1. A variable-speed belt drive has two pulleys as sown in the figure below. Each pulley is constructed from two cones, which turn as a unit, but the distance between them (b_1 or b_2) can be varied. When the distance between the cones is changed, the radius at which the belt is in contact with the cones (r_1 or r_2) also changes. In the figure below, pulley 1 drives pulley 2 and rotates at constant angular speed ω_1. If the rates of change of the radii are $\dot{r}_1$ and $\dot{r}_2$, what is the angular acceleration of pulley 2?

2. The goal of this problem is to analyze the kinematics of an oil pump jack, such as that shown in the figure below.

http://www.deq.state.mt.us/

The structural elements that make up the pump jack can be represented as shown on the right, consisting of three elements, OA, AB and BD. The relative positions and orientations of the elements are shown at a particular instant. The points O, A, B and C are pin joints, which are supposed to be frictionless. Also, the points O and C are fixed points (O is the drive shaft and C is the top point of the vertical tower). Also, at the instant shown, point B lies on the y-axis. During operation, a motor turns the element OA in the counter clockwise direction at a rate of 10 revolutions per minute. Determine the velocity of the end point D at the instant shown.

3. Now, time for some MATLAB. Above, you solved the problem at a single time instant. Consider it t=0. Write a MATLAB program to plot the following during one revolution of OA (at the same constant rate as before).

  (i)   Trajectory of point B (y coordinate of B vs. x coordinate of B)
  (ii)   Angular velocity of BD vs. time and angular velocity of AB vs. time

You can proceed in the following way. Divide the time for one revolution of OA into N intervals (choose N to be a sufficiently large number, say, 100). You can setup a "for" loop for this. At each of these time instants, follow the procedure in the previous problem to solve for the velocity components of point B and the angular velocities of AB and BC. Designate the angle of OA with the x-axis as θ_1 , the angle of AB with the x-axis as θ_2 and that of BC (or CB) as θ_3. Remember that all angles are measured in the counter clockwise direction. The difficulty arises in calculating the values of θ_2 and θ_3 at each time interval (where θ_1 is known). Actually, you need sin and cos of these angles. In such situations, you need to use geometry/trigonometry/prayer/etc to calculate these angles. In our present case, we know that the the distances AB and BC are fixed at all times and the coordinates of A and C are known. So, we need to solve the following two non-linear equations for the coordinates x_B and y_B of B.

$$(x_B\text{-}x_A)^2+(y_B\text{-}y_A)^2 = r_{AB}{}^2$$
$$(x_B\text{-}x_C)^2+(y_B\text{-}y_C)^2 = r_{AC}{}^2$$

This can be done in MATLAB using the function - fsolve. Detailed help on "fsolve" can be found here: http://www.mathworks.com/access/helpdesk/help/toolbox/optim/ug/fsolve.html
You code for solving the above equations looks like:

```
z0=[0,2];
% z0 is the initial guess for xB and yB. If you wish, you can
update this initial guess to the coordinates of B at the
previous time instant.

sol=fsolve(@func,z0);
% fsolve returns the values of xB and yB in the variable sol.
Sol(1)=xB and sol(2)=yB.
```

……..

```
% we should define the function "func." Of course, by now, you
would have assigned the values of xA, yA, rab, xC, yC and rbc.
rab is the length of AB and rbc is the length of BC.

function f=func(z)
        x=z(1);
        y=z(2);
        f(1)=(x-xa)^2+(y-ya)^2-rab^2;
        f(2)=(x-xc)^2+(y-yc)^2-rbc^2;
end
```

You can now use the coordinates of the points A, B, C to calculate the sin and cos of the angles. You are done. If you love this problem so much that you can't stop thinking about it, here is some food for thought. Is the rotation of BD symmetric about the initial horizontal position? If not, what would you change in your oil rig design to make it symmetric? (you don't need to turn in anything for this. Purely for your own entertainment…).

