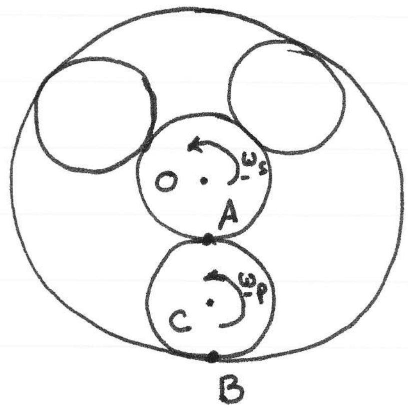
EN 40, Homework #8
Solutions

(1) (i) Consider the gears in the position shown



We know: $\underline{V}_o = 0$, $\underline{V}_B = 0$

$\underline{\omega}_s = -\omega_1 \underline{k}$

$$\underline{V}_A = \underline{V}_o + \underline{\omega}_s \times \underline{r}_{AO}$$

$$= 0 + (-\omega_1)\underline{k} \times (-r_s \underline{j})$$

$$= -\omega_1 r_s \underline{i}$$

Also, $\underline{V}_A = \underline{V}_B + \underline{\omega}_p \times \underline{r}_{AB} = 0 + \omega_p \underline{k} \times (2 r_p \underline{j}) = -2\omega_p r_p \underline{i}$

$$\Rightarrow -\omega_1 r_s = -2\omega_p r_p \Rightarrow \omega_p = \frac{\omega_1 r_s}{2 r_p}$$

$$\underline{V}_c = \underline{V}_B + \underline{\omega}_p \times \underline{r}_{cB} = 0 + \frac{\omega_1 r_s}{2 r_p} \underline{k} \times (r_p \underline{j}) = -\frac{\omega_1 r_s}{2} \underline{i}$$

Now, consider o as the center of the driven shaft and let it's angular velocity be $\underline{\omega}'$. Then

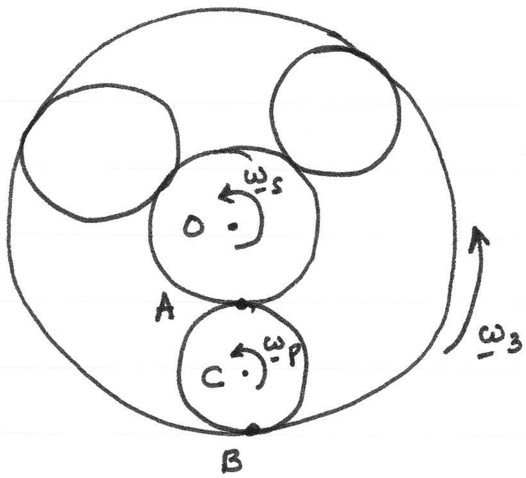
$$\underline{V}_c = \underline{V}_o + \underline{\omega}' \times \underline{r}_{co} = 0 + \omega' \underline{k} \times (r_s + r_p)(-\underline{j})$$

$$= \omega'(r_s + r_p)\underline{i}$$

$$\Rightarrow -\frac{\omega_1 r_s}{2} = \omega'(r_s + r_p) \Rightarrow \omega' = -\frac{\omega_1 r_s}{2(r_s + r_p)}$$

$$\Rightarrow \boxed{\underline{\omega}' = -\frac{\omega_1 r_s}{2(r_s + r_p)} \underline{k}} \longrightarrow \text{Clockwise}$$

(ii)



Given : $\underline{\omega}_S = -\omega_1 \underline{k}$

$\underline{V}_C = 0$

$\underline{V}_A = \underline{V}_O + \underline{\omega}_S \times \underline{r}_{AO}$

$= 0 + (-\omega_1)\underline{k} \times (-r_s \underline{j})$

$= -\omega_1 r_s \underline{i}$

$\underline{V}_A = \underline{V}_C + \underline{\omega}_P \times \underline{r}_{AC}$

$= 0 + \omega_p \underline{k} \times r_p \underline{j} = -\omega_p r_p \underline{i}$

$\Rightarrow -\omega_1 r_s = -\omega_p r_p \Rightarrow \omega_p = \dfrac{\omega_1 r_s}{r_p} \Rightarrow \underline{\omega}_p = \dfrac{\omega_1 r_s}{r_p}\underline{k}$

$\underline{V}_B = \underline{V}_C + \underline{\omega}_P \times \underline{r}_{BC} = 0 + \dfrac{\omega_1 r_s}{r_p}\underline{k} \times (-r_p \underline{j}) = \omega_1 r_s \underline{i}$

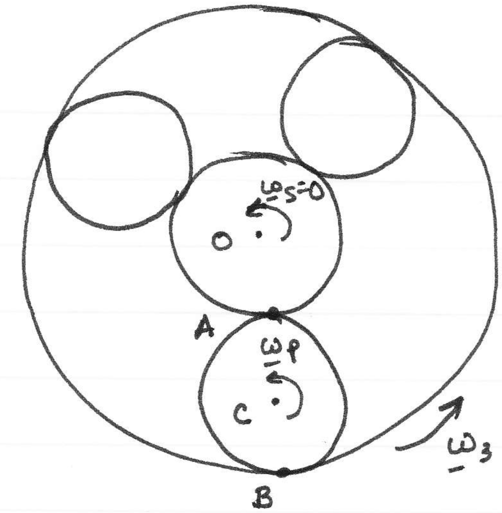
Now, consider O as the center of the ring gear.

