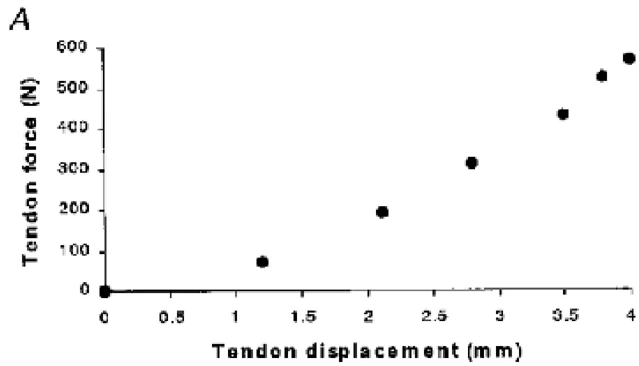
School of Engineering
Brown University

EN40: Dynamics and Vibrations

Homework 4: Work, Energy and Linear Momentum
Due Friday March 4th

1. The figure shows force-extension data for a human ankle tendon, measured in-vivo (from J Physiol. 1999 ; 521(Pt 1): 307–313.)

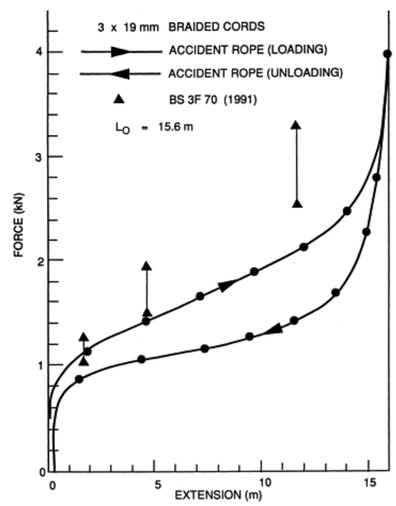
Estimate the total energy stored in the tendon when extended by 4mm. Compare the stored energy to the typical kinetic energy of a runner.



2. The figure shows a force-extension curve for a bungee rope (from J. Engineering Failure Analysis, **11**, 857 (2004)).

2.1 Estimate the energy required to stretch the cable to a tension of 2kN (use the loading curve)

2.2 Hence, estimate the maximum allowable weight of the jumper if the maximum force in the cable is not to exceed 2kN. Note that the un-stretched cable length is 15.6m (see fig).



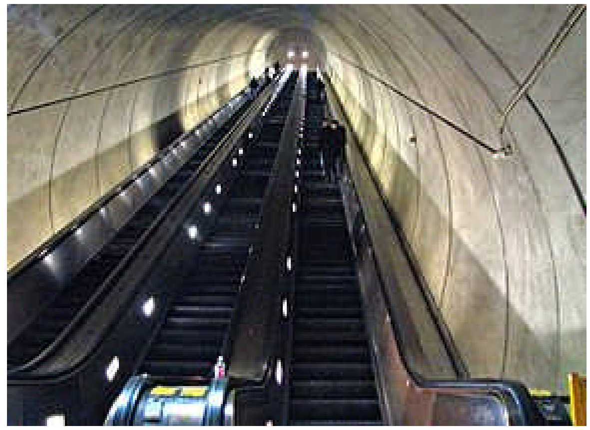
3. One procedure for measuring human power output is to ask the test subject to jump repeatedly for a set period of time (say T=60 sec), count the number of jumps, and also to measure the average time t that the subject is airborne during each jump (see, e.g. Eur J Appl Physiol Occup Physiol. 1983;50(2):273-82).

3.1 Use impulse-momentum to calculate the vertical speed of the subject at the instant he or she leaves the ground, in terms of the flight time t and the gravitational acceleration g.

3.2 Hence, calculate the subject's kinetic energy at the instant of take-off, in terms of g, t and the subject's mass m.

3.3 Assume that the body must supply this kinetic energy for each jump. Calculate the average power developed, in terms of the test time T, the number of jumps n, as well as g,t,m.

4. The longest single-span escalator in the Western hemisphere is located at the Washington Metro station in Montgomery County, Maryland. Some technical specifications for the escalator span can be found on Wikipedia (we have not checked this data!). Additional information concerning escalator standards can be found here.

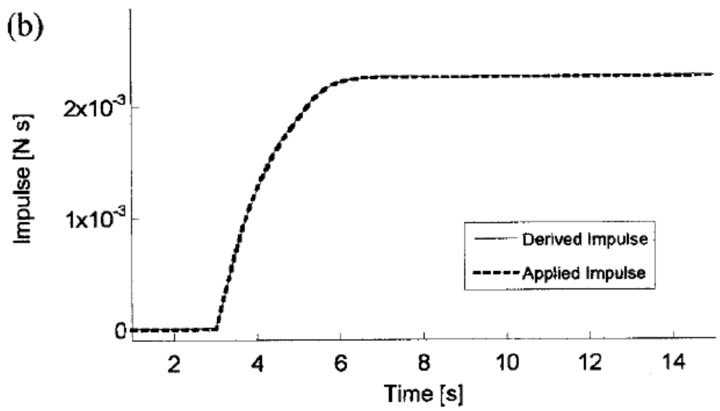


4.1 Use the Wikipedia data to calculate the kinetic energy of a single 80kg rider standing on the escalator

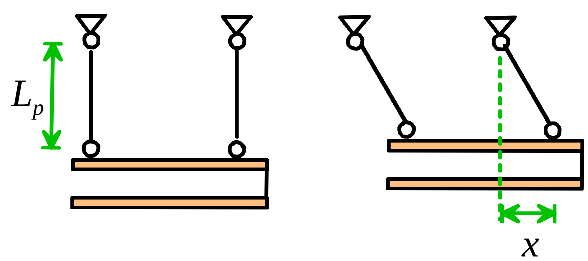
4.2 Calculate the change in potential energy of a single 80kg rider who travels the entire length of the escalator span.

