

EN 40: Introduction to Dynamics and Vibrations  
Homework # 8  
Due: Friday, April 30

1. Smoking aircraft

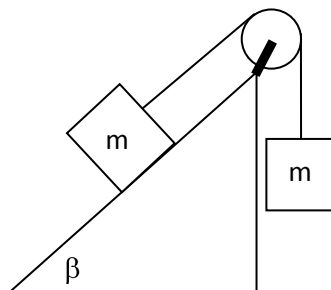


www.boeing.com

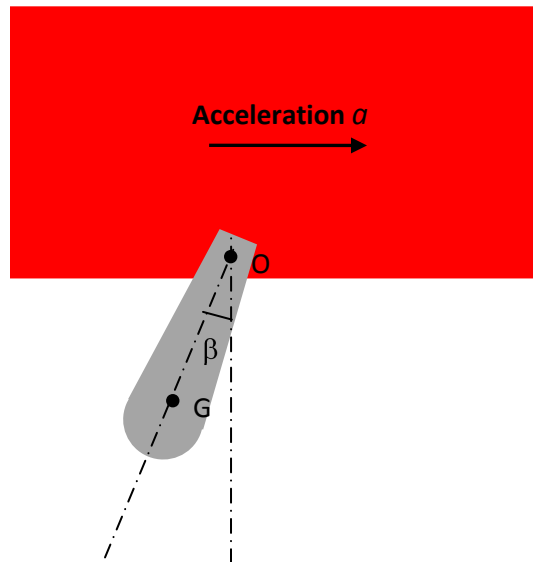
Consider a Boeing 747 landing at a speed of 260 km/hr. You may have seen the plume of smoke that is created when the aircraft lands; let's calculate the energy that goes into creating the smoke. The aircraft mass is 400 tonnes, assume that all 16 of the rear wheels contact the ground simultaneously; assume that the lift on the aircraft when the aircraft just lands is 95% of the aircraft weight. Assume that the coefficient of kinematic friction between the wheel and the runway is 0.5. The mass of a tire is 125 kg and its radius of gyration is 75 cm [moment of inertia = mass \* (radius of gyration)<sup>2</sup>]. Radius of the tire = 1 m.

- (i) Upon landing, how far would the tires skid before they begin to roll without slipping (this is also the length of the skid marks that the aircraft leaves on the runway)?
- (ii) What's the skidding time?
- (iii) What is the deceleration of the aircraft while it's skidding?
- (iv) What is the energy dissipated as heat due to friction?

2. **Pulleys and inclined planes revisited:** Consider the inclined plane problem that we solved in homework # 2. It has two equal masses connected on either side of a pulley; one of the masses rests on an inclined plane (angle of inclination =  $\beta$ ). The moment of inertia of the pulley about its axis of rotation (the center of mass lies on the axis of rotation) is  $I_G$ . Assume that the string is inextensible; it rolls without slipping on the pulley. Friction in the pulley bearing can be neglected and the inclined plane is frictionless. Determine (i) the acceleration of each mass; (ii) the tension in the string on each side; (iii) the angular acceleration of the pulley.



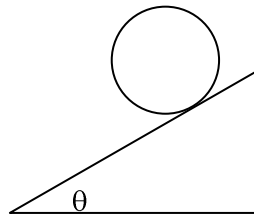
3. You have been charged with the task of determining the acceleration/deceleration of a London city bus in order to determine the energy cost of operating such a bus during a typical day. Having been equipped with EN40 Dynamics, you decide to (or, forced to) hang a rigid body pendulum at a convenient location off the bus and measure the changes in angle  $\beta$  in response to the vehicle acceleration  $a$  (in other words, you just made your own accelerometer). The pendulum oscillates freely about the pivot point O and its center of mass is located at G. The distance between O and G is R. The mass of the pendulum is  $m$  and its moment of inertia about the center of mass axis is  $I_G$ . At time  $t=0$ , the pendulum is at rest in the vertical position ( $\beta = 0$ ,  $\dot{\beta} = 0$ ) and a constant acceleration  $a$  is applied to the vehicle in the direction shown.



- (i) Derive an expression for the angular acceleration  $\ddot{\beta}$  (since  $\beta$  is measured in the clockwise direction from the vertical axis, this is actually the negative of our angular acceleration defined in the class) in terms of  $a$ ,  $g$  (acceleration due to gravity),  $R$ ,  $m$ ,  $I_G$ .
- (ii) What is the maximum value of  $\beta$  if the bus driver decides to do 0 – 100km/hr in 10 s (the bus certainly looks mean enough to do it)?
- (iii) Assume that the lubricant used in the pin joint at O applies a viscous damping moment on the pendulum which resists the motion and it can be expressed as  $M_v = c \dot{\beta}$ , where  $c = 0.1 \text{ kg m}^2/\text{s}$ . Write a MATLAB code to plot the time variations of  $\beta$  & horizontal and vertical reaction forces at the pivot O, during constant acceleration for 10 s. Use the value of acceleration from part (ii). Take  $m = 1\text{kg}$ ,  $I_G = 0.01 \text{ kg m}^2$ ;  $R = 0.2\text{m}$ .
- (iv) What is the value of  $\beta$  that the pendulum settles down to?

4. **Rolling disk:** Consider the solid homogeneous cylindrical disk released from rest at the top of the incline. Its radius is 15 cm, mass = 5 kg, incline angle =  $30^\circ$ . The static and kinetic coefficients of friction are 0.15 and 0.1 respectively.

- (i) What is the acceleration of the center of disk?
- (ii) What is the value of the friction force between the ramp and the cylinder?
- (iii) Repeat the above calculations for an incline angle of  $10^\circ$ .



5. You might remember the video of **Falkirk wheel** that Prof. Bower showed at the beginning of the course. If not, you can view it again at:

<http://www.youtube.com/watch?v=n61KUGDWz2A&feature=related>

The wheel is a lift system to carry boats up or down. During operation, the load on each side has a mass of 300 tonnes.



<http://upload.wikimedia.org/>

- (i) The wheel takes about 5.5 minutes to rotate through  $180^\circ$ . What is the average angular speed?
- (ii) At the beginning of its motion, let's idealize the wheel as shown in the figure below. The center of mass of each load is taken to be 1 m below the center of the corresponding circle. The wheel initially accelerates to the average angular speed calculated in part (i), maintains a constant speed and decelerates at the end. What is the total energy spent by the electric motor that runs the wheel

during the  $180^\circ$  turn of the wheel? Ignore the energy loss in bearing friction (this is probably the bigger quantity; let's ignore it anyway for now); ignore the deceleration phase (the energy of the wheel is dissipated in braking during deceleration); ignore the moment of inertia of the wheel structure itself. Also, assume that the acceleration phase is short enough that the radial positions of the centers of mass G1 and G2 do not change.

(iii) How many units of electrical energy is it equivalent to?