$\underline{V}_B = \underline{V}_O + \underline{\omega}_3 \times \underline{r}_{BO} = 0 + \omega_3 \underline{k} \times (2r_p + r_s)(-\underline{j}) = \omega_3 (2r_p + r_s)\underline{i}$

$\Rightarrow \omega_1 r_s = \omega_3 (2r_p + r_s) \Rightarrow \omega_3 = \omega_1 \left(\dfrac{r_s}{2r_p + r_s}\right)$

$\Rightarrow \boxed{\underline{\omega}_3 = \left(\dfrac{r_s}{2r_p + r_s}\right)\omega_1 \underline{k} \leftarrow \text{CCW}}$

(iii)



$\underline{\omega}_s = 0$

planet gear shaft $\rightarrow$ $\omega_2 \underline{k}$

[Note: The problem does not define direction of $\underline{\omega}_2$. Consider it to be CCW]

$$\underline{v}_C = \underline{v}_O + \underline{\omega}_2 \times \underline{r}_{CO}$$

$$= 0 + \omega_2 \underline{k} \times (r_s + r_p)(-\underline{j})$$

$$= (r_s + r_p)\,\omega_2\, \underline{i}$$

$$\underline{v}_A = 0$$

$$\underline{v}_C = \underline{v}_A + \underline{\omega}_p \times \underline{r}_{CA}$$

$$= 0 + \omega_p \underline{k} \times (-r_p \underline{j}) = \omega_p r_p \underline{i}$$

$$\Rightarrow (r_s + r_p)\,\omega_2 = r_p\,\omega_p \Rightarrow \underline{\omega}_p = \left(\frac{r_s + r_p}{r_p}\right)\omega_2 \underline{k}$$

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_p \times (2 r_p)(-\underline{j}) = 0 + \frac{r_s + r_p}{r_p}\,\omega_2 \underline{k} \times 2 r_p (-\underline{j})$$

$$= 2\omega_2 \left(\frac{r_s + r_p}{\cancel{r_p}}\right) \underline{i}$$

$$\underline{v}_B = \underline{v}_O + \underline{\omega}_3 \times \underline{r}_{BO} = 0 + \omega_3 \underline{k} \times (2 r_p + r_s)(-\underline{j})$$

$$= \omega_3 (2 r_p + r_s)\, \underline{i}$$

$$\Rightarrow \omega_3 = 2\omega_2 \left(\frac{r_s + r_p}{\cancel{r_p}}\right) \frac{1}{2 r_p + r_s} = 2\left(\frac{r_s + r_p}{2 r_p + r_s}\right)\omega_2$$

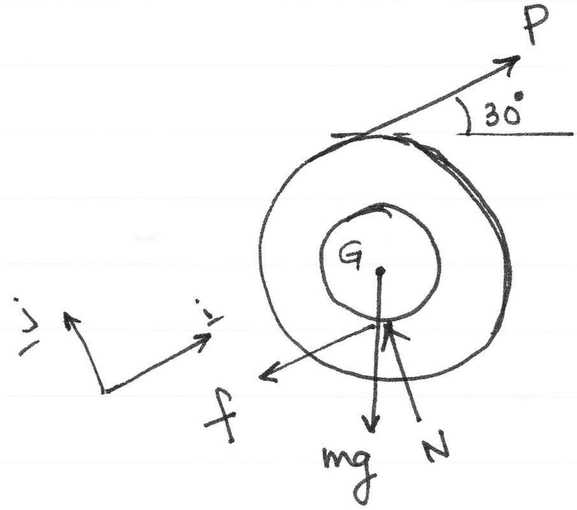
$$\Rightarrow \boxed{\underline{\omega}_3 = 2\left(\frac{r_s + r_p}{2 r_p + r_s}\right)\omega_2 \underline{k} \leftarrow \text{CCW}}$$

(2) (i) Moment of inertia of the spool I_G

$$I_G = \frac{1}{2}\left[\frac{m}{2}R^2 + 2\cdot\frac{m}{4}\left(\frac{R}{2}\right)^2\right] = \frac{5}{16} m R^2$$

Total mass $= \frac{m}{2} + 2\left(\frac{m}{4}\right) = m$

(ii)



(iii) $$P - f - mg\sin 30^\circ = m a_x \quad \text{—— (1)}$$

$$N - mg\cos 30^\circ = m a_y = 0 \quad \text{—— (2)}$$

$$-fr - PR = I_G \ddot{\theta} \quad \text{—— (3)}$$

(iv) Unknowns: $f, a_x, N, \ddot{\theta}$, So need another equation.

