Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 4: Work and Energy Methods
Due Friday March 5th

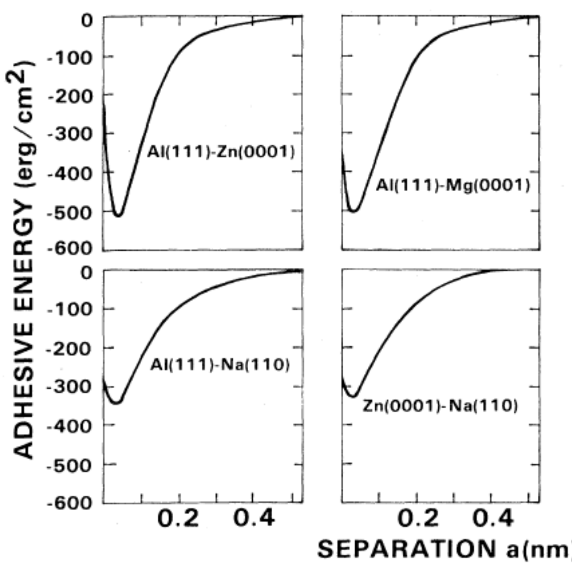
1. Do a simple experiment to measure the force-v-extension curve for a rubber band. Describe your experimental method briefly, and use MATLAB to plot a graph showing the force-v-extension curve. Calculate the maximum energy that can be stored in the elastic band before it breaks. (you can use the MATLAB 'trapz' function to integrate your force-extension curve).

OPTIONAL: Not for credit. Hence, estimate the maximum distance you can fire the elastic band when standing on level ground. Verify your prediction experimentally.

2. The work per unit area required to separate two atomic planes in a crystal by a distance *x* can be approximated by the 'Universal Binding Energy Relation' (Rose, Ferrante and Smith, Phys Rev Lett, **47** (1981)), given by

$$E(x) = E_0 - E_0\left(1+\frac{x}{d}\right)\exp\left(-\frac{x}{d}\right)$$

where E_0 is the total work of separation for the interface, and *d* is a characteristic length.

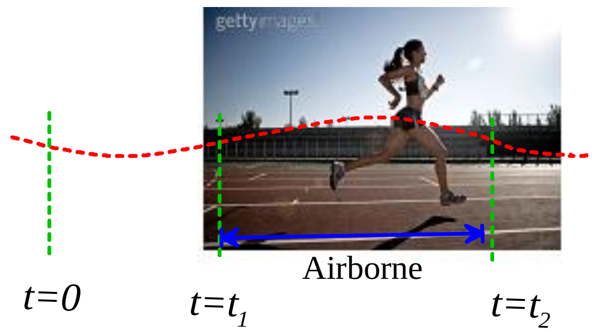


2.1 Find an expression for the force of attraction per unit area between the two crystallographic planes

2.2 Find an expression for the maximum force of attraction per unit area, in terms of E_0 and *d*.

2.3 Hence, estimate the strength of interplanar bond between Aluminum and Zinc, using the data from Rose, Ferrante and Smith (1981) shown in the figure. Express your answer in MNm^{-2} . (Note that Rose *et al* did not use our convention to define zero energy and zero distance. In our expression, the energy is zero for *x=0*, and increases to a value E_0 for large *x*. In the data shown in the figure, the energy is zero for large *x*, and has minimum energy $-E_0$. Their zero separation is arbitrary – the position of the minimum energy corresponds to *x=0* in our formula)

3. The goal of this problem is to estimate the power expended by a running athlete. The figure shows the trajectory of the center of mass of a runner. It consists of two portions: from $0 < t < t_1$ the runner's foot is in contact with the ground; while between $t_1 < t < t_2$ the runner is airborne. For simplicity, assume that the horizontal component of the runner's velocity *V* is constant.



