EN 40 Homework #7 Solutions

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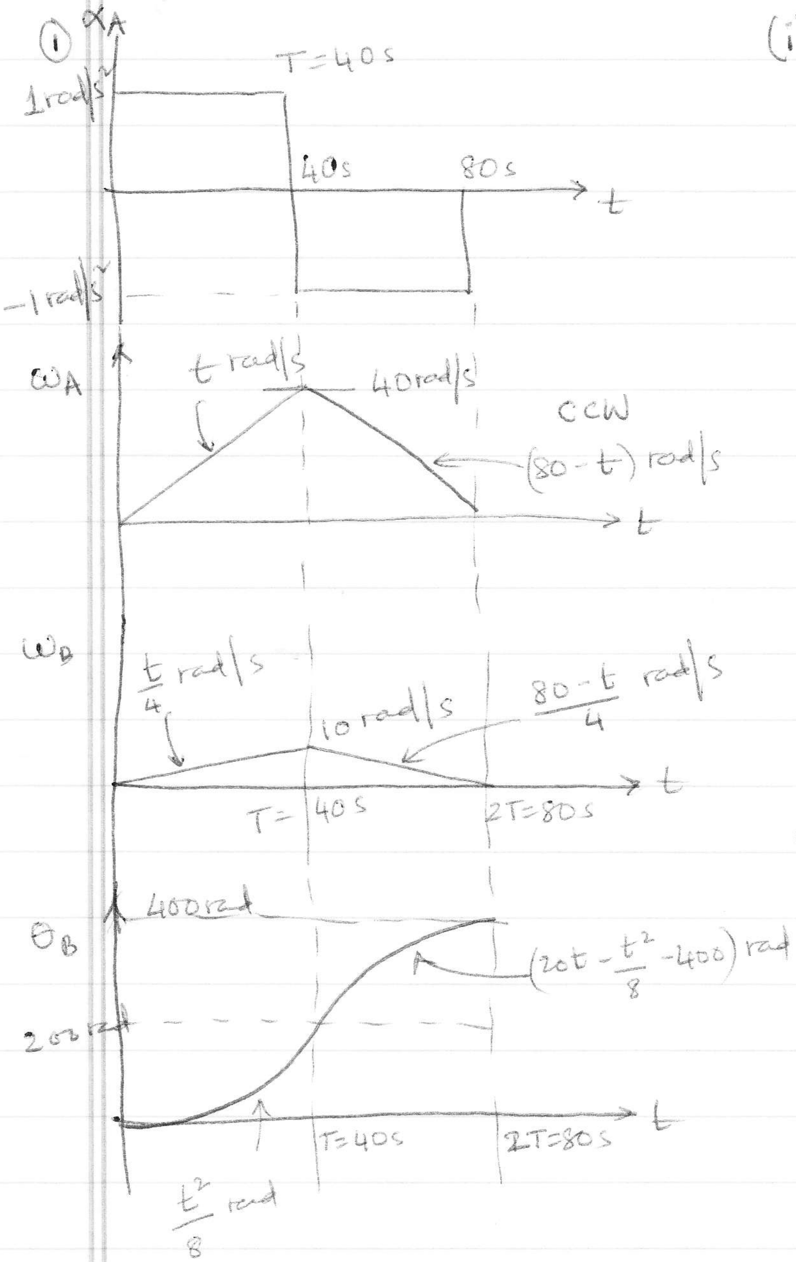

(i) $\frac{d\omega_A}{dt} = \alpha_A \Rightarrow \omega_A = \int_0^t \alpha_A \, dt$

For $0 \le t \le T$

$\omega_A = \alpha_A t = t$ rad/s

@ $t = T = 40$ s, $\omega_A = 40$ rad/s

For $T \le t \le 2T$

$$\omega_A = 40 + \int_{40}^{t} \alpha_A \, dt = 40 - (t - 40) = (80 - t) \text{ rad/s}$$

$$\frac{\omega_B}{\omega_A} = \frac{r_A}{r_B} = \frac{1}{4} \Rightarrow \omega_B = \frac{\omega_A}{4}$$

(clockwise)

(ii) $\frac{d\theta_B}{dt} = \omega_B \Rightarrow \theta_B = \int \omega_B \, dt$