Assume rolling without slip

$$a_x = -R\ddot{\theta} \quad \text{—— (4)}$$

From (2), $N = mg\cos 30^\circ = \frac{\sqrt{3}}{2} mg$ —— (5)

From (1) & (4)

$$P - f - \frac{mg}{2} = -mR\ddot{\theta} \quad \text{—— (6)}$$

$$\Rightarrow \ddot{\theta} = \frac{f + mg/2 - P}{mR}$$

Substitute in (3) along with $r = \frac{R}{2}$ & $I_G = \frac{5}{16} mR^2$ & $P = \frac{mg}{2}$

$$-\frac{fR}{2} - PR = \frac{5}{16} mR^2 \, \frac{\left(f + \frac{mg}{2} - P\right)}{mR}$$

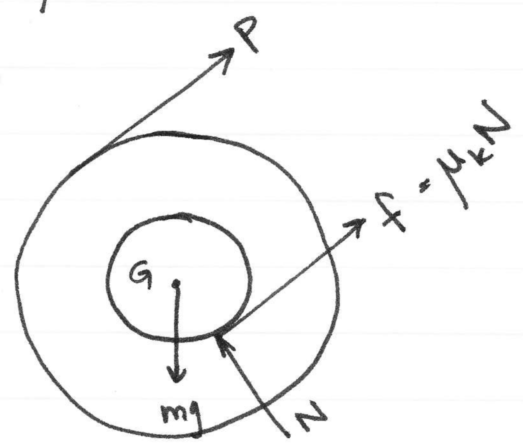
$$\Rightarrow f = -\frac{mg}{2}$$ (Note: friction is actually acting up the inclined plane)

$$\Rightarrow \left|\frac{f}{N}\right| = \frac{mg}{2} \cdot \frac{2}{\sqrt{3}\, mg} = 0.577 > \mu_s \cancel{0.2} \quad (\mu_s = 0.2)$$

$\Rightarrow$ The disk slips along the inclined plane.

(v) Need to re-do the problem by replacing eq. (4) with

$$f = \mu_k N$$



$$P + \mu_k N - mg \sin 30^\circ = m a_x$$

$$N - mg \cos 30^\circ = m a_y = 0 \Rightarrow N = \frac{\sqrt{3}}{2} mg$$

$$fr - PR = I_G \ddot{\theta}$$

$$f = \mu_k N$$

$$\rightarrow \frac{mg}{2} + \mu_k \frac{\sqrt{3}}{2} mg - \frac{mg}{2} = m a_x$$

$$\Rightarrow a_x = \frac{\sqrt{3}}{2} \mu_k g = 0.13 g = 1.274 \ m/s^2$$ (up the inclined surface)

$$\Rightarrow \underline{a}_G = 1.274 \, \underline{i} \ m/s^2$$

$$f r - P R = I_G \ddot{\theta}$$

$$\Rightarrow \mu_k \frac{\sqrt{3}}{2} mg \frac{R}{2} - \frac{mg}{2} R = \frac{5}{16} mR^2 \ddot{\theta}$$

Note : $R = 0.1\,m$

$$\Rightarrow \left[\frac{\sqrt{3}}{2}\mu_k - 1\right]\frac{g}{2} = \frac{5}{16} R\ddot{\theta} \Rightarrow \ddot{\theta} = -136.57\ rad/s^2 \text{ (clockwise)}$$

(vi) $V^2 - V_0^2 = 2a_x d \Rightarrow V^2 - 0 = 2 \times 1.274 \times 1$

$\Rightarrow V = 1.596\ m/s$

Translational kinetic energy $T_1 = \frac{1}{2}mV^2 = \frac{1}{2} \times 1 \times 1.596^2$

$= 1.274\ J$.

Time to move a distance of 1m, $t = \frac{V}{a_x} = \frac{1.596}{1.274} = 1.253\,s$

Angular speed at the end of time t $= \omega = \ddot{\theta} t$

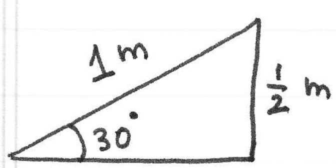
$= -136.57 \times 1.253 = -171.1\ rad/s$

$I_G = \frac{5}{16} mR^2 = \frac{5}{16} \times 1 \times 0.1^2 = 3.125 \times 10^{-3}\ kg\,m^2$.

