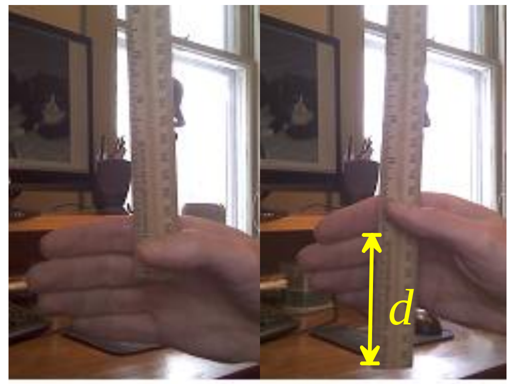
Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 2: Dynamics of Particles
Due Friday Feb 12, 2010

1. The figure illustrates a simple method for measuring a person's reaction time. A ruler is suspended vertically between the test subject's fingers. The ruler is then dropped, and the test subject catches it by closing his or her fingers. The reaction time can be determined from the distance traveled by the ruler before the subject brings it to rest.

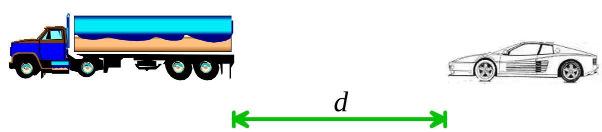


1.1 After the ruler is released, it drops vertically with constant acceleration of g=9.81 m/s^2. Write down an equation relating the distance d dropped by the ruler to the reaction time t_r

1.2 Find a friend, and have him or her measure your reaction time using this approach (if you have no friends, the faculty or TAs will be happy to help you do the experiment. Some of them may even be willing to be your friend)

1.3 Repeat the experiment, but this time determine your reaction time while sending a text message on your cell-phone (we assume you can text with only one hand, as you would while driving a car…)

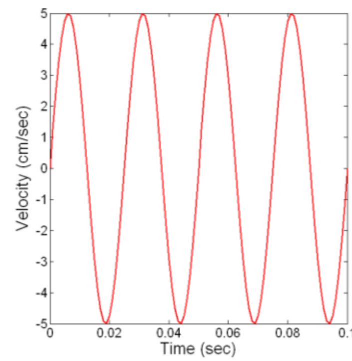
2. Two vehicles traveling at speed V are separated by a distance d as shown in the figure. At time $t=0$ the leading vehicle strikes an obstacle and is brought to a complete standstill. The driver of the trailing vehicle takes time t_r to react, and then applies the brakes, decelerating her car at rate $\mathbf{a} = -0.8g\mathbf{i}$, where g is the acceleration due to gravity.



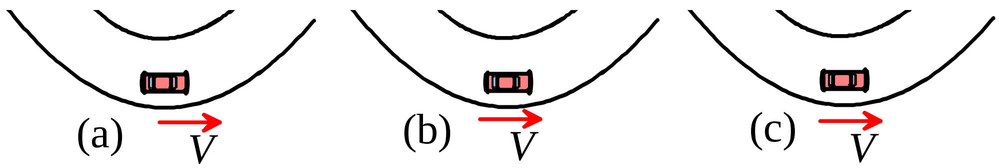
2.1 Find a formula for the minimum separation distance d that will prevent a collision.

2.2 Estimate values for d assuming that V=65MPH and using your answers to 1.2 and 1.3.

3. The figure shows a vibration measurement from a velocity transducer. The vibration may be assumed to be harmonic.



Estimate
(a) The period of oscillation
(b) The angular frequency of oscillation
(c) The amplitude of the velocity
(d) The amplitude of the displacement.
(e) The amplitude of the acceleration.



4. The figure shows a car that travels along a circular road.

In Fig 1(a), the car travels at constant speed.

In Fig 1(b), the driver is braking, and the car's speed is decreasing.

In Fig 1(c), the driver has her foot on the gas and the car's speed is increasing.

