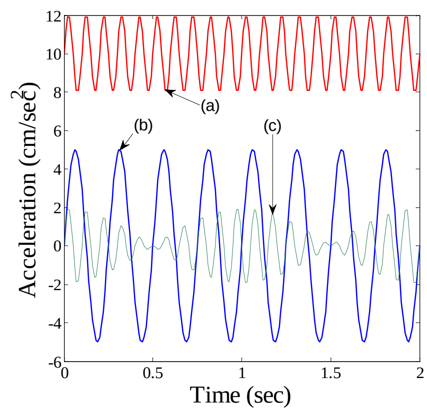
EN40: Dynamics and Vibrations

Homework 2: Dynamics of Particles
Due Friday Feb 11, 2011

School of Engineering
Brown University

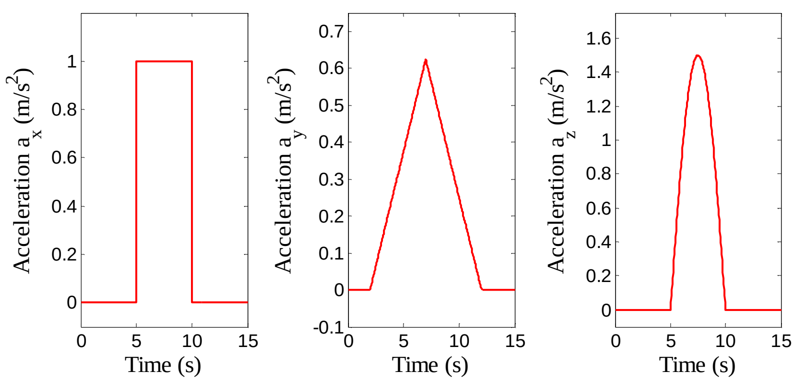
1. The figure shows three measurements from an accelerometer.



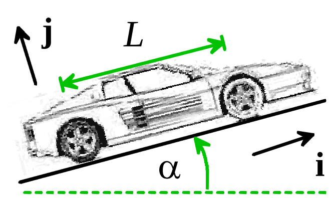
1.1 Which signal can be approximated as simple harmonic motion, and why?

1.2 Determine the amplitude of the velocity and displacement for the signal(s) which can be approximated as simple harmonic motion.

2. A three-axis accelerometer mounted on an inertial platform with fixed orientation measures three components of acceleration shown in the figure. The vertical acceleration component a_z can be approximated as $a_z = A\sin(2\pi(t-t_0)/T)$ while it is nonzero, with an appropriate choice of A, t_0 and T. If the platform is initially at rest, what is its velocity and speed after 15 sec?



3. The figure shows a vehicle climbing a slope of angle α. The interface between tyres and road has friction coefficient μ. The center of mass of the vehicle is midway between the two wheels, and a distance h above the ground.

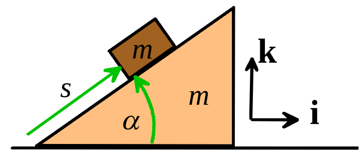


3.1 Draw a free body diagram showing the forces acting on the car. Assume that the vehicle has front wheel drive, and that the rear wheel rolls freely.

3.2 Suppose that the driver applies the maximum possible power so that the driven wheels slip on the road surface. Calculate the acceleration of the vehicle.

3.3 Repeat 3.1 and 3.2 for a rear-wheel drive vehicle. Which gives a greater acceleration (front or rear wheel drive)?

4. The figure shows a block of mass m on the surface of a wedge with identical mass. The interface between wedge and block has friction coefficient $\mu = \beta \tan\alpha$, where $0 < \beta < 1$. The wedge is supported by a frictionless horizontal surface. At time $t=0$ the system is at rest, with the block a distance $s=d$ from the base of the wedge. Find an expression for the time required for the block to reach the base of the wedge. Your solution should include the following steps



4.1 Let a_x denote the horizontal acceleration of the wedge, and let s denote the distance of the block from the base at some arbitrary time t. Write down the acceleration vectors for the wedge and block in terms of these variables (and their time derivatives)

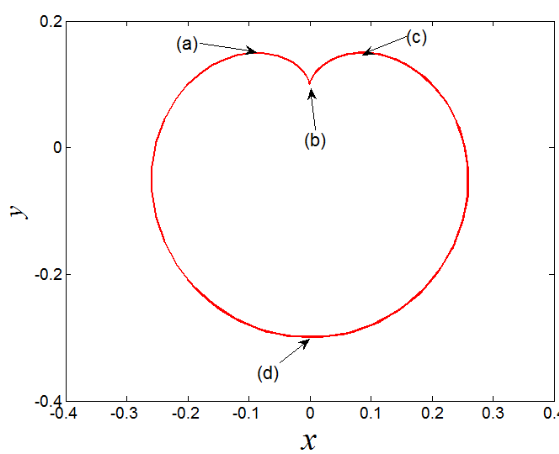
4.2 Draw free body diagrams for both the block and the wedge

4.3 Write down Newton's laws of motion for the block and the wedge

4.4 Hence, calculate a formula for $d^2 s / dt^2$, and hence find a formula for s as a function of time. The result will give the time.