Rotational kinetic energy $T_2 = \frac{1}{2} I_G \omega^2$

$= \frac{1}{2} \times 3.125 \times 10^{-3} \times 171.1^2 = 45.74\ J$

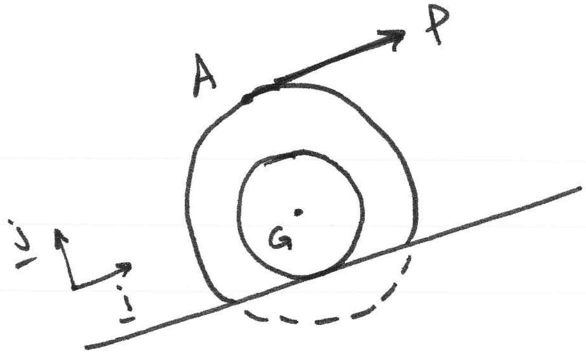
$\Rightarrow$ Total kinetic energy $T = T_1 + T_2 = 47.017\ J$



Increase in gravitational potential energy

$V = mg\Delta h = 1 \times 9.81 \times \frac{1}{2} = 4.905\ J$

(vii)



$$\underline{V_A} = \underline{V_G} + \underline{\omega} \times \underline{r_{AG}}$$

$$\underline{V_G} = a_x t \, \underline{i}$$

$$\left.\begin{aligned} \underline{\omega}(t) &= \ddot{\theta} t \, \underline{k} \\ \underline{r_{AG}} &= R \, \underline{j} \end{aligned}\right\} \begin{aligned} &\underline{\omega} \times \underline{r_{AG}} \\ &= -\ddot{\theta} R t \, \underline{i} \end{aligned}$$

$$\Rightarrow \underline{V_A} = a_x t \, \underline{i} - \ddot{\theta} R t \, \underline{i}$$

$$V_A = |\underline{V_A}| = (a_x - \ddot{\theta} R) t$$

Power exerted by force P, $\mathcal{P}_P = P V_A = \frac{mg}{2}(a_x - \ddot{\theta}R)t$

Work done by P $= W_P = \int_0^t \mathcal{P}_P \, dt = \int_0^{1.253 s} \frac{mg}{2}(a_x - \ddot{\theta}R)t \, dt$

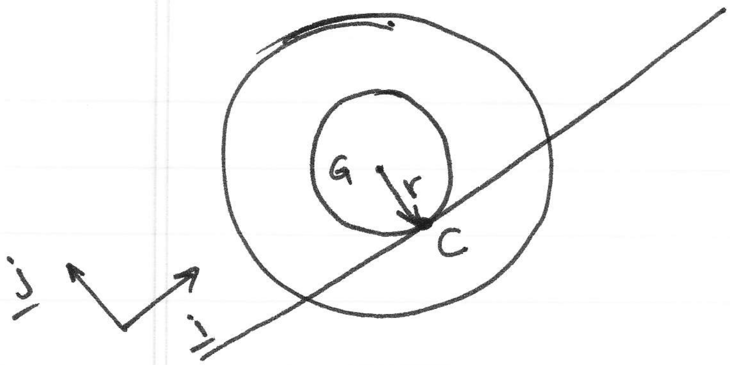
$$= \frac{mg}{2}(a_x - \ddot{\theta}R)\frac{t^2}{2}\Big|_0^{1.253} = 57.487 \text{ J}$$

(viii) Total change in mechanical energy $= T + V = 51.922$ J

$$\Rightarrow W_P > T + V$$

Because some of the work done is dissipated as heat at the disk-surface contact due to friction.

Since the problem did not ask for it, you need not do the following. But, we will proceed to calculate the energy dissipation due to friction.



C is the contact point.

$$\underline{v}_C = \underline{v}_G + \underline{\omega} \times \underline{r}_{CG}$$

$$= (a_x t)\underline{i} + (\ddot{\theta} t)\underline{k} \times r(-\underline{j})$$

$$= (a_x t + \ddot{\theta} r t)\underline{i}$$

$$= \left(a_x + \frac{\ddot{\theta} R}{2}\right) t \, \underline{i}$$

Frictional dissipative power $\mathcal{P}_f = |f|\,|\underline{v}_C|$

$$= \left(\mu_k \frac{\sqrt{3}}{2} mg\right) \left| a_x + \frac{\ddot{\theta} R}{2} \right| t$$

Frictional dissipation

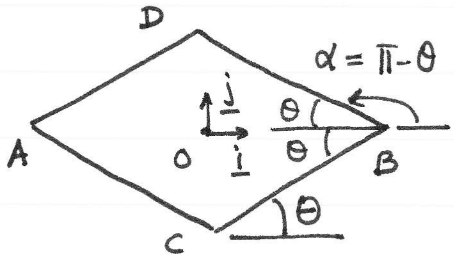
$$W_f = \int_0^t \mathcal{P}_f \, dt = \int_0^{1.253\,s} \mu_k \frac{\sqrt{3}}{2} mg \left| a_x + \frac{\ddot{\theta} R}{2} \right| t \, dt$$

$$= 5.56\,J .$$

$\Rightarrow \quad T + V + W_f = 51.922 + 5.56 = 57.482\,J$

$\therefore \quad W_P = T + V + W_f$ ✓ That's relief!

