



School of Engineering  
Brown University

## EN40: Dynamics and Vibrations

### Homework 1: Introduction to MATLAB Due 12:00 noon Friday February 3rd

- YOUR SOLUTION TO THIS HOMEWORK SHOULD CONSIST OF A COMMENTED MATLAB .m FILE
- THE ASSIGNMENT SHOULD BE SUBMITTED ELECTONICALLY BY EMAILING THE FILE AS AN ATTACHMENT TO [Stephanie\\_gesualdi@brown.edu](mailto:Stephanie_gesualdi@brown.edu)
- PLEASE PUT en4\_hw1 IN THE SUBJECT OF YOUR EMAIL
- PLEASE NAME THE ATTACHED FILE *lastname\_firstname.m*

You should make your file a function, so that when the file is executed, it will solve all the homework problems. For example:

```
function my_amazing_homework
    Solutions to problems 1-6
    Functions for the differential equations in problems 5&6
end
function number_of_vars = count_variables(vector)
...
end
```

If you get stuck, you might find the solutions to homework 1, 2009-2011 helpful. If you find this homework too easy, you could make it more interesting by looking for more sophisticated solutions to the problems that make use MATLAB's ability to manipulate vectors.

1. Using a loop, create a vector called  $x$  that contains 401 equally spaced points, starting at -10 and ending at +10 (there is an example like this on page 7 of the MATLAB tutorial...)
2. Using the solution to problem 1, plot a graph of  $y = \sin(5x) + \sin(5.5x)$ , for  $-10 < x < 10$ . You can create the vector  $y$  using a loop, or else use the dot notation. See Section 9 of the Matlab tutorial for a discussion of both approaches.
3. By using a loop and a conditional ('if...end') statement to create the arrays  $v, w$ , create a plot of

$$w = \begin{cases} \cos(v) & \cos(v) \geq \sin(v) \\ \sin(v) & \cos(v) \leq \sin(v) \end{cases}$$

as a function of  $v$  for  $0 \leq v \leq 4\pi$ . See Section 13 of the MATLAB tutorial for an example using a conditional statement.

4. Write a MATLAB function that will count the number of entries in a vector whose value exceeds a critical threshold value.

Your function could look something like this:

```
function number_of_vars = count_variables(vector, threshold)
% Function to count the number of entries in vector with
% value exceeding threshold
    enter your calculation here (Hint: use the function length(vector)
to compute the number of elements in v,
```

and use a loop and a conditional statement to count the values of  $v$ ). Advanced programmers might like to explore MATLAB for a more elegant solution...

```
end
```

and should appear at the end of your homework file. Use your function to

- (i) count the number of values in the vector  $x$  defined in problem 1 with value exceeding 0.
- (ii) Count the number of the first 17 digits of  $\pi$  that exceed 7. You can use the following method to create a vector containing the first 17 digits of  $\pi$

```
for i=1:17
    pivec(i) = floor(pi*10^(i-1))-10*floor(pi*10^(i-2));
end
```

If you are up for a challenge, you could try counting more digits – but you need a more sophisticated code to compute the digits of  $\pi$  than the example here, because MATLAB only stores 17 digits.

## 5. The differential equation

$$\frac{dN}{dt} = \frac{rN(K - N)}{K}$$

is called the ‘Logistic equation’ or the ‘Verhulst’ model of population growth in mathematical biology.  $N$  is the population of a species;  $r$  is the reproduction rate and  $K$  is the maximum population.

Following the method in Section 15.1 of the Matlab tutorial, write a function in your M-file that will calculate  $dN / dt$  given values of  $t, N$ .

Then use the MATLAB ode solver to compute, and plot the variation of  $N$  with time (as usual, the MATLAB solver won’t give you a function for  $N$ , but instead will return two vectors, one containing values of time, and the other containing values of  $N$ . You can plot these.) Use the following parameters (modeling human population growth in the USA, with a wild guess for the maximum population):  $r = 0.225$  per annum  $K = 1000000$  thousand people (i.e. 1 billion). Assume  $N = 310384$  (thousands of people) at  $t = 0$ . Plot the solution with  $N$  in thousands of people and time in years, for a period of 30 years. (I will probably not be around by then to know if my guess for  $K$  was correct or not, but hopefully you will!)

6. The famous ‘[Belousov-Zhabotinskii](#)’ reaction is an example of a chemical reaction that results in cyclic variations in concentration of the reagents (you can find [movies](#) showing the reaction). Reactions of this type are of interest in biology, and a number of theoretical models have been developed to describe them. One example is the ‘cubic autocatalator’ reaction, which is described by differential equations

$$\frac{dx}{dt} = \mu - xy^2 - rx \quad \frac{dy}{dt} = xy^2 - y + rx$$

Here,  $x$  and  $y$  represent the concentrations of two reacting species, and  $\mu, r$  are two numbers that characterize the rate at which the reactions take place. Write a MATLAB ‘equation of motion’ function that will compute the value of the vector  $dw/dt = [dx/dt; dy/dt]$  given the value of the vector  $w = [x, y]$  at some time  $t$ .

Then, use the MATLAB ode solver to plot  $x, y$  as functions of time for  $0 < t < 100$ . Try the following cases:

6.1  $r = 0.2, \mu = 0.6$  (this approaches a steady solution)

6.2  $r = 0.1, \mu = 0.6$  (this a periodic solution)

Take  $x = y = 1$  at  $t = 0$  for both cases.