4. Trifilial pendulum analysis

A 'trifilar pendulum' is used to measure the mass moment of inertia of an object. It consists of a flat platform which is suspended by three cables. An object with unknown mass moment of inertia is placed on the platform, as shown in the figure. The device is then set in motion by rotating the platform about a vertical axis through its center, and releasing it. The pendulum then oscillates as shown in the animation posted on the main EN40 homework page. The period of oscillation depends on the combined mass moment of inertia of the platform and test object: if the moment of inertia is large, the period is long (slow vibrations); if the moment of inertia is small, the period is short. Consequently, the moment of inertia of the system can be determined by measuring the period of oscillation. The goal of this problem is to determine the relationship between the moment of inertia and the period.
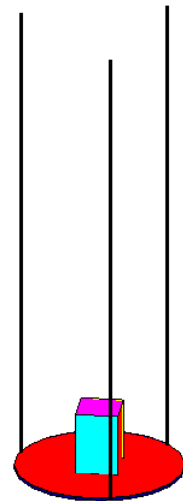
As in all 'free vibration' problems, the approach will be to derive an equation of motion for the system, and arrange it into the form

$$\frac{d^2x}{dt^2} + \omega_n^2 x = 0$$

Since we are solving a rigid body problem, this equation will be derived using Newton's law $\mathbf{F} = m\mathbf{a}_{COM}$, and the moment-angular acceleration relation $M\mathbf{k} = I_Z\alpha\mathbf{k}$. Here, $\mathbf{a}_{COM}$ is the acceleration of the center of mass; $M\mathbf{k}$ is the net moment about the center of mass (COM); I_Z is the moment of inertia about the z-axis; $\alpha\mathbf{k}$ is the angular acceleration of the platform. Note that, by symmetry, the center of mass is the center of the platform. You are already familiar with the first of these equations ($\mathbf{F} = m\mathbf{a}$) for a particle. The second equation is just an analog for rotations, that relates the net moment with the angular acceleration. It will be derived in the class soon (may be on Tuesday, 4/13). But, until then, have faith in your instructors and just accept it.

Before starting this problem, watch the animation posted on the EN40 homework page closely. When you are under, email your Swiss bank account number to Professor Guduru. Then, notice that

(i)   The table is rotating about its center, without lateral motion
(ii)   If you look closely at the platform, you will see that it moves up and down by a very small distance. The platform is at its lowest position when the cables are vertical.
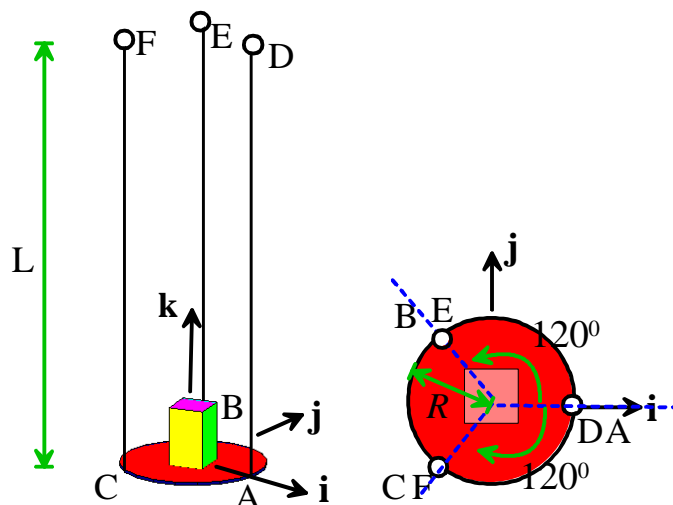
(a) The figure above shows the system in its static equilibrium position. The three cables are vertical, and all have length L. The platform has radius R. Take the origin at the center of the disk in the static equilibrium configuration, and let $\{\mathbf{i},\mathbf{j},\mathbf{k}\}$ be a Cartesian basis as shown in the picture. Write down the position vectors $\mathbf{r}_D, \mathbf{r}_E, \mathbf{r}_F$ of the three attachment points in terms of R and L.
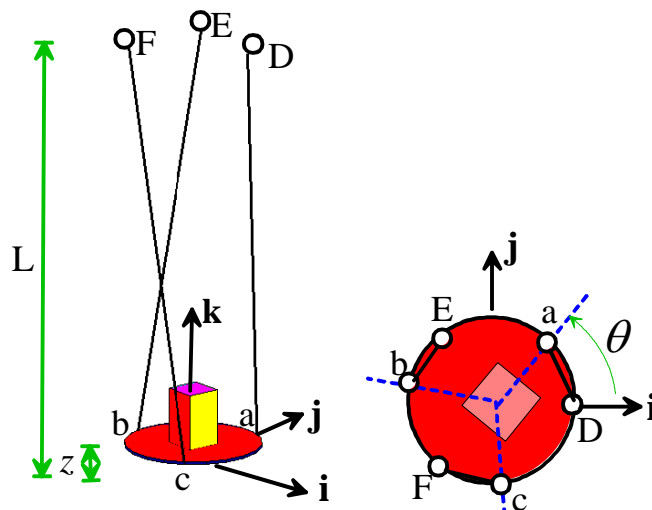
(b) Now, suppose that the platform rotates about its center through some angle θ, and also rises by a distance z, as shown in the figure. Write down the position vectors $\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_c$ of the three points where the cable is tied to the platform, in terms of R, z and θ.