③ P = load supported by jack $= \dfrac{1500}{4} g = 3678.75\,N$



B moves to left by distance p in time T for one rotation of the threaded rod

$$\Rightarrow T = \frac{2\pi}{\omega}$$

$$\text{Speed of } B = \frac{p}{T} = \frac{p\omega}{2\pi}$$

$$\Rightarrow \underline{V}_B = -\frac{p\omega}{2\pi}\underline{i} + V_{By}\underline{j}$$

We know $\underline{V}_c = 0$, $\underline{V}_B = \underline{V}_c + \dot{\theta}\underline{k} \times (L\cos\theta\,\underline{i} + L\sin\theta\,\underline{j})$

$$= 0 + L\dot{\theta}(\cancel{+}\cos\theta\,\underline{j} - \sin\theta\,\underline{i})$$

$$= -L\dot{\theta}\sin\theta\,\underline{i} + L\dot{\theta}\cos\theta\,\underline{j}$$

$$\Rightarrow \frac{p\omega}{2\pi} = L\dot{\theta}\sin\theta \Rightarrow \dot{\theta} = \frac{p\omega}{2\pi L\sin\theta}$$

$$\Rightarrow V_{By} = L\dot{\theta}\cos\theta = \frac{p\omega}{2\pi\tan\theta}$$

$$\underline{V}_D = \underline{V}_B + \underline{\omega}_{DB} \times \underline{r}_{DB} = \underline{V}_B + \dot{\alpha}\underline{k} \times (L\cos\alpha\,\underline{i} + L\sin\alpha\,\underline{j})$$

$$= \underline{V}_B - \dot{\theta}\,\underline{k} \times L(-\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$$

$$= \cancel{L\dot{\theta}\sin\theta}\ \underline{V}_B - L\dot{\theta}(-\cos\theta\,\underline{j} - \sin\theta\,\underline{i})$$

$$= -L\dot{\theta}\sin\theta\,\underline{i} + L\dot{\theta}\cos\theta\,\underline{j} + L\dot{\theta}\cos\theta\,\underline{j} + L\dot{\theta}\sin\theta\,\underline{i}$$

$$= 2L\dot{\theta}\cos\theta\,\underline{j} = \cancel{2}\cancel{L}\cos\theta \cdot \frac{p\omega}{\cancel{2}\pi\cancel{L}\sin\theta}\,\underline{j} = \frac{p\omega}{\pi\tan\theta}\,\underline{j}$$

$$\Rightarrow \boxed{V_D = |\underline{V}_D| = \frac{p\omega}{\pi\tan\theta}}$$

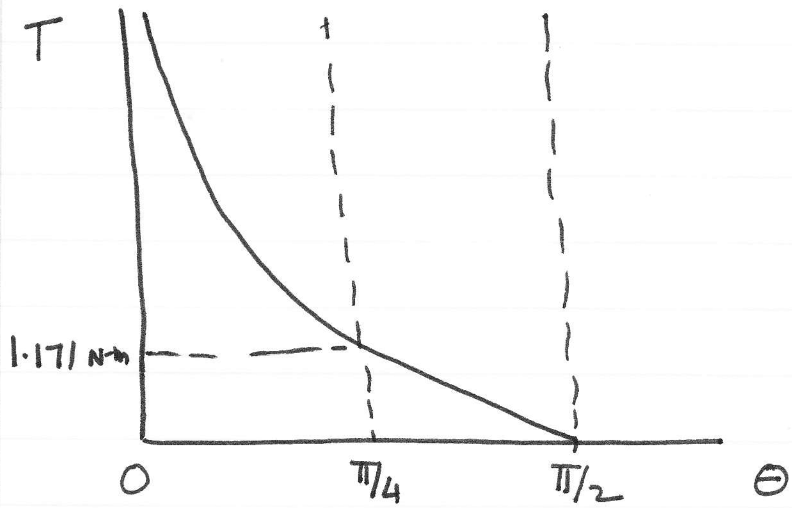
(ii) Conservation of energy $\Rightarrow T\omega = P V_D$

$$\Rightarrow T = \frac{PV_D}{\omega} = \frac{P}{\omega} \cdot \frac{p\omega}{\pi \tan\theta} = \frac{Pp}{\pi \tan\theta}$$

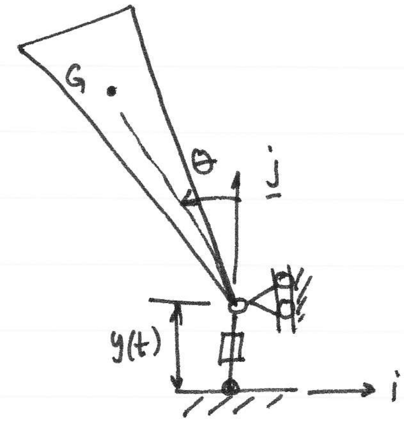
$$= \frac{3678.75 \text{ N} \times 10^{-3} \text{ m}}{\pi \tan\theta} = \frac{1.171}{\tan\theta} \text{ N-m}$$

