ENGN 40: Introduction to Dynamics and Vibrations
Homework # 8
Due: Friday, April 29, 2011

1. Planetary gears

A Planetary gear system consists of one or more *planet* gears (three, in this problem), revolving about a central, or *sun* gear. The planet gears are connected rigidly to a shaft through a structure consisting of three arms, as shown in the figure below. In addition, the three planet gears mesh with an outer ring gear with teeth on the inner surface, as shown. Advantages of planetary gears over parallel axis gears include high power density, large gear ratio in a small volume, multiple kinematic combinations, pure torsional reactions, and coaxial shafting.

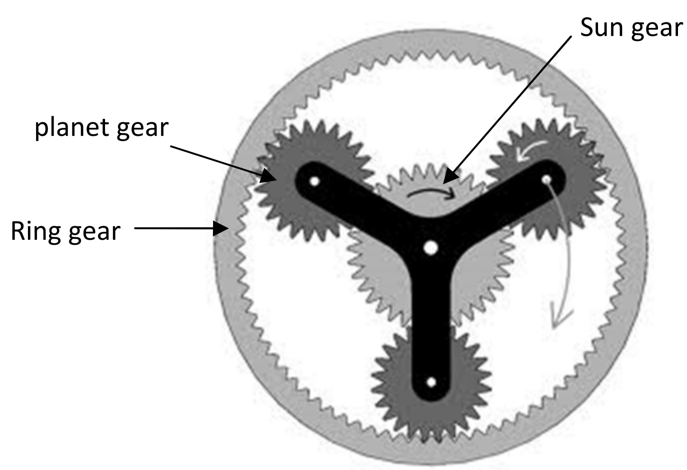
In the figure below, the radius of the sun gear is r_s and that of the planet gear is r_p. This gear system can work in one of three ways. To visualize the operation of the planetary gears, click on the link below.

http://www.youtube.com/watch?v=ECljAo1q1RQ

(i) Suppose that the sun gear is connected to the driver shaft, the planetary gears are connected rigidly to the driven shaft and the ring gear is held fixed. If the driver shaft rotates at an angular speed of ω_1 clockwise, what is the angular speed of the driven shaft? Does the driven shaft rotate clockwise (CW) or counter-clockwise (CCW)?

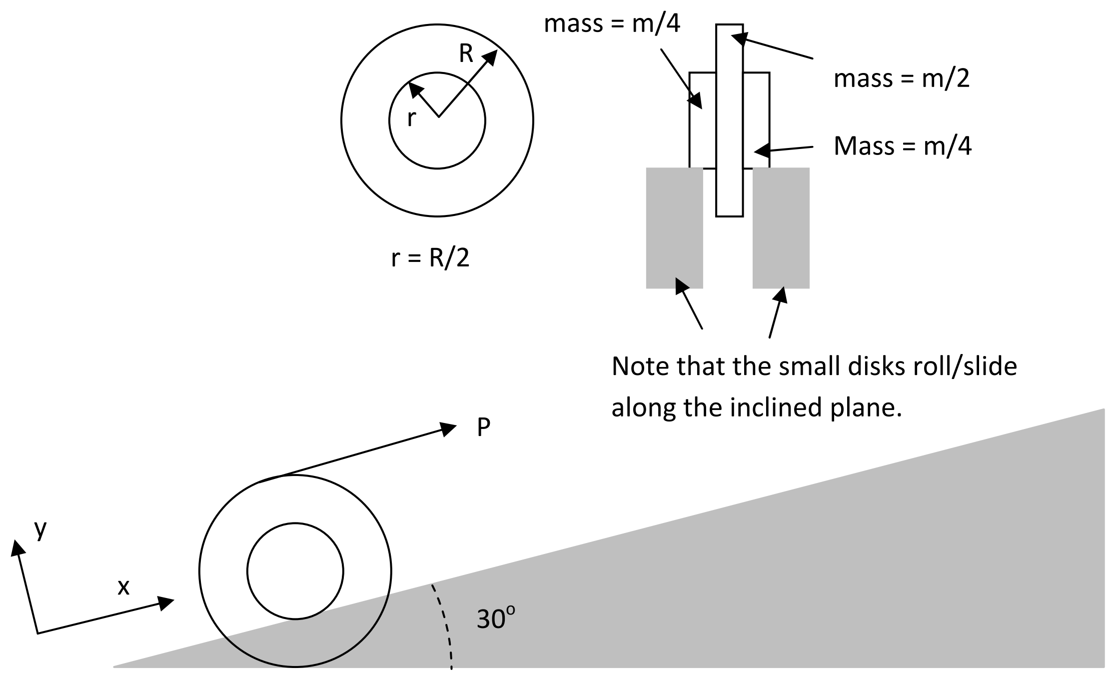
(ii) Suppose that the sun gear is the driver and the ring gear is the driven gear, while the shaft connected to the planet gears is prevented from rotating. If the sun gear rotates at angular speed of ω_1 clockwise, what is the angular speed of the ring gear and what is its direction (CW or CCW)?