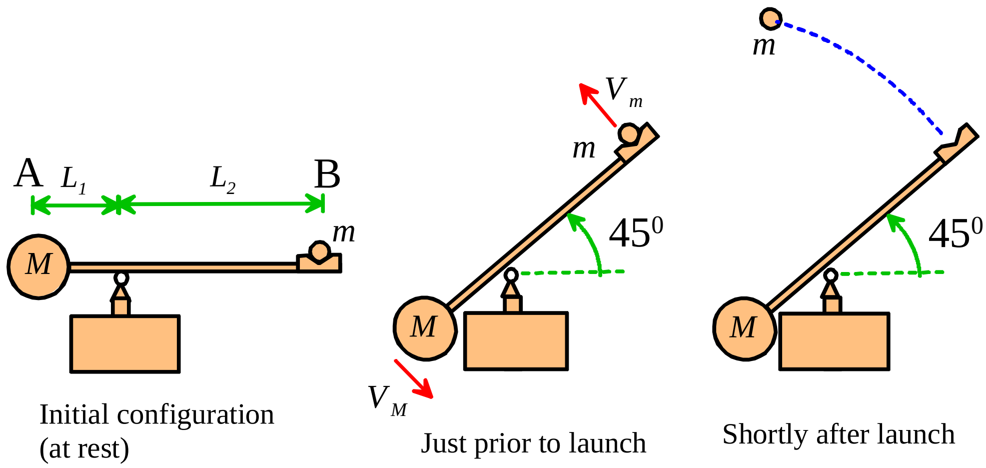
3.1 By considering the motion of the runner while she is airborne, calculate a formula for the vertical component of her velocity at the instant when she leaves the ground, in terms of the gravitational acceleration *g* and t_1 and t_2 (neglect air resistance) and the runner's mass *m*. Hence, write down a formula for the runner's kinetic energy at this instant.

3.2 Write down a formula for the runner's kinetic energy at the instant that her center of mass reaches its minimum height, in terms of her mass *m* and *V*.

3.3 Assume that the additional kinetic energy required for the athlete to become airborne is dissipated as heat during the subsequent impact of her foot with the ground. Hence, calculate a formula for the power lost to impacts, in terms of *g*, t_1 and t_2 and *m*.

3.4 Write down a formula for the power expended by the runner to overcome air resistance (you can neglect the runner's vertical motion in making this estimate).

3.5 In a representative stride, a runner has a foot in contact with the ground for 0.1sec and is airborne for 0.2 sec. (see, e.g. Weyand *et al* 'Ambulatory estimates of maximal aerobic power from foot-ground contact times and heart rates in running humans,' *J. Appl. Physiol*, **91**, 451-458 2001). Taking the drag coefficient $C_D = 0.3$, and air density 1.02 kg/m^3, estimate the total energy that you would expend if you were to complete a 5k in 20 mins.



4. A proposed design for a catapult that launches a projectile with mass *m* is shown in the figure. The mass of the arm AB can be neglected, and the masses *m* and *M* can be idealized as particles.

4.1 Take the potential energy of the initial configuration to be zero. Find an expression for the potential energy of the system at the instant of launch, in terms of m, M , L_1, L_2 and the gravitational acceleration g.

4.2 Write down the total kinetic energy of the system at the instant just prior to launch, in terms of m, M, V_m and V_M

4.3 By noting that the two masses move around circular paths, find a relationship between magnitudes of the velocities V_m and V_M at the instant just prior to launch.