@ $\theta = 15^\circ$, $T = 4.37$ N-m

@ $\theta = 45^\circ$, $T = 1.171$ N-m



(4)

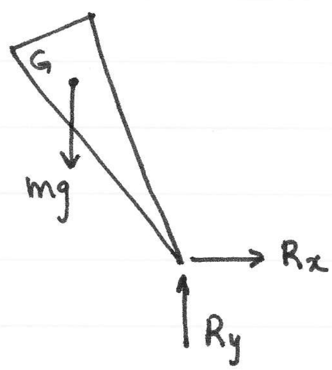


(i) $\underline{r} = -L\sin\theta\,\underline{i} + \left(y(t) + L\cos\theta\right)\underline{j}$

(ii) $\underline{v} = \dfrac{d\underline{r}}{dt} = -L\dot{\theta}\cos\theta\,\underline{i} + \left[\dot{y}(t) - L\dot{\theta}\sin\theta\right]\underline{j}$

$$\underline{a} = \frac{d^2\underline{r}}{dt^2} = \left(+L\dot{\theta}^2\sin\theta - L\ddot{\theta}\cos\theta\right)\underline{i} + \left[\ddot{y}(t) - L\dot{\theta}^2\cos\theta - L\ddot{\theta}\sin\theta\right]\underline{j}$$

(iii)



(iv)
$$R_x = m a_x$$
$$R_y - mg = m a_y$$
$$R_y L\sin\theta + R_x L\cos\theta = I_G\ddot{\theta}$$

(v) $(mg + m a_y)L\sin\theta + m a_x L\cos\theta = \dfrac{1}{10} m L^2\ddot{\theta}$

$$\cancel{m}\left[g + \ddot{y} - L\dot{\theta}^2\cos\theta - L\ddot{\theta}\sin\theta\right]\sin\theta + \cancel{m}\left[L\dot{\theta}^2\sin\theta - L\ddot{\theta}\cos\theta\right]\cos\theta = \frac{1}{10}\cancel{m}L\ddot{\theta}$$

$$(g+\ddot{y})\sin\theta - L\dot{\theta}^2 \cancel{\cos\theta}\sin\theta - L\ddot{\theta}\sin^2\theta + L\dot{\theta}^2 \cancel{\sin\theta}\cos\theta$$

$$- L\ddot{\theta}\cos^2\theta = \frac{L\ddot{\theta}}{10}$$

$$\Rightarrow \quad \frac{g+\ddot{y}}{L}\sin\theta - L\ddot{\theta} = \frac{L\ddot{\theta}}{10}$$

$$\Rightarrow \quad \frac{11}{10}\ddot{\theta} - \frac{g+\ddot{y}}{L}\sin\theta = 0 \quad \text{or} \quad \ddot{\theta} - \left(\frac{10}{11}\frac{g+\ddot{y}}{L}\right)\sin\theta = 0$$

(vi)

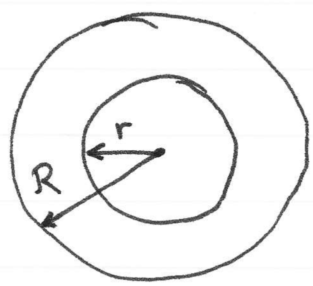
$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}\omega \\ \frac{10}{11}\frac{g+\ddot{y}}{L}\sin\theta\end{bmatrix} \qquad \text{where} \quad \omega = \frac{d\theta}{dt}.$$

5. Stopping a car

Total mass of the car = 1000 kg.

Mass of the car without wheels = $M = 920\,kg$

Radius of gyration of wheel = $R_g = 0.15\,m$



$R = 0.25\,m$

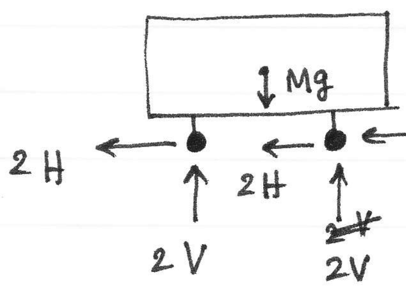
$r = 0.1\,m$

Brake pad friction coefficient $\mu_b = 0.5$

Between wheel & road : $\mu_s = 0.4$

$\mu_k = 0.3$

(i) FBD for the car without the wheels. Assume that the car weight is distributed equally on all 4 wheels.

