

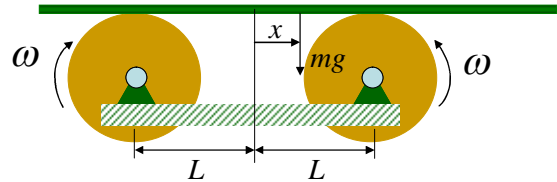


School of engineering  
Brown University

## EN 4 Dynamics and Vibrations

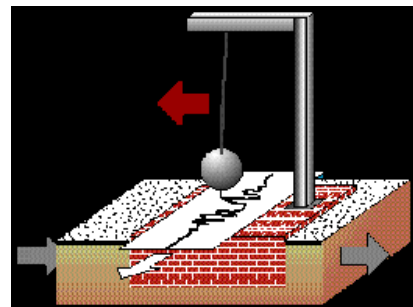
### Homework 6: Forced Vibrations

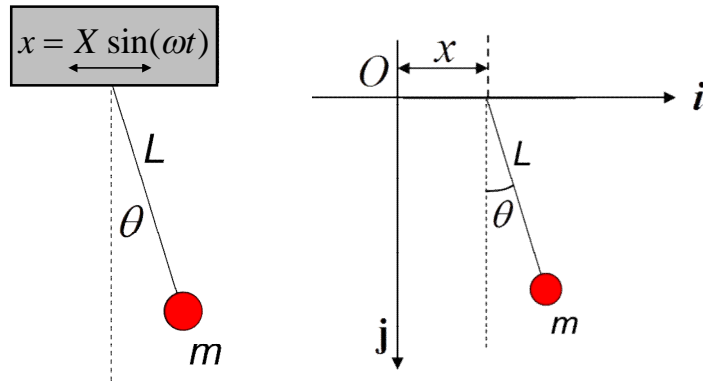
- When a homogenous bar of mass  $m$  is accidentally placed on two rotating drums, as shown in the figure, it is found that the bar performs periodic oscillation. As an engineer, you have been asked to analyze this phenomenon.



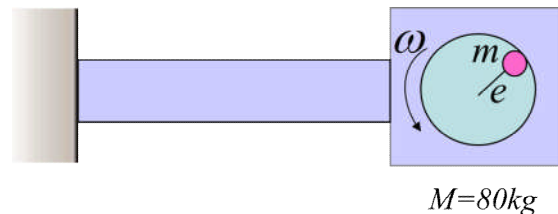
Neglect the thickness of the bar in your analysis. The interface between the bar and the rotating drums has friction coefficient  $f$ .

- 1.1 Draw a free body diagram showing all the forces on the bar as its center of mass is shifted by a distance  $x$  toward one of the rotating drums.
  - 1.2 Determine the normal and tangential forces acting on the bar as a function of  $x$ .
  - 1.3 Derive the governing equation for the horizontal motion of the bar. Determine the natural frequency of the oscillation.
  - 1.4 Suppose that, at  $t=0$ , the bar is placed on the drums with its center of mass at  $x=0.2$  m. The length of the bar is  $L=1$  m and the friction coefficient is  $f=0.3$ . Calculate the position and velocity of the bar at  $t=1$  s.
2. A quick way to estimate the damping coefficient of a machine part is to apply a harmonic force with a tunable frequency that can induce the machine part to vibrate in resonance. An inspector applied a harmonic force of 2.4 N to an airplane wing and found that the wing vibrates with amplitude of 12 cm and a period of 0.1 s at resonance. Estimate the damping coefficient of the wing.
3. A simple seismograph consists of a pendulum with  $m=500$  g and  $L=20$ cm attached to a rigid frame, as shown in the figure. Assume that the base is excited by a simple harmonic motion  $x = X \sin \omega t$  with  $X=1$ cm.