5. The figure shows the trajectory of a charged particle in a '[Penning Trap]', for a particular choice of the electric and magnetic fields that trap the particle. The particle remains in the (x,y) plane at all times, and move from a to b to c to d…

- At point (a), the particle's speed is decreasing
- At point (c), the particle's speed is increasing
- At point (d), the particle's speed is a maximum

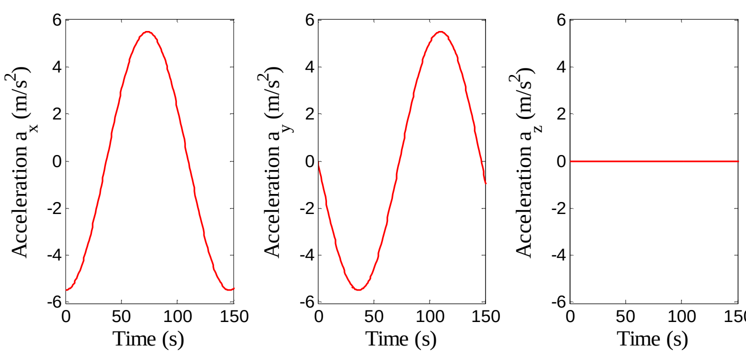


1.1 Draw arrows on a copy of the figure at points (a), (c), and (d) to show the approximate direction of the particle's acceleration vector.

1.2 What is the particle's speed at point (b)?

1.3 How would the acceleration change if the direction of motion of the particle was reversed?

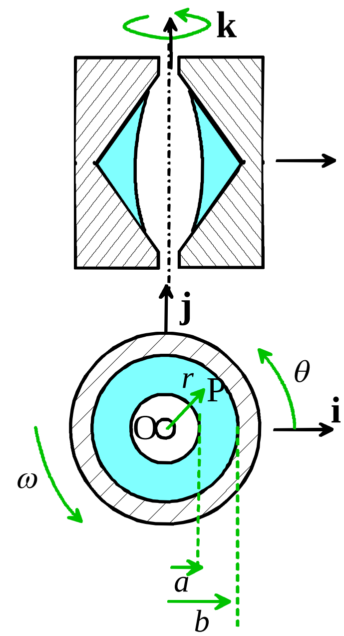
6. An aircraft flies at constant altitude around a circular path. Its navigation system includes an inertial platform (a device which maintains a fixed orientation with respect to space, regardless of the motion of the aircraft). The accelerations measured by three mutually perpendicular accelerometers mounted on this platform are shown in the figure.



6.1 Let $\mathbf{r}=R\mathbf{i}$ and $\mathbf{v}=V\mathbf{j}$ denote the position and velocity vectors of the aircraft at time $t=0$. Find formulas for the position, velocity and acceleration vectors of the aircraft in terms of R, V and t.

6.2 Hence, use the measurements to determine the radius of the aircraft's path, and the aircraft's speed.

7. The figure shows a micro-centrifuge (see Marziali *et al* PNAS, 1999 for a detailed description). It is used to extract particles from a fluid suspension.



7.1 Consider a point in the fluid that is a (constant) distance r from the axis of the centrifuge, and subtends an angle $\theta(t)$ to the fixed **i** direction as shown in the figure. The point spins with the fluid at constant angular speed $d\theta / dt = \omega$. Write down the position vector of the point, and hence determine its velocity and acceleration vector, expressing your answer as components in the fixed {**i,j,k**} basis.

7.2 Write down the components of a unit vector parallel to the (rotating) line OP. Hence, find an expression for the component of acceleration parallel to OP.

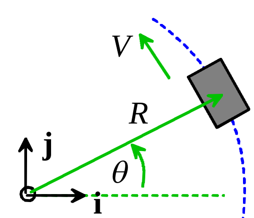
7.3 The radial velocity of a particle suspended in the fluid is usually approximated as

$$v_r = -S a_r$$

where a_r is the radial acceleration of the fluid (note – not the particle!), and S is a constant (with units of time) called the 'Sedimentation coefficient.' Find an expression for the time for a particle to sediment from position $r=a$ at the surface of the fluid to $r=b$ at the circumference of the centrifuge.

7.4 E Coli has a sedimentation coefficient of 70×10^{-13} s, the centrifuge spins at 100000rpm, and the fluid radii are $a = 0.1$ inches and $b = 0.29$ inches. Determine the time required to sediment E coli from suspension.

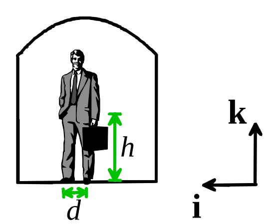
8. An airport 'people mover' travels at constant speed V around a circular path with radius R. The figure shows a plan view of the path.



8.1 Write down the position vector of the vehicle in terms of R and the angle θ shown in the figure.

8.2 Hence, calculate formulae for the velocity and acceleration vectors for the vehicle, in terms of R, V, and θ, expressing your answer as components in the basis shown.

8.3 The figure shows a passenger inside the car, at the instant when $\theta = 0$. His center of mass is a height h above the floor, and he stands with feet a distance d apart, facing in the direction of motion of the vehicle. There is sufficient friction between the floor and his feet to prevent slip. Draw a free body diagram showing the forces acting on the passenger.



8.4 By considering the motion of the passenger at the instant when $\theta = 0$, determine formulae for the reaction forces exerted on the passenger by the floor of the vehicle, in terms of m, g, V, R, d and h. Not all the forces can be determined uniquely.

8.5 Finally, calculate an expression for the minimum allowable radius of the path for the passenger to remain standing, in terms of V, g, h and d.