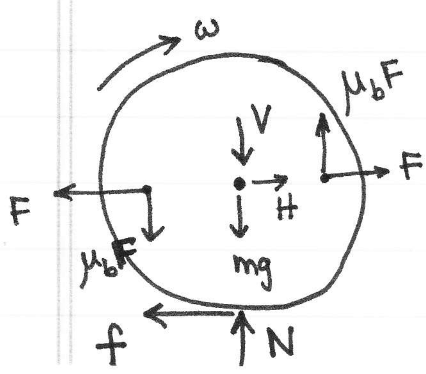


axles(4), only two axles are shown in side view

$$4V - Mg = Ma_y = 0 \Rightarrow V = \frac{Mg}{4}$$

$$-4H = Ma_x \Rightarrow a_x = -\frac{4H}{M}$$

FBD for a wheel (mass of wheel $m = 20\,kg$)



Initial velocity of the car $\underline{V}_o = 90 \frac{km}{hr} \underline{i} = 25\, m/s\, \underline{i}$

$V_o = -\omega_o R \Rightarrow \omega_o = -\frac{25}{0.25} = -100\, rad/s$.

$\Rightarrow \underline{\omega_o} = -100\, \underline{k}\, rad/s$.

EOM for the wheel

$H - f = m a_x$

$N - mg - V = m a_y = 0 \Rightarrow N = mg + V = mg + \frac{Mg}{4} = \left(m + \frac{M}{4}\right) g$

$2\mu_b F r - f R = I_G \ddot{\theta}$

Assume rolling without slip $\Rightarrow a_x = -R\ddot{\theta}$

$H - f = -mR\ddot{\theta} \Rightarrow \cancel{\ddot{\theta} = \frac{f-H}{mR}}$

$H = -\frac{M a_x}{4}$

$= -\frac{M}{4}(-R\ddot{\theta}) = \frac{MR\ddot{\theta}}{4}$

$\cancel{2\mu_b F r - f R = \frac{I_G}{mR}(f - H)}$

$\Rightarrow \frac{MR\ddot{\theta}}{4} - f = -mR\ddot{\theta} \Rightarrow f = \left(\frac{M}{4} + m\right) R\ddot{\theta}$

$\Rightarrow 2\mu_b F r - \left(\frac{M}{4} + m\right) R^2 \ddot{\theta} = I_G \ddot{\theta}$

$$\Rightarrow \ddot{\theta} = \frac{2\mu_b F r}{\left(\frac{M}{4} + m\right) R^2 + I_G}$$

$$\Rightarrow f = \frac{\left(\frac{M}{4} + m\right) 2\mu_b F r R}{\left(\frac{M}{4} + m\right) R^2 + I_G}$$

where $I_G = m R_g^2$

$$\frac{f}{N} = \frac{2\mu_b F r R}{g\left[I_G + \left(\frac{M}{4} + m\right) R^2\right]}$$

$I_G = 20 \times 0.15^2 = 0.45\ kg\,m^2$ | $I_G + \left(\frac{M}{4}+m\right)R^2 = 16.075\ kg\,m^2$

$\frac{M}{4} + m = \frac{920}{4} + 20 = 250\ kg$

$$\frac{f}{N} = \frac{2 \times 0.5 \times 1000 \times 0.1 \times 0.25}{9.81\left[0.45 + 250 \times 0.25^2\right]} \approx 0.159$$

$\frac{f}{N} < \mu_s \Rightarrow$ Rolling without slip OK.

$$\ddot{\theta} = \frac{2 \times 0.5 \times 1000 \times 0.1}{16.075} = 6.22\ rad/s^2$$

$$a_x = -R\ddot{\theta} = -1.555\ m/s^2$$

$\Rightarrow \boxed{\text{Deceleration} = 1.555\ m/s^2}$

Time to stop $T_{stop} = \frac{V_0}{-a_x} = \frac{25}{1.555} = \boxed{16.075\,s}$

Distance traveled before stopping $d_{stop} = \frac{0^2 - 25^2}{2 \times -1.555} = \boxed{200.96m}$

Initial kinetic energy of the car

$$T = \frac{1}{2} \times 1000 \times 25^2 + 4\left(\frac{1}{2}\right) I_G \omega_0^2$$

$$= \frac{1}{2} \times 1000 \times 25^2 + 4\left(\frac{1}{2}\right) 0.45 \times 100^2 = \boxed{3.215 \times 10^5\ J}$$

Since there is no slip, all the ~~en~~ kinetic energy is dissipated at the brake pad, i.e. $\boxed{3.215 \times 10^5\ J}$.

No slip $\Rightarrow$ Energy dissipated at the wheel-road contact $= \boxed{0}$.