4.4 Show that the projectile velocity at the instant of launch satisfies the equation

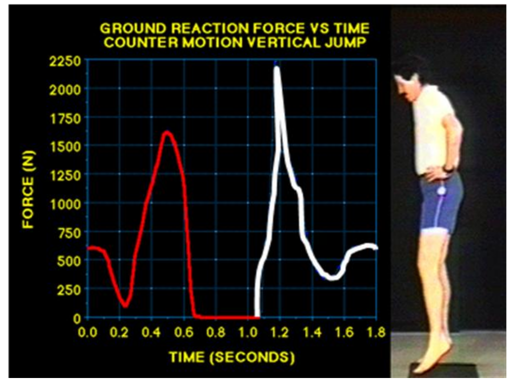
$$V_m^2 = \frac{\sqrt{2}g(ML_1 - mL_2)}{m + M\frac{L_1^2}{L_2^2}}$$

4.5 The solution to 4.4 can be written $V_m^2 = \dfrac{\sqrt{2}gL_1(1-\mu\rho)\rho^2}{1+\mu\rho^2}$ where $\mu = m/M \qquad \rho = L_2/L_1$. Show that the launch velocity is maximized by the value of $\rho = L_2/L_1$ satisfying the equation

$$\mu^2\rho^3 + 3\mu\rho - 2 = 0$$

4.6 Plot a graph showing the optimal value of ρ as a function of μ for some sensible range. (Hint: it is easier to find the value of μ for a given value of ρ than the reverse).

5. The figure shows experimental data for the reaction forces acting on a human test subject during a vertical jump (the red curve corresponds to the jump; the white is the force during the subsequent landing. The person leaves the ground at the point where the red curve drops to zero).



5.1 Assume that at time $t=0$ the subject is just standing on the pressure mat. Use the experimental data to determine the test subject's mass.

5.2 By dividing the force-time curve into a series of straight line segments, calculate (a) the impulse exerted on the person during takeoff by the reaction force at the ground; (b) the impulse exerted on the person by gravity during the period before takeoff, and (c) the total impulse exerted on the person during the period before takeoff.

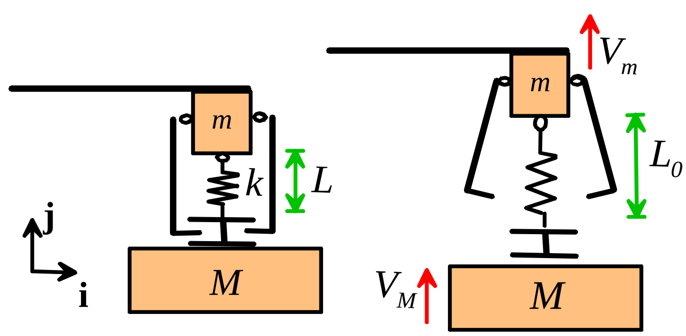
5.3. Using the results of 6.1 and 6.2, calculate the vertical velocity of the test subject at the instant of take-off

5.4 Hence, determine the height of his jump.

5.5 Check your answer by using the time that the jumper is airborne.

6. An 'Impulse Hammer' (see e.g. http://www.dytran.com/img/tech/a11.pdf for a detailed description of one design) is a device that is used to subject a test specimen or structure to a known impulse. The motion of the test structure caused by the impulse can then be used to deduce properties or parameters of interest. For example, impulse response measurements have a number of biomedical applications – see, e.g. Holi and Radhakrishnan, 'In Vivo assessment of osteoporosis in women by impulse response,' TENCON, **4**, 1395, (2003).

The figure shows a proposed design for an impulse hammer. It consists of a mass m attached to a spring with unstretched length L_0 and stiffness k. Before operation, the spring is pre-compressed to a length L. The base AB is then placed in contact with the specimen, which has mass M. The clamps are then released, allowing the spring to extend. The spring exerts equal and opposite impulses on the specimen and the test mass m.



6.1 Write down the total linear momentum of the system before the catch is released.

6.2 Write down the total potential and kinetic energy of the system before the catch is released. Gravity may be neglected.