For $0 \le t \le T$

$$\theta_B = \int_0^t \frac{t}{4} \, dt = \frac{t^2}{8} \text{ rad}$$

for $T \le t \le T$

$$\theta_B = \frac{T^2}{8} + \int_T^t \left(20 - \frac{t}{4}\right) dt$$

$$= \frac{T^2}{8} + 20(t - T) - \frac{t^2 - T^2}{8}$$

$$= 200 + 20(t - 40) - \frac{t^2}{8} + 200$$

$$= \left(20t - \frac{t^2}{8} - 400\right) \text{ rad}$$

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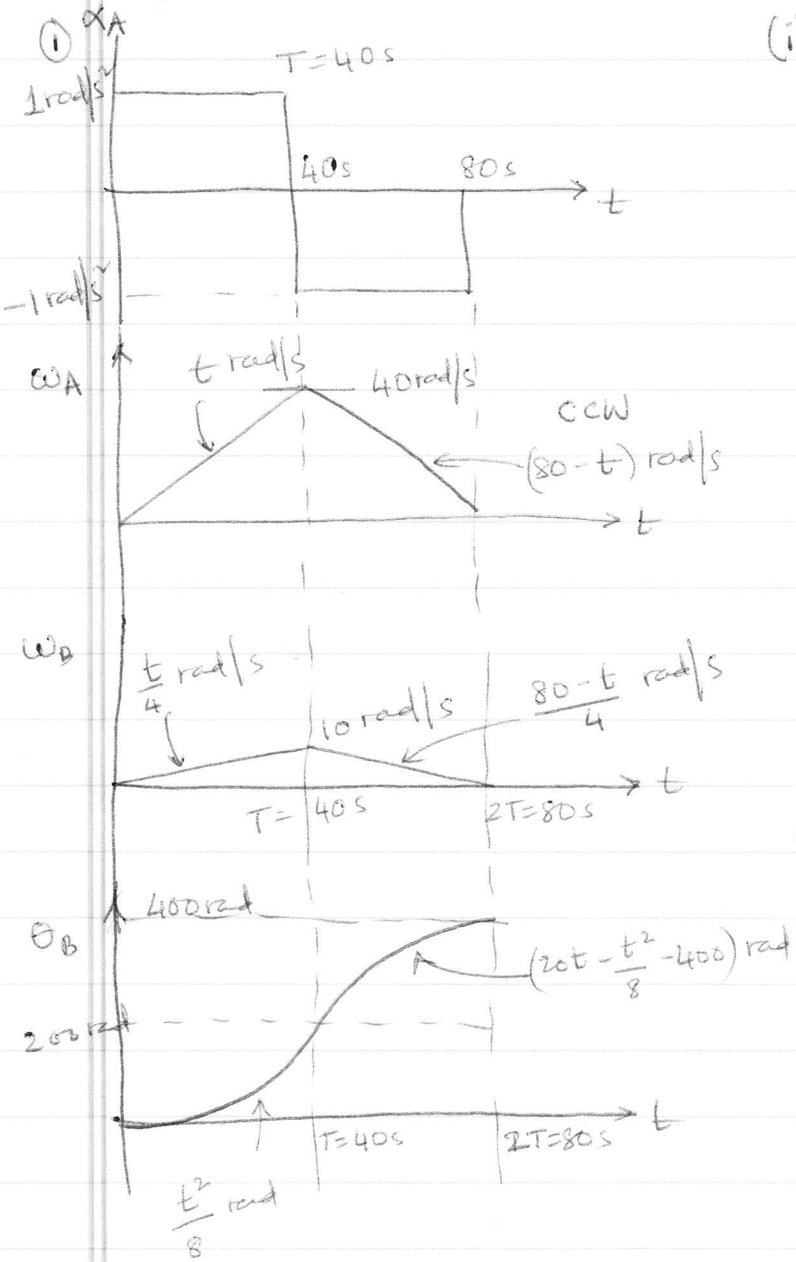


(i) $\frac{d\omega_A}{dt} = \alpha_A \Rightarrow \omega_A = \int_0^t \alpha_A \, dt$

For $0 \le t \le T$

$\omega_A = \alpha_A t = t$ rad/s

@ $t = T = 40$ s, $\omega_A = 40$ rad/s

For $T \le t \le 2T$

$\omega_A = 40 + \int_{40}^{t} \alpha_A \, dt = 40 - (t - 40)$
$= (80 - t)$ rad/s

$\frac{\omega_B}{\omega_A} = \frac{r_A}{r_B} = \frac{1}{4} \Rightarrow \omega_B = \frac{\omega_A}{4}$ (clockwise)

(ii) $\frac{d\theta_B}{dt} = \omega_B \Rightarrow \theta_B = \int \omega_B \, dt$

For $0 \le t \le T$

$\theta_B = \int_0^t \frac{t}{4} \, dt = \frac{t^2}{8}$ rad

for $T \le t \le T$

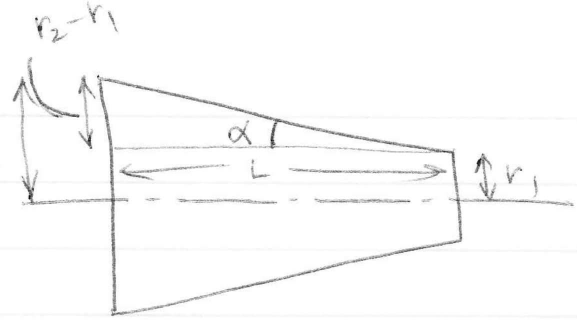
$\theta_B = \frac{T^2}{8} + \int_T^t \left(20 - \frac{t}{4}\right) dt$