Fraction of energy dissipated at the brake pad $= \boxed{100\%}$

(ii) Assuming Rolling without slip

$$\frac{f}{N} = \frac{2\mu_b F r R}{g\left[I_G + \left(\frac{M}{4} + m\right) R^2\right]}$$

$$= \frac{2 \times 0.5 \times 3000 \times 0.1 \times 0.25}{9.81 \times 16.075} = 0.476$$

$\frac{f}{N} > \mu_s \Rightarrow$ The wheel slips. Need to re-do the solution with slip condition.

$$\left.\begin{aligned} H - f &= m a_x \\ N &= \left(m + \frac{M}{4}\right) g \\ 2\mu_b F r - f R &= I_G \ddot{\theta} \\ f &= \mu_k N \end{aligned}\right\} \Rightarrow -\frac{M}{4} a_x - \mu_k \left(\frac{M}{4} + m\right) g = m a_x$$

(*) — $2\mu_b F r - f R = I_G \ddot{\theta}$

$$\Rightarrow a_x = -\mu_k g = \boxed{-2.943\ m/s^2}$$

$$f = \mu_k \left(\frac{M}{4} + m\right) g = 735.75\,N$$

$$\ddot{\theta} = \frac{2\mu_b F r - \mu_k \left(\frac{M}{4} + m\right) g R}{I_G}$$

$$= \frac{2 \times 0.5 \times 3000 \times 0.1 - 0.3 \times 250 \times 9.81 \times 0.25}{0.45}$$

$$= 257.9\ rad/s^2.$$

Time to stop $T_{stop} = \frac{V_0}{-a_x} = \frac{25}{2.943} = \boxed{8.495\ s}$

Distance traveled before stopping $d_{stop} = \frac{0^2 - 25^2}{2 \times (-2.943)} = \boxed{106.18\ m}$

Time for wheel to lock-up $= \left|\frac{\omega_0}{\ddot{\theta}}\right| = \frac{100}{257.9} = 0.388\ s$

So, after 0.388 s, the wheel is no longer turning, it's locked.

Beyond this point, the car is just skidding; it also stops responding to the steering wheel and loses maneuverability. That's why ~~t~~ modern cars come with ABS, i.e. anti-lock braking systems, which sense when the wheels are about to lock up and automatically lower F (~~the~~ normal force between brake pad and wheel ~~at~~ inner rim). If you have driven a car with ABS system and had to slam on the brake, you would have experienced the brake pedal pulsating, which is the system's attempt to lower F so that the ~~car~~ wheels don't lock up and the vehicle stays under your control.

Now, let's calculate the energy dissipation at brake pad.

$$\omega(t) = \omega_0 + \ddot{\theta} t$$

Frictional dissipative power $P_b = \left| 2 \times \mu_b F r \times \omega(t) \right|$

Total energy dissipation at brake pad $= \int_0^t P_b \, dt$

$$= \int_0^{0.388\,s} \left| 2 \mu_b F r (\omega_0 + \ddot{\theta} t) \right| dt$$

$$= \left| 2 \mu_b F r \left[\omega_0 \times 0.388 + \ddot{\theta} \times \frac{0.388^2}{2} \right] \right|$$

$$= \left| 2 \times 0.5 \times 3000 \times 0.1 \left[-100 \times 0.388 + 257.9 \times \frac{0.388^2}{2} \right] \right|$$

$$= \boxed{5816.2 \text{ J}}$$

For 4 wheels, energy dissipation $= 4 \times 5816.2 = 23264.8$ J

The rest of the kinetic energy dissipated at wheel-road contact $= 3.215 \times 10^5 - 4 \times 5816.2 = \cancel{315683}\ \cancel{3.157 \times 10^5}\ J.$

$2.982 \times 10^5\ J \longrightarrow (**)$

Fraction of initial K.E. dissipated at brake pad

$$= \frac{5816.2 \times 4}{3.215 \times 10^5} = \cancel{0.0181}\ 0.0724 \quad \text{or} \quad \boxed{\cancel{1.81\%}} \ \boxed{7.24\%}$$

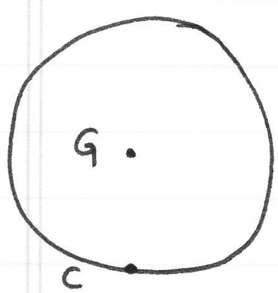
You need not turn in/grade the following.

Let's double check the above calculation by independently calculating the energy dissipation at wheel-road contact.

f is constant whether the wheel is locked up or not

$$f = 735.75\ N.$$

Before the wheel locks up



$$\underline{V}_C = \underline{V}_G + \omega \underline{k} \times R(-\underline{j})$$

$$= \underline{V}_G + \omega R\, \underline{i}$$

$$\Rightarrow V_c = (V_o + a_x t) + (\omega_o + \ddot{\theta} t)$$

After the wheel locks up, $V_c = V_o + a_x t$

$$\Rightarrow W_f = 4\left\{ f \int_0^{0.388\,s} \left[(V_o + a_x t) + (\omega_o + \ddot{\theta} t) \right] dt + f \int_{0.388\,s}^{8.495\,s} \left[V_o + a_x t \right] dt \right\}$$

$= 2.98235 \times 10^5\ J$ ✓ Matches with $(**)$ above!