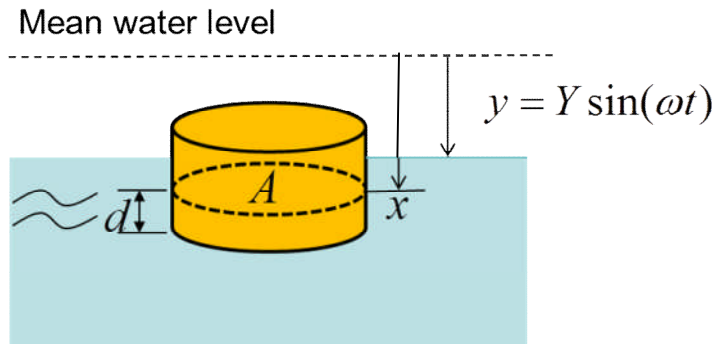


- 3.1 Given the coordinates shown above, write down the position, velocity and acceleration vectors of the mass  $m$ .
  - 3.2 Write down the equation of motion of  $m$ . Do not linearize with respect to  $\theta$ , except to show that the natural frequency of the pendulum is  $\omega_n = \sqrt{g/L}$  in the case of small  $\theta$ .
  - 3.3 Write a MATLAB code to calculate the motion of the pendulum for  $\omega = 0.1\omega_n, \omega_n, 10\omega_n$  for a time period of  $0 < t < 50$ s. Assume the pendulum is at rest at  $t=0$ .
4. The figure shows a tail rotor of a helicopter used to provide yaw control and torque balance. During a flight, a bird of mass  $m=500$  g accidentally hit the rotor blade and got stuck on one of the blades at a distance of 15cm from the axis of rotation. This problem can be modeled as an unbalanced rotor with stiffness  $k = 1 \times 10^5$  N/m (provided by the tail section) and an equivalent mass of  $M=80$  kg (including the bird mass).

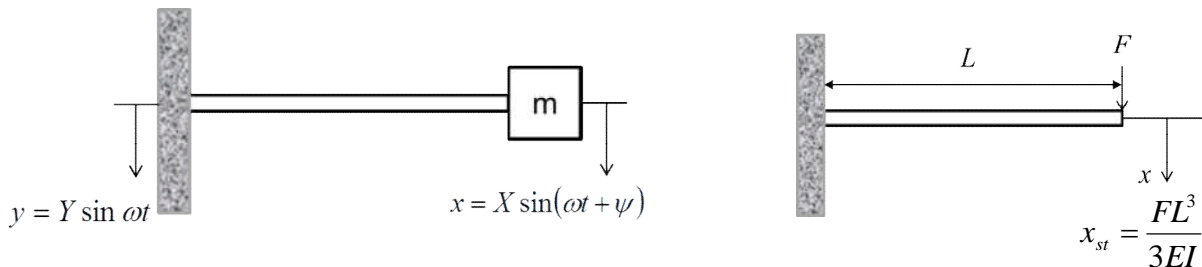


- 4.1 Calculate the magnitude of the vibration of the tail section of helicopter as the tail rotor rotates at 1800 rpm. Assume damping factor  $\zeta = 0.01$ .
  - 4.2 Calculate the vibration amplitude at the resonance frequency.
5. The oscillation of a ship on rough sea is modeled as a floating cylinder on water, with the water level itself also rising and falling in simple harmonic motion  $y = Y \sin \omega t$ . Assume the friction between the ship and water can be represented as a damper with damping coefficient  $c=0.4$ . The cross-section area

of the ship is  $200 \text{ m}^2$ , the mass of the ship  $m = 2 \times 10^5 \text{ kg}$ , and the density of the water is  $\rho_{\text{water}} = 1000 \text{ kg/m}^3$ .  $d$  is the draft of this ship in still water.