(iii) Suppose that the sun gear is prevented from rotating and the planet gear shaft rotates at an angular speed of ω_2. What is the angular speed of the ring gear and what is its direction (CW or CCW)?



2. The Rolling Spool: Consider a spool with a string wound on it as shown in the figure. The spool consists of three concentric cylindrical disks, fastened to each other to make a rigid spool. The central disk has radius of R and mass of m/2. Each of the smaller disks on the side has a mass of m/4 and a radius of r = R/2. The string is wound on the central disk. The static coefficient of friction $\mu_s = 0.2$ and the kinematic coefficient of friction $\mu_k = 0.15$.

The spool is pulled up along an inclined plane by applying a constant force P at the end of the string, as sown in the figure. The direction of P is parallel to the inclined plane. Note that the two smaller disks are in contact with the inclined planes. The angle of inclination is 30°.



(i) What is the total moment of inertia of the spool about its center of mass axis?

(ii) Draw a free body diagram for the spool.

(iii) Write the equations of motion along the x and y directions (x-direction is parallel to the inclined plane) and the rotational equation of motion.

(iv) For P = mg/2, show that the spool is slipping on the inclined plane. Use P = mg/2 for all parts of the question below.

(v) What is the acceleration of the center of mass of the spool?

(vi) The spool starts from rest; let R = 0.1 m and m = 1 kg (use these values for all parts of the question below); when it moves plane a distance of 1 m along the inclined, what is its kinetic energy? What is the change in its gravitational potential energy of the spool?

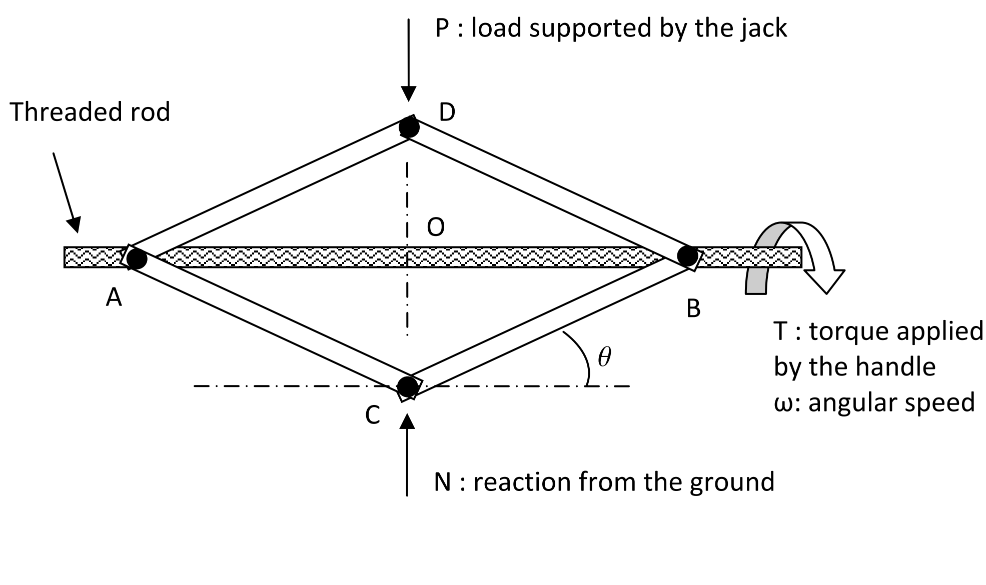
(vii) What is the work done by P in moving the spool along the inclined plane by 1 m?

(viii) Is the work done by P equal to the change in the total mechanical energy (kinetic energy+potential energy)? If yes, why? If no, why not?