$= \frac{T^2}{8} + 20(t - T) - \frac{t^2 - T^2}{8}$

$= 200 + 20(t - 40) - \frac{t^2}{8} + 200$

$= \left(20t - \frac{t^2}{8} - 400\right)$ rad

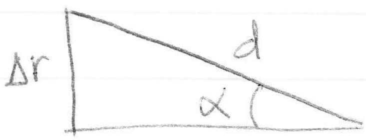
(iii)



$$\tan\alpha = \frac{r_2 - r_1}{L} = \frac{2R}{10R} = \frac{1}{5}.$$

$$\Rightarrow \sin\alpha = \frac{1}{\sqrt{26}}$$

With each revolution, the cable advances by d along the drums surface.



$$\Delta r = d \sin\alpha.$$

The corresponding increase in radius $\Delta r = d \sin\alpha$ per revolution.

$$\Rightarrow \frac{dr}{d\theta_B} = \frac{1}{2\pi} d\sin\alpha \Rightarrow \boxed{r = \frac{d\sin\alpha}{2\pi}\theta_B + r_1}$$

(iv) Speed of the mass $v = \omega_B r$.

for $0 \le t \le T$, $v_1 = \frac{t}{4}\left[\frac{d\sin\alpha}{2\pi}\frac{t^2}{8} + 4R\right]$ m/s

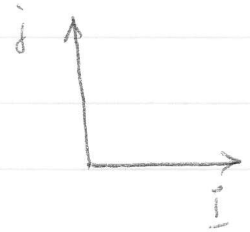
Where $d = 0.01$m, $R = 0.1$m, $\sin\alpha = \frac{1}{\sqrt{26}}$.

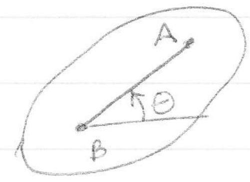
for $T \le t \le 2T$, $v_2 = \left(\frac{80 - t}{4}\right)\left[\frac{d\sin\alpha}{2\pi}\left(20t - \frac{t^2}{8} - 400\right) + 4R\right]$ m/s

(v) $$\delta = \delta_1 + \delta_2 = \int_0^{40s} v_1\,dt + \int_{40s}^{80s} v_2\,dt = 86.24 + 98.3$$

$$= \underline{184.97\text{m}}$$

② The main equations we are going to use are:





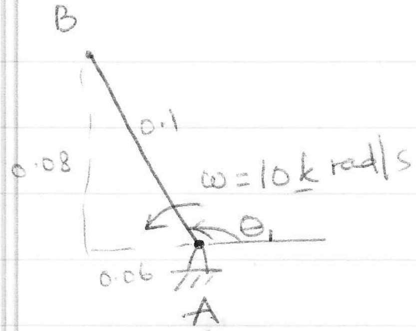
$$\dot{x}_A = \dot{x}_B - r_{AB}\,\dot{\theta}\sin\theta$$

$$\dot{y}_A = \dot{y}_B + r_{AB}\,\dot{\theta}\cos\theta$$

$$\ddot{x}_A = \ddot{x}_B - r_{AB}\,\ddot{\theta}\sin\theta - r_{AB}\,\dot{\theta}^2\cos\theta$$

$$\ddot{y}_A = \ddot{y}_B + r_{AB}\,\ddot{\theta}\cos\theta - r_{AB}\,\dot{\theta}^2\sin\theta$$

(i)



Apply the velocity equation to AB

$$\dot{x}_B = \dot{x}_A - r_{AB}\,\dot{\theta}_1\sin\theta_1$$

$$= 0 - 0.1\times10\times0.8 = -0.8\ \text{m/s}$$

$\sin\theta_1 = 0.8$ | $\dot{x}_A = 0$

$\cos\theta_1 = -0.6$ | $\dot{y}_A = 0$

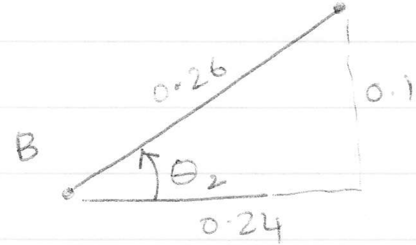
$\dot{\theta}_1 = \omega = 10$ rad/s

$$\dot{y}_B = \dot{y}_A + r_{AB}\,\dot{\theta}_1\cos\theta_1$$

$$= 0 + (0.1)\times 10\times(-0.6) = -0.6\ \text{m/s}$$

$$\Rightarrow \underline{v}_B = -(0.8\,\underline{i} + 0.6\,\underline{j})\ \text{m/s}$$