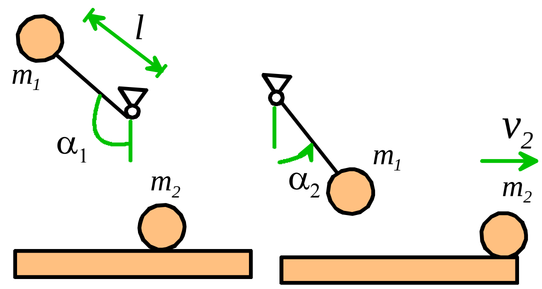
6.3 Let V_m, V_M denote the velocities of the two masses after the spring has fully extended. Write down the total linear momentum of the system at this time.

6.4 Write down the total potential and kinetic energy of the system at the instant that the spring has fully extended.

6.5 Hence, calculate formulas for V_m, V_M in terms of *m*, *M*, *k*, *L* and L_0.

6.6 Finally, find an expression for the impulse exerted on the specimen.

7. The figure shows an experimental apparatus for measuring the restitution coefficient of, e.g. a golf-ball or a bowling ball. It uses the following procedure



- A pendulum (a golf-club head, e.g.) is swung to a known initial angle α_1 and then dropped from rest so as to strike the ball
- The angle of follow-through α_2 of the pendulum is recorded

Your goal is to derive a formula that can be used to determine the restitution coefficient *e* from the measured data.

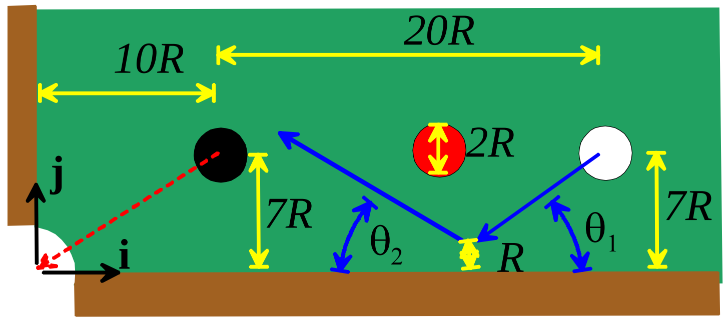
7.1 Using energy conservation, derive an expression for the speed V of the mass on the end of the pendulum just before it strikes the ball, in terms of α_1, *l* and the gravitational acceleration.

7.2 Similarly, derive an expression for the speed v_1 of the mass on the end of the pendulum just after it strikes the ball, in terms of α_2, *l* and the gravitational acceleration.

7.3 Use momentum conservation to calculate an expression for the velocity v_2 of the ball just after it is struck, in terms of *V* and v_1, and any other necessary parameters.

7.4 Hence, deduce a formula for the coefficient of restitution, in terms of α_1, α_2, *l*, and *g*, and any other necessary parameters.

8. **Virtual Snooker**. The figure shows three balls on a snooker table. All balls have radius *R*. The impact of the cue ball (white) with the cushion has a restitution coefficient of 0.8; the impact of two balls has a restitution coefficient of 0.9. Your goal is to calculate an initial velocity for the (white) cue ball that will pot the black ball. Note that you are not allowed to hit the red ball, and so can't make a direct shot – you will have to play the shot off the cushion.



8.1 Write down the coordinates of the cue ball at the instant that it strikes the black ball, assuming that it is on a trajectory to knock the black ball directly into the pocket.

8.2 Consider the rebound of the cue ball off the cushion, as shown in the figure. Show that the angle of incidence is related to the angle of rebound by $\tan\theta_2 = e\tan\theta_1$.

8.3 Hence, calculate the initial direction of motion of the cue ball in order to make the shot (it's sufficient to calculate the value of $\tan\theta_1$)

8.4 **Not for credit** To check your calculation click here to download a MATLAB simulation of the shot. Save the code in a file called *potit.m* and then run it with

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>>potit(Vx,Vy)
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where Vx, Vy are the initial velocities of the cue ball (the magnitude is arbitrary – in fact the code will scale your velocity to have unit magnitude for convenience in doing the animation – so just choose Vx=-1, Vy=$-\tan\theta_1$ to launch the cue ball in the correct direction).