School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 1: Introduction to MATLAB
Due 12:00 noon Friday February 4th

- YOUR SOLUTION TO THIS HOMEWORK SHOULD CONSIST OF A COMMENTED MATLAB .m FILE
- THE ASSIGNMENT SHOULD BE SUBMITTED ELECTONICALLY BY EMAILING THE FILE AS AN ATTACHMENT TO Stephanie_gesualdi@brown.edu
- PLEASE PUT en4_hw1 IN THE SUBJECT OF YOUR EMAIL
- PLEASE NAME THE ATTACHED FILE *lastname_firstname.m*

You should make your file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function my_amazing_homework
       Solutions to problems 1-6
       Functions for the differential equations in problems 5&6
end
function rms = compute_rms(vector)
…
end
```

If you get stuck, you might find the solutions to homework 1, 2009 and 2010 helpful.

1. Using a loop, create a vector called x that contains 401 equally spaced points, starting at -6 and ending at +6.

2. Using the solution to problem 1, plot a graph of $y = \sin(8x)\exp(-x^2)$, for -6<*x*<6. You can create the vector y using a loop, or else use the dot notation. See Section 9 of the Matlab tutorial for a discussion of both approaches.

3. By using a loop and a conditional ('if…end') statement to create the arrays *v*,*w*, create a plot of
$$w = \begin{cases} 0.7071 & \sin(v) \geq 0.707 \\ \sin v & -0.707 < \sin(v) < 0.707 \\ -0.7071 & -0.7071 \geq \sin(v) \end{cases}$$
as a function of *v* for $0 \leq v \leq 2\pi$. See Section 13 of the MATLAB tutorial for an example using a conditional statement.

4. Write a MATLAB function that will compute the root mean square value of all the elements in a vector. The root mean square is
$$\rho = \sqrt{\frac{1}{n}\sum_{i=1}^{n} v_i^2} = \sqrt{\frac{v_1^2 + v_2^2 + \dots}{n}}$$

Your function should look something like this:

```
function rms = compute_rms(vector)
% Function to compute the rms of a vector
```

```
        enter your calculation here (Hint: use the function
    length(vector) to compute the number of elements in v )
    end
```

and should appear at the end of your file. Use your function to compute the rms value of the vector *w* calculated in problem 3.

5. The differential equation

$$\frac{dC}{dt} = \frac{\beta (C/K)^n}{\left[1+(C/K)^n\right]} - \alpha C$$

is used in systems biology to calculate the concentration of an auto-repressing transcription factor protein produced by a cell. In this equation, C is the concentration of the protein (which varies with time), K is a constant called the 'activation coefficient' for the reaction; β is a second constant, known as the 'maximal expression rate,' and α is a constant that determines the rate of degredation or dilution of the protein, and n is also a constant.

Following the method in Section 15.1 of the Matlab tutorial, write a function in your M-file that will calculate dC/dt given values of t, C.

Then use the MATLAB ode solver to compute, and plot the variation of C with time (as usual, the MATLAB solver won't give you a function for C, but instead will return two vectors, one containing values of time, and the other containing values of C. You can plot these.) Use the following parameters: $\beta = 0.8$ mol/m^{3}s $\alpha = 0.6$ s^{-1} K=0.25 mol/m^3 n=4.

Plot (on the same graph) the solutions for $C = 0.1$ at time $t=0$ and $C = 0.2$ at time $t=0$, for the interval 0<t<15 sec.

Notice that there are two possible stable steady-state values for the protein concentration, depending on the initial value of *C*. Will this behavior still occur if *n=1?* Why?

6. The differential equations

$$\frac{dx}{dt} = 10(y-x) \qquad \frac{dy}{dt} = x(\rho - z) - y \qquad \frac{dz}{dt} = xy - \frac{8}{3}z$$

describe a 'Lorenz oscillator.' They were originally developed to model the earth's climate, but they have since been found to describe other systems, including lasers 10.1016/0375-9601(75)90353-9. They are famous because they led to the discovery of chaos. The three variables (*x,y,z*) are functions of time. In the case of a laser, *x* represents the amplitude of the electric field intensity; *y* is the polarization; and *z* is the 'inversion,' which quantifies the energy states of electrons in the laser responsible for emitting photons. The parameter ρ is a constant (in the laser application, it is a characteristic dimension).

Write a MATLAB 'equation of motion' function that will compute the value of the vector dwdt =[*dx/dt, dy/dt, dz/dt*] given the value of the vector w = [*x,y,z*] at some time *t*.

Then, use the MATLAB ode solver to plot x,y,z as functions of time for 0<t<20. Try the following cases:
6.1 $\rho = 99.96$ ($x = -0.2706$ $y = -3.3019$ $z = 70.3189$) at $t = 0$ (this gives a periodic solution)
6.2 $\rho = 28$ ($x = -0.2706$ $y = -3.3019$ $z = 70.3189$) at $t = 0$ (this gives chaotic behavior)
Plot your solutions on separate graphs. You might also like to plot *trajectories* of the solution (a 3D plot of (*x,y,z)* as a path). The trajectory for 6.2 shows the 'Strange Attractor' for the Lorenz equations.