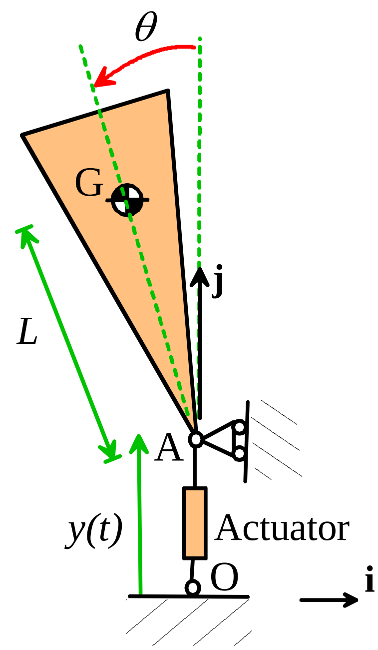
3. Car Jack: Consider a car jack, that's lifting a corner of a car and assume that a fourth of the car's weight is supported by the jack. The car has a total mass of 1500 kg. The car jack structure is symmetric and consists of 4 bars of equal length (L) connected to each other at the pin joints A, B, C, D. The Point C rests on the ground and point D moves up or down, lifting or lowering the load. As you turn the handle, it turns the threaded rod, which forces the two pins A and B approach each other. For every full turn of the handle, each pin A and B moves by a distance p towards the center of the threaded rod, where p is the pitch of the threaded rod (pitch is the distance between two adjacent threads).

(i) As the handle is turned at a constant angular speed ω , derive an expression for the speed with which the load is lifted, in terms of L, θ, ω, p?

(ii) Let L = 0.2 m, p = 1 mm and ω = 10 RPM (revolutions per minute). Plot the required torque T as a function of θ, between $\theta = 5^{\circ}$ and 85°. What torque T should be applied when $\theta = 15^{\circ}$ and when $\theta = 45^{\circ}$? Assume that the frictional losses at the pins and the threaded rod connections are negligible and can be ignored.



4. The figure shows an inverted pendulum supported by a frictionless pivot at A. The pendulum is a rigid body with mass *m*, and moment of inertia $I_G = \frac{1}{10} mL^2$. Its center of mass is a distance *L* from the pivot. An actuator causes the pivot to move vertically with a displacement $y(t)$. The goal of this problem is to derive a differential equation of motion relating the angle θ to $y(t)$.



(i) Write down the position vector $\mathbf{r}$ of the center of mass in {**i**,**j**} components in terms of L, θ, and y.

(ii) Hence, calculate the acceleration vector of the center of mass in terms of θ, y, and their time derivatives.

(iii) Draw a free body diagram for the rigid body pendulum on the figure shown below.

(iv) Write down Newton's law of motion and the equation of rotational motion for the pendulum.

(v) Combine these equations appropriately to obtain a *single* differential equation of motion for θ, in terms of *m, L, g,* and $y(t)$ and its time derivatives.

(vi) Rearrange the equation into a form that could be solved by MATLAB.

5. Stopping a car: Consider a passenger car of mass 1000 kg moving on a horizontal road at a speed of 90 km/hr. The car has 4 wheels and the mass of each wheel is 20 kg (note that the total mass of the car, 1000 kg, includes the mass of the wheels). The wheel radius is 0.25m and its radius of gyration is 0.15m. At time t = 0, brakes are applied on all four wheels by pushing brake pads against the inner rim as shown in the figure below (assume that each wheel has two brake pads as shown). The coefficient of kinetic friction between the brake pad and the inner rim is 0.5.

The coefficient of static friction between the outer surface of the wheel and the road is 0.4 and the coefficient of kinetic friction is 0.3.

(i) The driver applies the brakes such that the normal force between the brake pad and the inner rim of the wheel is 1000 N. What is the deceleration of the car? How long does it take for the car to stop and how far does the car travel before it stops? What is the initial kinetic energy of the car? What is the energy dissipated due to friction between the brake pad and the rim? What is the energy dissipated due to friction between the wheel and the road? What fraction of the initial kinetic energy is dissipated at the brake pad?

(ii) The driver slams on the brake such that the normal force between the brake pad and the inner rim of the wheel is 3000 N. What is the deceleration of the car? How long does it take for the car to stop and how far does the car travel before it stops? What is the energy dissipated due to friction between the brake pad and the rim? What is the energy dissipated due to friction between the wheel and the road? What fraction of the initial kinetic energy is dissipated at the brake pad?