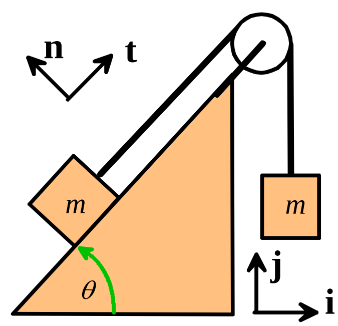
Draw an arrow on a copy of each of figures (a), (b), (c) to show the approximate direction of the car's acceleration vector.

5. The Pilot's Operating Handbook for a Cessna-172 (see http://www.redskyventures.org/doc/cessna-poh/C172N-1978-POH-S1to7-scanned.pdf) includes the following information:

- Maximum aircraft takeoff weight 2300lb
- Minimum takeoff distance 805ft
- Short-field takeoff speed 55 knots

Use this information to estimate the thrust produced by the aircraft's engine (neglect drag). Express your answer in Newtons.

6. The two blocks shown in the figure have the same mass. The contact between the mass and wedge is frictionless, and the mass of the pulley can be neglected.



6.1 Draw a free body diagram showing the forces acting on each mass

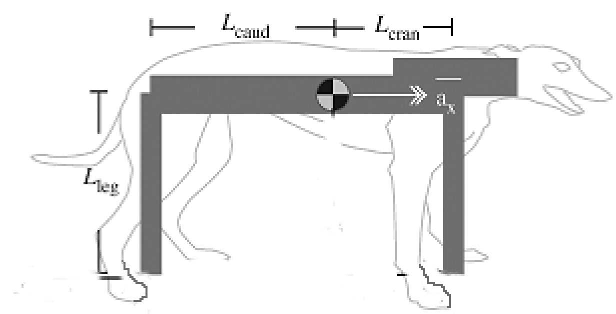
6.2 Write down a relationship between the magnitudes a_1, a_2 of the accelerations of the two masses

6.3 Write down unit vectors parallel to the direction of the acceleration of each mass. You can use whichever set of basis vectors you find most convenient.

6.4 Write down Newton's laws of motion for both masses (use either basis)

6.5 Finally, determine the tension in the cable, and the acceleration of each mass

7. Experimental measurements suggest that athletes, and running animals, cannot accelerate faster than a critical limit (see, e.g. [Williams *et al* 2009](#)). At high speeds, the maximum possible acceleration is limited by the maximum power that the animal's muscles can develop. At low speeds, the maximum possible acceleration is limited by basic physics: if the animal tries to accelerate too fast, it tips over. The goal of this problem is to determine the maximum low-speed acceleration of a quadruped.

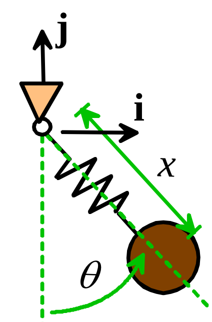


7.1 Draw the forces acting on a copy of the idealized stick-animal shown in the figure during its initial acceleration

7.2 Assume that the center of mass of the animal remains at a fixed height, so the animal's acceleration vector is $a\mathbf{i}$, where a is to be determined. Use Newton's laws **F**=m**a** and the rotational equation of motion **M=0** to calculate formulas for the reaction forces acting on the animal's feet, in terms of a and the dimensions shown in the figure.

7.3 Hence, find a formula for the maximum possible value of a that can be achieved without the animal's front legs leaving the ground.

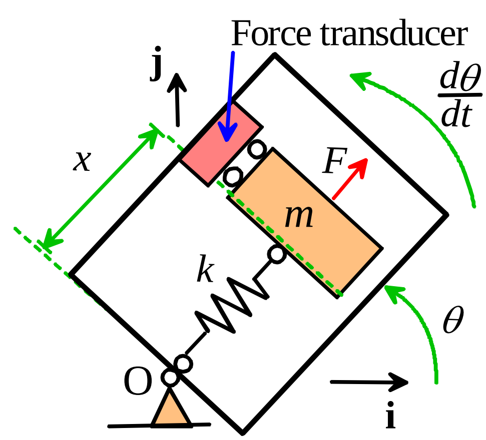
8. The figure shows a mass at the end of a flexible cable, which is idealized as a spring. The motion of the system is characterized by the length of the spring, x, and the angle θ.