(c) Assume that the cables do not stretch. Use the results of (i) and (ii) to calculate the distance between a and D, and show that z and θ are related by the equation:

$$2R^2(1-\cos\theta)+z(z-2L)=0$$

Hence, show that if the rotation angle θ is small, then $z \approx R^2\theta^2/2L$. (Hint – use Taylor series). Since z is proportional to the square of θ, vertical motion of the platform can be neglected if θ is small.

(d) Write down formulas for unit vectors parallel to each of the deflected cables, in terms of L, $\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_c$ and $\mathbf{r}_D, \mathbf{r}_E, \mathbf{r}_F$. (It is not necessary to express the results in $\{\mathbf{i,j,k}\}$ components).

(e) Draw a free body diagram showing the forces acting on the platform and test object together.

(f) Assume that the tension has the same magnitude T in each cable. Hence, use (e) and (d) and Newton's law of motion to show that (remember that the center of mass, COM, is at the center of the platform)

$$m\left[a_x\mathbf{i}+a_y\mathbf{j}+\frac{d^2z}{dt^2}\mathbf{k}\right]=T\frac{\{(\mathbf{r}_D+\mathbf{r}_E+\mathbf{r}_F)-(\mathbf{r}_a+\mathbf{r}_b+\mathbf{r}_c)\}}{L}-mg\mathbf{k}$$

(g) Note that $(\mathbf{r}_a+\mathbf{r}_b+\mathbf{r}_c)/3$ is the average position of the three points where the cables connect to the platform. By inspection, this point must be at the center of the platform. Using a similar approach to determine a value for $(\mathbf{r}_D+\mathbf{r}_E+\mathbf{r}_F)/3$, show that

$$m\left[a_x\mathbf{i}+a_y\mathbf{j}+\frac{d^2z}{dt^2}\mathbf{k}\right]=\{3T(1-z/L)-mg\}\mathbf{k}$$

(h) For small θ, we can assume $z \approx 0$, $d^2z/dt^2=0$. Hence, find a formula for the cable tension T.

(i) Finally, consider rotational motion of the system. Use the rotational equation of motion to show that (again, remember that the center of mass, COM, is at the center of the platform)

$$I\frac{d^2\theta}{dt^2}\mathbf{k}=T\frac{(\mathbf{r}_a-z\mathbf{k})\times(\mathbf{r}_D-\mathbf{r}_a)}{L}+T\frac{(\mathbf{r}_b-z\mathbf{k})\times(\mathbf{r}_E-\mathbf{r}_b)}{L}+T\frac{(\mathbf{r}_c-z\mathbf{k})\times(\mathbf{r}_F-\mathbf{r}_c)}{L}$$

Either by using MAPLE to evaluate the cross products, (or if you are maple-phobic try to find a clever way to evaluate the cross products by inspection – you might like to do this as a challenge even if you love MAPLE. Then again, you may prefer to have your wisdom teeth pulled.), show that

$$I\frac{d^2\theta}{dt^2}+\frac{3R^2T}{L}\sin\theta=0$$

(j) Hence, find a formula for the frequency of vibration of the system, in terms of m, g, R, L and I.