(ii)

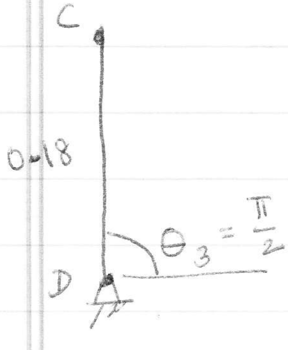


$$\dot{x}_C = \dot{x}_B - r_{CB}\,\dot{\theta}_2\sin\theta_2$$

$$= -0.8 - 0.26\,\dot{\theta}_2\,\frac{0.1}{0.26} = 0.8 - 0.1\,\dot{\theta}_2 \quad (*)$$

$$\dot{y}_C = \dot{y}_B + r_{CB}\,\dot{\theta}_2\cos\theta_2$$

$$= -0.6 + 0.26\,\dot{\theta}_2\,\frac{0.24}{0.26} = -0.6 + 0.24\,\dot{\theta}_2 \quad (**)$$



$$\dot{x}_C = \dot{x}_D - r_{CD}\,\dot{\theta}_3\sin\theta_3$$

$$= 0 - 0.18\,\dot{\theta}_3 = -0.18\,\dot{\theta}_3 \quad\text{——} (+)$$

$$\dot{y}_C = \dot{y}_D + r_{CD}\,\dot{\theta}_3\cos\theta_3 = 0 + r_{CD}\,\dot{\theta}_3\,(0) = 0 \quad (++)$$

From (++) and (* *)

$$-0.6 + 0.24\,\dot{\theta}_2 = 0 \Rightarrow \dot{\theta}_2 = \frac{0.6}{0.24} = 2.5 \text{ rad/s}.$$

$$\Rightarrow \underline{\omega}_{BC} = 2.5\,\underline{k} \text{ rad/s}.$$

From (*) & (+) $\quad -0.8 - 0.1\,\dot{\theta}_2 = -0.18\,\dot{\theta}_3$

$$\Rightarrow \dot{\theta}_3 = \frac{0.1 \times 2.5 + 0.8}{0.18} = 5.83 \text{ rad/s}.$$

$$\Rightarrow \underline{\omega}_{CD} = 5.83\,\underline{k} \text{ rad/s}.$$

$$\dot{x}_c = -0.18\,\dot{\theta}_3 = -0.18 \times 5.83 = -1.05 \text{ m/s}.$$

$$\dot{y}_c = 0.$$

$$\Rightarrow \underline{v}_c = -1.05\,\underline{i} \text{ m/s}$$

(iii) Apply the acceleration equations for AB

$$\ddot{x}_B = \ddot{x}_A - 0.1\,\ddot{\theta}_1 \sin\theta_1 - 0.1\,\dot{\theta}_1^2 \cos\theta_1$$

$$= 0 - 0 - 0.1 \times 100 \times (-0.6) = 6 \text{ m/s}^2.$$

$$\ddot{y}_B = \ddot{y}_A + r_{BA}\,\ddot{\theta}_1 \cos\theta_1 - r_{BA}\,\dot{\theta}_1^2 \sin\theta_1$$

$$= 0 + 0 - 0.1 \times 100 \times 0.8 = -8 \text{ m/s}^2.$$

Now, consider BC

$$\ddot{x}_c = 6 - \cancel{0.26}\,\ddot{\theta}_2 \frac{0.1}{\cancel{0.26}} - \cancel{0.26} \times 2.5^2 \times \frac{0.24}{\cancel{0.26}}$$

$$= (4.5 - 0.1\,\ddot{\theta}_2) \text{ m/s}^2 \quad \text{—— (1)}$$

$$\ddot{y}_c = \cancel{8} -8 + 0.\cancel{26}\,\ddot{\theta}_2 \frac{0.24}{0.\cancel{26}} - 0.\cancel{26}\, 2.5^2 \frac{0.1}{0.\cancel{26}}$$

$$= -8.625 + 0.24\,\ddot{\theta}_2 \quad \text{——— (2)}$$

Now apply for CD

$$\ddot{x}_c = 0 - 0.18\,\ddot{\theta}_3 \quad \text{——— (3)}$$

$$\ddot{y}_c = 0 - 0.18(5.83)^2$$

$$= -6.124 \text{ m/s}^2 \quad \text{——— (4)}$$

From (2) & (4), $\ddot{\theta}_2 = 10.42 \text{ rad/s}^2$

From (1) & (3)

$$4.5 - 0.1 \times 10.42 = -0.18\,\ddot{\theta}_3 \Rightarrow \ddot{\theta}_3 = -19.21 \text{ rad/s}^2$$

$\Rightarrow$ Angular acceleration of BC $= 10.42\,\underline{k}$ rad/s^2

" " CD $= -19.21\,\underline{k}$ rad/s^2