



EN40: Dynamics and Vibrations

Homework 1: Introduction to MATLAB Due 12:00 noon Friday February 5th

Division of Engineering
Brown University

- **YOUR SOLUTION TO THIS HOMEWORK SHOULD CONSIST OF A COMMENTED MATLAB .m FILE**
- **THE ASSIGNMENT SHOULD BE SUBMITTED ELECTONICALLY BY EMAILING THE FILE AS AN ATTACHMENT TO Stephanie_gesualdi@brown.edu**
- **PLEASE PUT en4_hw1 IN THE SUBJECT OF YOUR EMAIL**
- **PLEASE NAME THE ATTACHED FILE *lastname_firstname.m***

You should make your file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function my_amazing_homework
    Solution to problems 1-6
    Functions for the differential equations in problems 5&6
end
function avg = compute_average(vector)
...
end
```

If you get stuck, you might find the solutions to homework 1, 2009 helpful.

1. Using a loop, create a vector called x that contains 201 equally spaced points, starting at -3 and ending at $+3$.
2. Using the solution to problem 1, plot a graph of $y = \exp(-x^2)$, for $-3 < x < 3$. You can create the vector y using a loop, or else use the dot notation. See Section 9 of the Matlab tutorial for a discussion of both approaches.

3. By using a loop and a conditional ('if...end') statement to create the arrays v, w , create a plot of

$$w = \begin{cases} \sin v & \sin(v) < 0.707 \\ 0.7071 & \sin(v) \geq 0.707 \end{cases}$$

as a function of x for $0 \leq v \leq 2\pi$. See Section 12 of the MATLAB tutorial for an example using a conditional statement.

4. Write a MATLAB function that will compute the average of all the elements in an array. Your function should look something like this:

```
function av = compute_average(vector)
% Function to compute the average of a vector
    enter your calculation here. Hint: you can compute
    the sum of all the elements in the vector using a loop
    total=0; for i=1:length(vector) ; total=total+vector(i); end
end
```

and should appear at the end of your file. Use your function to compute the average value of the vector w calculated in problem 3.

5. The fractional charge of a laptop battery during charging is governed approximately by the differential equation

$$\frac{dC}{dt} = \frac{1}{t_0} \left(\frac{1}{C} - 1 \right)$$

where $0 < C < 1$ is the fractional charge of the battery (it is completely discharged if $C=0$, and fully charged if $C=1$), and t_0 is a constant.

Following the method in Section 14.1 of the Matlab tutorial, write a function in your M-file that will calculate dC/dt given values of t, C . Then add a line to your file that calculates C as a function of time. Take $t_0 = 8$ hours, and assume $C = 0.1$ at time $t=0$. Plot the solution for $0 < t < 15$ hrs. How long does it take for the battery to reach 90% charge?

6. The differential equations

$$\frac{dm}{dt} = m(\alpha - \beta c) \qquad \frac{dc}{dt} = -c(\gamma - \delta m)$$

are the famous ‘predator-prey’ equations used by mathematical biologists to predict the populations of two interacting species. In these equations

- m (mice) represents the population density of the prey
- c (cats) represents the population density of the predators
- α is the rate at which mice reproduce – for large α , the mice reproduce very fast. The model assumes that mice have lots of food, so their population will be infinite if they don’t get eaten.
- β specifies how fast the cats eat the mice – for large β , each cat eats lots of the available mice.
- γ is the rate at which cats perish if they can’t catch any mice
- δ is the rate at which cats reproduce if they have a supply of mice. Note that the cats reproduce faster if there are lots of mice.

It is impossible to find formulas for the numbers of cats and mice $c(t), m(t)$ satisfying these equations. You can easily find a numerical solution using MATLAB, however. Section 14.4 of the Matlab tutorial describes how to do this.

Following the example in the tutorial, write a function that will calculate dm/dt and dc/dt given values of t and $[m, c]$. Then add commands to your MATLAB file that will integrate the equations and plot the solution $m(t), c(t)$ as functions of time. Find and plot the solution up to time $t=15$ days, for the following parameter values:

Initial conditions: $m = 20, c = 1$ at time $t=0$

Parameters $\alpha = 4 \quad \beta = 3 \quad \gamma = 1 \quad \delta = 0.1$ (in days⁻¹)

One application of this model is to predict the effects of external factors on the population cycle. For example, write commands in your MATLAB file that will find and plot the solution to the following tests:

1. Suppose that a large number of mice are released into the environment at time $t=0$, so that $m = 200, c = 1$ at time $t=0$. Run this simulation for a longer time – say 30 days.
2. Suppose that a few extra kittens are acquired in an attempt to control the mouse population, so that $m = 20, c = 2$ at time $t=0$. Notice that this actually *increases* the maximum mouse population!
3. **OPTIONAL EXTRA CREDIT PROBLEM** – What initial mouse & cat density would you expect to give a constant population? Is the constant population stable?