4.3 Assuming the escalator operates at its theoretical capacity of 9000 passengers per hour, estimate the power required to operate the escalator.

5. The figure shows an experimental measurement of the impulse exerted by an electrostatic thruster, as a function of time (from Rev. Sci Instr, **76**, 015105 (2005)). Estimate the maximum force exerted by the thruster.



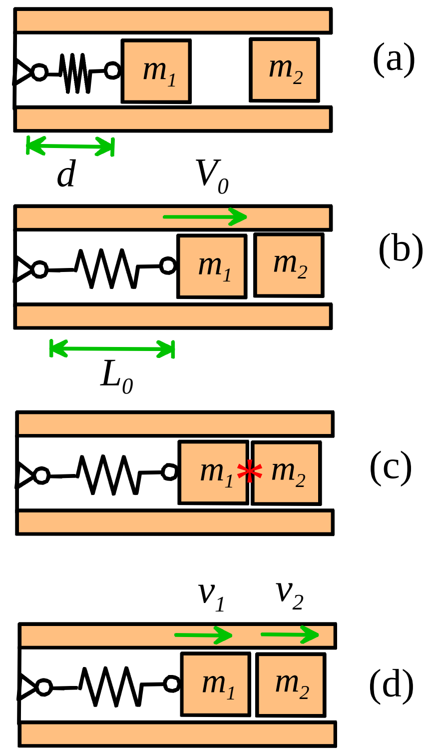
6. The figure shows an experiment used by researchers at Caltech to measure the impulse caused by detonating a volume of combustible gas in a tube. The tube is mounted on a pendulum, which is at rest before the experiment starts. The gas mixture is then ignited, and the horizontal deflection x at the instant when the pendulum just comes to rest is measured. Their paper states that "From elementary mechanics, the impulse is given by



$$I = m\sqrt{2gL_p\left(1-\sqrt{1-\left(\frac{x}{L_p}\right)^2}\right)}$$

Derive this equation (assume that the duration of the impulse is vanishingly small).

7. The figure shows a proposed design for a spring-loaded catapult. It operates as follows: (a) The spring is compressed to a length d and then released from rest; (b) the spring returns to its unstretched length, accelerating mass m_1 to a speed V_0; (c) immediately after this point masses m_1 and m_2 collide; (d) causing mass m_2 to be expelled from the muzzle with speed v_2



The spring has stiffness k and un-stretched length L_0, and the collision between the two masses can be characterized by a restitution coefficient e.

7.1 Write down the potential energy of the system in state (a).

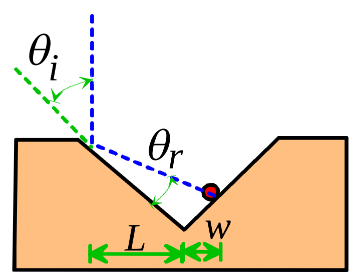
7.2 Hence, calculate a formula for the speed of mass m_1 just before impact (b), in terms of k, L_0, m_1 and d.

7.3 Deduce expressions for the speeds v_1 and v_2 of the two masses just after the collision (d), in terms of k, L_0, m_1, m_2, e and d.

7.4 Show that the speed of mass m_2 is optimized if $m_1 = m_2$

7.5 Finally, compute a formula for the energy efficiency of the optimal design.

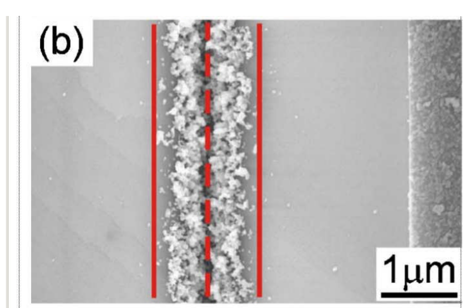
8. The figure shows an experiment designed to measure the restitution coefficient of nanoparticles incident on a surface (Ayesh et al, Physical Review B **81** 195422 (2010)). Bismuth nanoparticles are fired onto a V groove in a silicon wafer. The particles bounce once on impact with the sides of the groove, and then stick to the Si surface on the opposite side of the groove.



8.1 Let θ_i denote the groove angle (which is equal to the angle of incidence) and θ_r denote the angle of reflection. Neglecting friction, show that the restitution coefficient is related to θ_i and θ_r by

$$e = \frac{\tan\theta_r}{\tan\theta_i}$$

8.2 The figure shows an experimental result. The flat surface is the (100) plane of an Si wafer, while the groove exposes the (111) planes – this means the angle of incidence is $\theta_i = 35^0$. The dashed red line shows the center of the groove; the right hand edge can be seen as the boundary between the light and dark gray regions. Use the figure to estimate the coefficient of restitution for the particles on Si. **Neglect gravity**



9. Two balls with identical mass collide on a pool table as shown in the picture. The black ball is at rest before the collision, and the collision is frictionless with restitution coefficient $e=1$. Calculate the angle θ_2 between the paths followed by the two balls after the collision (idealize the balls as particles) (*note – in general the angle depends on the direction of motion of A before collision, but e=1 is a special case.)*