, by combining the previous EOMs, write down the two EOMs for y and z.

d. Considering all possible directions of motion, how many total degrees of freedom are there for Carbon Dioxide motion?

If we constrain the molecule to motion along the linear axis, how many degrees of freedom are there?

If one of the linear d.o.f.'s corresponds to rigid motion (relative positions of all three atoms held fixed) along the line of the molecule axis, how many vibrational modes are there for vibrations of the atoms along the linear axis?

Is the number of d.o.f.s consistent with the number of EOMs in part **c**?

e. Now determine the natural frequencies for this molecule. To do this, follow the strategy discussed in class – write the equations of motion in matrix form and find the Eigenvalues of the stiffness matrix. This will yield the two vibrational frequencies and modes of the CO_2 molecule corresponding to vibrations along the O-C-O axis of the molecule in terms of m (we did not distinguish the masses in this problem) and k (which is related to D and α ; see last week's HW).

f. Using values found last week and results in e., compute the wavenumbers corresponding to these vibrational frequencies and compare to the experimental values above. Convert to a wavelength, and determine what range these vibrations are in (Infrared? Visible? Ultraviolet? Microwave?) by consulting references on the electromagnetic spectrum.

g. Thinking about the possible motions physically, sketch the type of atomic motions corresponding to each of the two natural frequencies determined in f. above. That is, determine the "modes" of vibration for each of the frequencies, i.e. the relative motions of the atoms at each frequency. See ChemTube3d if you have trouble with this.

h. Now consider an electromagnetic field impinging on the CO_2 molecule. The electromagnetic field is a sinusoidal field of the form $\vec{E} = \vec{E}_o \sin(\omega t)$. Let the polarization (direction) of $\vec{E}_o$ be parallel to the O-C-O line of the molecule. The C and O atoms each have an electric charge (the molecule forms by transferring electrons from the C to the O atoms). Take the charges to be +2e on the C and –e on each O, with e the charge of an electron. The forces acting on the individual atoms are then qE where q is the charge and E is the magnitude of the electric field. Draw a F.B.D. for the molecule showing the *applied forces due to the electric field acting on each atom*. Determine which type of vibration motion is consistent with the electromagnetic forces exerted on the atoms.

i. What frequency of electromagnetic radiation ω will cause resonance of the molecule? This resonance corresponds to absorption of energy from the electromagnetic field into the molecule.

j. Based on the above analysis, why are oxygen and nitrogen, O_2 and N_2, not "Greenhouse Gases"? What about methane, CH_4?