8.1 Write down the position vector of the mass in terms of these variables, expressing your answer as components in the basis {**i,j,k**} shown in the figure.

8.2 Find formulas for the velocity and acceleration vectors of the mass, in terms of x, θ and their time derivatives. You can use MAPLE to do this if you like – but you will need to be able to do this kind of calculation by hand in exams, so make sure you understand what MAPLE is doing!

9. The figure shows a design for a MEMS gyroscope, which is used to sense rotations (for more information, see, e.g. http://www.sensorsmag.com/sensors/acceleration-vibration/an-overview-mems-inertial-sensing-technology-970) The device consists of a small mass m inside a rigid case. The entire assembly rotates at a **constant** rate $d\theta/dt$ about the pivot shown: the purpose of the sensor is to measure this rotation. The mass is supported by a spring with stiffness k, and is subjected to a time-varying force $F(t)=F_0 \sin\omega t$ by an electrostatic actuator. This causes the spring/mass system to vibrate, so that the length of the spring has the form $x(t)=L+X_0 \sin\omega t$. A force transducer then measures the reaction force $N(t)$ acting on the mass: this signal is then used to determine the rotation rate $d\theta/dt$. The goal of this problem is to derive the formula relating N to $d\theta/dt$



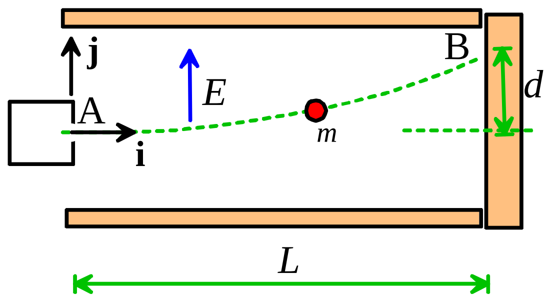
9.1 Write down an expression for the position vector of the mass relative to the pivot at O, in terms of x and θ

9.2 Hence, write down the acceleration vector of the mass, in terms of x, θ and their time derivatives. (you can just use the solution to problem 8…).

9.3 Draw a free body diagram showing the forces acting on the mass. Gravity can be neglected.

9.4 Hence, write down **F**=*m***a** for the mass.

9.5 Finally, show that $d\theta / dt = N / (2m dx / dt)$



10. The figure shows a proposed design for an electrostatic mass spectrometer, which is intended to be used as a carbon-13 urea breath test (used to detect the bacteria that cause stomach ulcers). In this test, the patient swallows urea containing carbon-13. The mass spectrometer is then used to test the patient's breath for the presence of isotope labeled CO2. If the isotope is detected, the urea was split in the patients stomach, indicating that H. pylori bacteria are present. See, e.g. http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1766662/ for more information.

- Positively charged CO2 ions are emitted with speed $V = \sqrt{2e\Phi / m}$ at A, where e is the charge of an electron, Φ is the acceleration voltage, and m is the mass of the ion.
- Between A and B, an electric field E exerts a vertical force with magnitude eE (where e is the charge of an electron) on the ions
- The ions strike detectors at B.

Because the carbon-13 isotope has a different mass to carbon 12, in a properly designed spectrometer the two isotopes will strike the detector at different positions. The goal of this is to determine whether the design shown here is feasible.

10.1 The motion of an ion can be described by its (x,y) coordinates. Write down the acceleration vector of an ion in terms of these variables.

10.2 Draw a free body diagram showing the forces acting on the ion. Gravity can be neglected.

10.3 Write down Newton's law of motion for the ion

10.4 Hence, calculate formulas for x and y as a function of time.

10.5 Finally, find a formula for the vertical deflection h of an ion when it reaches the detectors, in terms of E, Φ and L. Will the design work?