- 5.1 Draw a free body diagram showing all the forces on the ship.
  - 5.2 Write down the equation of motion for the ship and determine the amplitude of vibration.
  - 5.3 Suppose the ship is designed to sustain an acceleration of  $0.3g$  without running the risk of capsizing and the water level is oscillating with a period of 6s. Determine the maximum amplitude of water level oscillation that the ship can sustain.
  - 5.4 During a storm, the water level changes with amplitude  $Y = 2\text{m}$  and  $\omega = 2 \text{ rad/s}$ , determine the vibrating amplitude of the ship.
6. The figure shows a mass  $m=20\text{g}$  attached to the end of a cantilever (whose own weight is negligible compared to  $m$ ). The base of the cantilever is subject to simple harmonic motion  $y = Y \sin \omega t$  and the mass is found to vibrate as  $x = X \sin(\omega t + \psi)$ . It is known that, under static loading, the deflection of the cantilever  $x$  can be related to an applied force  $F$  as  $x_{st} = \frac{FL^3}{3EI}$  where  $L=10 \text{ cm}$  is the length of the cantilever and  $EI$  is the bending stiffness of the beam.

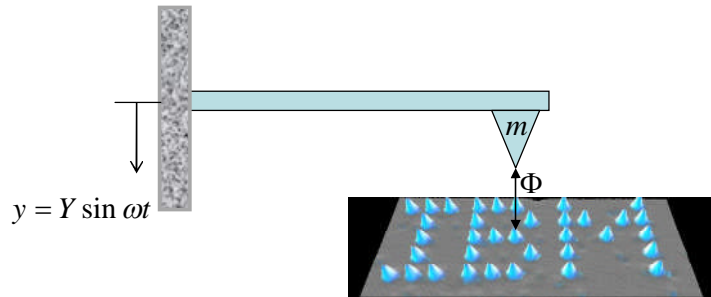


- 6.1 At resonance, it is found that the mass is vibrating at amplitude  $X = 2Y$  and period 0.2 s. Determine the damping coefficient and bending stiffness of the cantilever.

6.2 Plot  $X/Y$  as a function of the base excitation frequency  $\omega$  and determine the range of  $\omega$  for which  $X/Y \leq 1$ .

6.3 Assume the damping factor does not change with the length of the cantilever beam. Plot  $X/Y$  as a function of the beam length  $L$  for  $\omega = 20$  rad/s. Determine the minimum length of the cantilever beam required to isolate vibration such that  $X/Y$  be kept below 0.1.

**7. Optional – For Extra Credit.** The design of an Atomic force microscope (AFM) is based on a cantilever as discussed in Problem 6. In the tapping mode of AFM ([http://en.wikipedia.org/wiki/Atomic\\_force\\_microscopy](http://en.wikipedia.org/wiki/Atomic_force_microscopy)), the cantilever is driven to oscillate up and down at its resonance frequency. A simple cantilever is used to model an AFM, where the mass  $m$  represents the tip which is interacting with the surface atoms of a sample. Neglect the mass of the rest of the system in your calculation. The interactive force between the tip and sample surface obeys the following interatomic potential



$$\Phi = 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right]$$

where  $r$  is the distance between the AFM tip and the surface of the sample. Assume  $\epsilon = 1.6 \times 10^{-19}$  J and  $\sigma = 0.3$  nm.

7.1 Calculate the interaction force  $F = \frac{\partial \Phi}{\partial r}$  and instantaneous stiffness  $k_s = \frac{\partial F}{\partial r}$  between the sample and the cantilever tip. Plot  $F$  and  $k_s$  as a function of  $r$ .

7.2 During an AFM scan, the height of the tip is tuned to an appropriate distance from the sample so that the tip vibrates in resonance. This allows the AFM to detect the surface topography of the sample as its tip scans over the entire sample surface. Write down the equation of motion for the AFM tip corresponding to the figure on the right.

7.3 At resonance, the mass vibrates at frequency  $0.75 \times 10^6$  Hz due to the interaction with the sample. Given that the mass of the tip is  $10^{-11}$  kg, and the stiffness  $k = 200$  N/m, determine the critical height of the tip when resonance occurs.

